Information, Incentives and Institutions

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The Faculty of Business, Economics and Informatics of the University of Zurich hereby authorizes the printing of this dissertation, without indicating an opinion of the views expressed in the work.

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Nineteen years ago, the *Lingnan Youth Daily* published a short article written by a then 11-year-old boy. In that article, the little boy dreamed that he would become a professor in 2020, winning the Nobel prize by inventing a spaceship which can travel at the speed of light, and a robot which can fly to the sky to fix the ozone depletion. Now, given that 2020 is less than a year away, the part about the Nobel prize and the two awesome inventions seems very unlikely to come true. However, as the author of the article (surprise!), I am glad that I was not totally naive: I will soon start my career as a professor after this dissertation is submitted.

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Contents

Part I

Dissertation Overview

Dissertation Overview

This dissertation consists of five separate essays on asymmetric information, strategic behavior, and institution design. In Chapter 1, Harry Pei and I abstract from the exact institutional details and consider an important class of games with asymmetric information: signalling games. In particular, we study the monotonicity of sender's equilibrium strategy with respect to her type in signalling games. We use counterexamples to show that when the sender's payoff is non-separable, the Spence-Mirrlees condition cannot rule out equilibria in which the sender uses non-monotone strategies. We provide sufficient conditions under which the sender's strategy is monotone in every Nash equilibrium. Our conditions require the sender's payoff to have strictly increasing differences between the state and the action profile and to be monotone with respect to each player's action. Our sufficient conditions fit into a number of applications, including advertising, warranty provision, education and job assignment.

Chapter 2 focuses on the market institution and studies how firms strategically interact with boundedly rational consumers through it. In this chapter, Andreas Hefti, Armin Schmutzler and I ask whether firms seek to make a market transparent or they want to manipulate the perception of product characteristics. We show that, contrary to the well-studied case of homogeneous goods, obfuscation is not necessarily an equilibrium phenomenon in markets with differentiated goods. In particular, if the taste distribution is polarized, so that indifferent consumers are relatively rare, firms seek to educate consumers. However, if the taste distribution features a concentration of indecisive consumers, confusion is beneficial for firms and obfuscation is an equilibrium strategy. The adverse welfare consequences of obfuscation are more severe than with homogeneous goods, as consumers may not only pay higher prices, but also buy the wrong product. Our model can also be adapted to offer new insights on the incentives for political candidates to induce polarized opinions by confusing voters.

An important class of non-market institutions through which strategic agents interact is that of voting mechanisms. In Chapter 3, I study the effect of public information on the voting outcomes in committees, where members can have both common and conflicting interests. In the presence of public information, the simple and efficient vote-your-signal strategy profile no longer constitutes an equilibrium under the commonly-used simultaneous voting rules, while the intuitive but inefficient follow-the-expert strategy profile almost always does. Although more information may be aggregated if agents are able to coordinate on more sophisticated equilibria, inefficiency can persist even in large elections if the provision of public information introduces general correlation between the signals observed by the agents. We propose simple voting procedures that can indirectly implement the outcomes of the optimal ex post incentive compatible mechanisms with public information. Our voting procedures also have additional advantages when there is a concern for strategic disclosure of public information.

The results in Chapter 3 show that the performance of an institution depends crucially on the incentives provided to its participants, and these incentives in turn depend on how the institution is designed. In Chapter 4, Dimitri Migrow and I consider another dimension of the design of an institution. Motivated by the observation that multinational and multiproduct firms often experience uncertainty in the relative return of conducting activities in different markets (due to, for example, exchange rate volatility or the changing prospects of different products), we study how a multi-divisional organization should optimally allocate decision-making authority to its managerial members when operating in such volatile markets. To be able to adapt its decisions to local conditions, the organization has to rely on self-interested division managers to collect and disseminate the relevant information. We show that if communication takes the form of verifiable disclosure, then centralized decision-making does not suffer from information asymmetry and it allows the headquarter of the organization to better cope with the inter-market uncertainty. However, a downside of centralization is that it can discourage information acquisition, and this negative effect is amplified by the need for coordinating the activities of different divisions. As a result, the optimality of decentralized decision-making can actually be driven by a large coordination motive.

In Chapters 3 and 4, the institution designer is constrained as monetary transfers between the involving parties are either limited or entirely ruled out. Chapter 5 considers a general setting where the designer is free to use monetary transfers. In this chapter, Alexey Kushnir and I extend the equivalence between Bayesian and dominant strategy implementation (Manelli and Vincent in *Econometrica* 78: 1905-1938, 2010; Gershkov et al. in *Econometrica* 81: 197-220, 2013) to environments with nonlinear utilities satisfying a property of increasing differences over distributions and a convex-valued assumption. The new equivalence result produces novel implications to the literature on the principal-agent problem with allocative externalities, environmental mechanism design, and public good provision.

Part II

Research Papers

1 Monotone Equilibria in Signalling Games[1]

Joint with Harry Pei

1.1 Introduction

Starting from the seminal contribution of Spence (1973), signalling games have become powerful tools to study strategic interactions under asymmetric information. In a typical signalling model, an informed *sender*, who has private information about the payoff environment (or her *type*), takes an action that can influence the behavior of an uninformed *receiver*. This game theoretic model helps researchers to understand phenomena such as education, limit pricing, the peacock's tail, etc.

In virtually all applications of signalling games, players' payoffs satisfy a well-known Spence-Mirrlees condition: The sender's actions and types are ranked, such that a higher type has a comparative advantage in taking higher actions compared to a lower type. For example, it is less costly for a talented worker to receive more education (Spence, 1973), it is more profitable for an efficient firm to cut prices (Milgrom and Roberts, 1982), etc. Under this condition, it may seem intuitive to predict that the sender's action is non-decreasing in her type, or in other words, the game's outcome is *monotone*. Indeed, researchers often focus on such *monotone equilibria* in many applications of signalling games, see for example, Spence (1973, 1977), Ross (1977), etc.

In this paper, we assess the robustness of this monotonicity prediction. We focus on signalling games that satisfy a generalized version of the Spence-Mirrlees condition: the set of types and actions are complete lattices and the sender's payoff exhibits strictly increasing differences between the state and her own action. An equilibrium is monotone if a higher type sender never plays a strictly lower action than a lower type.

We examine the monotonicity of *all* (Bayes) Nash equilibria in these signalling games. Our motivation for this is twofold. First, as pointed out by Fudenberg et al. (1988) and Weinstein and Yildiz (2012), refinements in extensive form games are sensitive to the modeling details. Therefore, it is important to evaluate the robustness of the monotonicity prediction against equilibrium selection. Second, monotone equilibria have desirable properties, making them straightforward to interpret, tractable to analyze and easy to compute (Athey, 2001). Therefore, a result establishing the monotonicity of all equilibria can facilitate the characterization of the set of equilibrium strategies and outcomes.[2]

[1]This paper should be cited as Liu, S. and H. Pei (2018): "Monotone Equilibria in Signalling Games," Mimeo.

[2]To address the concern that there is a plethora of equilibria in signalling games, we apply the following "double standard": For counterexamples, we adopt stronger solution concepts such as sequential equilibrium (Kreps and Wilson, 1982), equilibria that can survive the refinements proposed in Kohlberg and Mertens (1986), Cho and Kreps (1987), Banks and Sobel (1987), etc. When presenting positive results, we adopt weaker solution concepts such as Nash equilibrium.

We start with a counterexample showing that the Spence-Mirrlees condition *cannot* guarantee the monotonicity of the sender's equilibrium strategy. These non-monotone equilibria exist even when both players' payoffs are strictly supermodular functions with respect to the sender's type, the sender's action and the receiver's action. Furthermore, they can survive standard refinements as both players have strict incentives and the sender plays every action with strictly positive probability.

Compared to Spence (1973), non-monotone equilibria arise due to the *non-separability* of the sender's payoff, that is, her return from the receiver's action depends on her type. This occurs in a number of applications, such as a worker's preference over jobs depends on her talent. Intuitively, when jobs and talents are horizontally differentiated (Roy, 1951) and the worker chooses which job-specific human capital to acquire (her action), a high type sender has an incentive to play a low action if doing so can induce the receiver to play a high action. The receiver has an incentive to play his high action after observing the sender's low action as he believes that the sender's type is high, making his belief self-fulfilling.

We then proceed to provide sufficient conditions under which all Nash equilibria are monotone. At the heart of our analysis is a *monotone-supermodular* condition, which requires in addition to the Spence-Mirrlees condition, that (1) the sender's payoff is strictly decreasing in her own action and is strict increasing in the receiver's action, (2) the sender's payoff has increasing differences between the state and the receiver's action.[3] This fits into a number of applications, including the education signalling game where an employer assigns workers job positions after observing their years of education, the warranty provision game where a firm chooses the length of warranty and the amount of refund before a consumer chooses the quantity to purchase, etc.

Our first result (Theorem 1.1) shows that every equilibrium is monotone when the sender's payoff is monotone-supermodular and the receiver's action choice is binary. This fits into the warranty provision game when the consumer has unit demand. Intuitively, thanks to the binary choice assumption, every pair of distributions over the receiver's action can be ranked according to first-order stochastic dominance (FOSD). Since playing a higher action is more costly for the sender, she only has an incentive to do so when it can induce a more favorable response from the receiver. This implies that the ranking over the sender's equilibrium actions must coincide with the ranking over the distributions of the receiver's action that they induce. Since a high type sender has a stronger preference towards higher action profiles, she will never play a strictly lower action than a low type in any Nash equilibrium.

However, in games where the receiver has three or more actions, non-monotone equilibria can arise despite the sender's payoff being monotone-supermodular. This is because not every pair of the receiver's mixed actions can be ranked according to FOSD. Consequently, one cannot draw inference about the ranking of the receiver's responses based on the ranking of the sender's equilibrium actions.

We introduce two sets of sufficient conditions to address this issue. First, we show in Theorem 1.2 that every equilibrium is monotone if the sender's payoff is monotone-supermodular, the ranking over the receiver's action set is complete and the receiver's payoff satisfies a *quasiconcavity-*

[3] Despite the monotone-supermodularity condition is not necessary in general, we use counterexamples to address why every component of this condition is not superfluous in subsection 1.4.3

preserving property (QPP). QPP requires the receiver's payoff to be strictly quasi-concave in his own action under every belief about the state. A sufficient condition for QPP is that the receiver's payoff being strictly concave in his own action, which fits into the warranty provision game when the consumer faces decreasing marginal returns to quantity.[4] QPP implies that the receiver has at most two pure best replies in every circumstance, which must be adjacent elements in his action set. As a result, every pair of his *mixed best replies* can be ranked according to FOSD.

Second, we identify a novel condition on the sender's payoff under which every pair of the receiver's mixed actions can be ranked endogenously. We call this property *increasing absolute differences over distributions* (IADD). Theorem 1.3 shows that if the sender's payoff is monotone-supermodular and satisfies IADD, then every Nash equilibrium is monotone. Unlike Theorem 1.2, Theorem 1.3 (as well as Theorem 1.1) makes no reference to the receiver's payoff function and incentives, so the conclusion extends to richer environments such as the sender signalling to a population of heterogeneous receivers, the receiver having private information about his payoff, etc. We also establish a representation result that characterizes IADD (Proposition 1.1), which facilitates the application of our Theorem 1.3 to economic modeling.

Literature Review. Starting from Spence (1973), the monotonicity of outcome has become a natural prediction in various applications of signalling games to labor economics, industrial organization, corporate finance and biology. Our paper points out that even in environments where the Spence-Mirrlees condition is satisfied, such a prediction is *not* without loss when the sender's return from the receiver's action depends on her type. This contrasts to the alternative channels suggested in the literature through which non-monotone equilibria can arise, as the latter often require various departures from the standard signalling model. For example, Feltovich et al. (2002) and Araujo et al. (2007) show that if the receiver observes an exogenous signal that is informative about the sender's type, then there exist *countersignalling equilibria* in which the high type plays a low action to distinguish herself from the medium type. As shown in several papers on warranty provision (e.g., Balachander, 2001; Gal-Or, 1989), non-monotone equilibria can also arise when the sender's actions cannot be perfectly observed.

This paper also contributes to the literature on supermodular incomplete information games. Most of the papers in this literature focus on simultaneous-move games and provide sufficient conditions for the *existence* of monotone pure strategy Nash equilibrium (e.g., Athey, 2001; McAdams, 2003; Reny, 2011; Van Zandt and Vives, 2007).[5] Several papers establish the monotonicity of all equilibria in simultaneous move supermodular games, for example, Morris and Shin (1998) and the vast literature on global games. In multi-dimensional environments, McAdams (2006) shows that every equilibrium is outcome-equivalent to a monotone equilibrium in multi-unit uniform price auctions with risk neutral bidders and independent values. In contrast, we analyze the monotonicity of equilibria in *sequential move* games where players' payoff functions

[4]In Appendix A.2, we relate QPP to a strict version of the signed-ratio monotonicity condition in Quah and Strulovici (2012) and provide a full characterization.

[5]Necessary conditions for the existence of monotone equilibrium are non-tractable and is beyond the scope of this literature as well as the current paper. Mensch (2018) studies dynamic incomplete information games with strategic complementarities and establishes the existence of monotone perfect Bayesian equilibrium.

are supermodular, with one-shot signalling games a natural starting point.[6]

To the best of our knowledge, the question when it is without loss to focus on monotone equilibria has not been systematically analyzed in signalling game contexts. We take a step to close this gap by providing sufficient conditions under which all equilibria are monotone. Our conditions highlight the economic forces behind equilibrium monotonicity and our counterexamples illustrate how monotonicity can fail once they are relaxed. Furthermore, these conditions are easy to verify given the functional forms of players' payoffs, which are useful for future applied works.

We close this section by commenting on two specific results in the literature that are related to our sufficient conditions. The first one is obtained by Cho and Sobel (1990). While the main focus of their paper is to analyze the uniqueness and strategic stability of the universally divine equilibrium outcomes, they also establish a result stating that the sender's strategy must be monotone in every equilibrium (Lemma 4.1, p. 393 - 394). Different from ours, their result *assumes* that the receiver's best response is always a pure strategy. The second related result is obtained in a recent work by Kartik et al. (2018). Similar to our Theorem 1.3, they also identify a class of utility functions under which the sender will use a monotone strategy in every equilibrium. We will show in Section 1.5 that our results neither nest theirs nor are nested by theirs.

1.2 The Model

Consider the following two-player signalling game. Player 1 (or sender, she) privately observes the realization of a payoff relevant state $\theta \in \Theta$ (call it her *type*) and then chooses an action $a_1 \in A_1$. Player 2 (or receiver, he) has a prior belief $\pi \in \Delta(\Theta)$ about θ. He chooses $a_2 \in A_2$ after observing a_1. Player i's payoff is $u_i(\theta, a_1, a_2)$ with $i \in \{1, 2\}$. Both players are expected utility maximizers. Let $u_1(\theta, a_1, \alpha_2) \equiv \int_{a_2} u_1(\theta, a_1, a_2) d\alpha_2$ for every $\alpha_2 \in \Delta(A_2)$.

Throughout the paper, we assume that Θ, A_1 and A_2 are finite lattices and π has full support.[7] We will comment on cases in which Θ, A_1 and A_2 are infinite after stating our main results. We use $\succ$ and $\succsim$ to denote strict and weak orders on lattice sets. For two lattices X and Y, a mapping $f : X \times Y \to \mathbb{R}$ exhibits *increasing differences* if for every $x, x' \in X$ and $y, y' \in Y$ with $x \succ x'$ and $y \succ y'$:

$$f(x, y) - f(x', y) \geq f(x, y') - f(x', y'), \tag{1.1}$$

and it exhibits *strictly increasing differences* if the above inequality is strict (Topkis, 1998). We introduce a condition on the sender's payoff, which generalizes the Spence-Mirrlees condition to discrete lattices:

[6]Complementarities in dynamic games are explored by Echenique (2004a,b), that explain how intertemporal incentives can weaken the implications of complementarity and supermodularity. Despite we have established the monotonicity of all Nash equilibria in a signalling game context, a signalling game with supermodular payoff functions is not necessarily supermodular in its normal form and, therefore, the other attractive properties of simultaneous move supermodular games, such as the existence of extremal equilibria, monotone comparative statics, the tâtonnement algorithm to compute the set of equilibria, etc. cannot be applied to our setting.

[7]A set X is a lattice if there exists a partial order $\succeq$ such that for every $x, x' \in X$, $x \vee x', x \wedge x' \in X$, where $x \vee x'$ is the smallest element above both x and x', $x \wedge x'$ is the largest element below both x and x'.

Definition 1.1 (Generalized Spence-Mirrlees Condition)**.** *u_1 satisfies the generalized Spence-Mirrlees condition if it exhibits strictly increasing differences in (θ, a_1).*

Intuitively, this generalized Spence-Mirrlees condition requires that a higher type sender has a comparative advantage in playing higher actions compared to a lower type.[8] This fits into most applications of signalling theory, including the education game in which receiving education is less costly for a more talented worker (Spence, 1973), the beer-quiche game in which drinking beer is more pleasant for the strong sender (Cho and Kreps, 1987), the warranty provision game in which providing lengthier warranty is less costly for a high quality firm (Gal-Or, 1989), etc. Our condition is also satisfied in many multi-dimensional signalling models, such as Araujo et al. (2007), Quinzii and Rochet (1985), etc.

Strategies & Equilibrium. The sender's strategy is $\sigma_1 : \Theta \to \Delta(A_1)$ and the receiver's strategy is $\sigma_2 : A_1 \to \Delta(A_2)$. Let $\sigma_1^\theta \in \Delta(A_1)$ be the (possibly mixed) action played by type θ, which gives $\sigma_1 = \left(\sigma_1^\theta\right)_{\theta \in \Theta}$.

The solution concept is Nash equilibrium (henceforth equilibrium), which consists of a strategy profile $\sigma \equiv (\sigma_1, \sigma_2)$ such that σ_i best responds to σ_{-i} for every $i \in \{1, 2\}$. Since the game is finite, an equilibrium exists. Next, we introduce the definitions of *monotone strategy* and *monotone equilibrium*:

Definition 1.2 (Monotone Strategy & Monotone Equilibrium)**.** *σ_1 is a monotone strategy if for every $\theta \succ \theta'$, there exist no $a_1 \in supp(\sigma_1^\theta)$ and $a_1' \in supp(\sigma_1^{\theta'})$ such that $a_1 \prec a_1'$. An equilibrium (σ_1, σ_2) is monotone if σ_1 is a monotone strategy.*

In words, a strategy is monotone if a low type sender never plays a strictly higher action than a high type. When the order on A_1 is complete (or A_1 is one-dimensional), the monotonicity of σ_1 is equivalent to the following:

$$\min_{a_1} \left\{\operatorname{supp}(\sigma_1^\theta)\right\} \succsim \max_{a_1} \left\{\operatorname{supp}(\sigma_1^{\theta'})\right\} \text{ for every } \theta \succ \theta'. \tag{1.2}$$

That is to say, if type θ' plays a_1 with positive probability, then every type higher than θ' must be playing actions that are higher or equal to a_1 with probability 1.

We are interested in examining the monotonicity of *all* Nash equilibria in signalling games, and in particular, games in which the sender's payoff satisfies the generalized Spence-Mirrlees condition.[9] The choice of a weak solution concept makes our positive results presented in Section 1.4 (Theorems 1.1-1.3) robust against equilibrium selection. However, due to the plethora of equilibria in signalling games, one might argue that we should restrict attention to a subset of equilibria that can survive standard refinements instead of all Nash equilibria. To address this concern, we will adopt the more stringent solution concept of sequential equilibrium (Kreps

[8] For alternative versions of the Spence-Mirrlees condition, see Engers (1987), Cho and Sobel (1990), etc.

[9] The existence of monotone equilibria is often trivial given our weak solution concept. For example, when u_1 is strictly decreasing in a_1 (which is satisfied in many costly signalling games, such as Spence education signalling and the beer-quiche game), let $\underline{a}_1$ be the smallest element in A_1 and let $\hat{a}_2 \in \arg\max \int_\theta u_2(\theta, \underline{a}_1, a_2) d\pi(\theta)$, then the strategy profile σ with $\sigma_1(\theta) = \underline{a}_1$ $\forall \theta \in \Theta$ and $\sigma_2(a_1) = \hat{a}_2$ $\forall a_1 \in A_1$ constitutes a Nash equilibrium. This is monotone according to Definition 1.2.

and Wilson, 1982) when presenting counterexamples. To make it even more convincing, we also require them to survive the refinements proposed in Kohlberg and Mertens (1986), Cho and Kreps (1987), Banks and Sobel (1987), etc.

Remark on Monotonicity. Our notion of monotonicity can be viewed as a strong one. For example, when A_1 is one-dimensional, (1.2) implies that $\mathrm{supp}(\sigma_1^{\theta})$ dominates $\mathrm{supp}(\sigma_1^{\theta'})$ in strong set order for every $\theta \succ \theta'$. Nevertheless, it is worth to note that all counterexamples in our paper (Examples 1.1 - 1.4) will violate the weaker notion of monotonicity based on strong set order, making them more convincing. In contrast, all positive results in this paper (Theorems 1.1 - 1.3) apply under our more demanding monotonicity requirement, which strengthens their implications.

When A_1 is multi-dimensional, our definition of monotone strategies is no longer equivalent to but is only implied by condition (1.2). This is due to the incompleteness of the order on A_1. Thus, one may be interested in further strengthening the robust prediction about the sender's strategy by insisting that (1.2) holds even when A_1 is multi-dimensional. In Appendix A.3, we use an example to illustrate the difficulty of obtaining positive results when Definition 1.2 is replaced by (1.2) in multi-dimensional environments.

1.3 Counterexample: Existence of Non-Monotone Equilibria

In this section, we present a counterexample which shows that the generalized Spence-Mirrlees condition cannot guarantee the monotonicity of the sender's equilibrium strategy even in $2\times2\times2$ games.

Example 1.1. Consider the following $2 \times 2 \times 2$ game:

$\theta = \theta_1$	h	l
H	$2, 2$	$0, 0$
L	$1, 1$	$0, 0$

$\theta = \theta_0$	h	l
H	$-1, -1/2$	$1, 0$
L	$0, -1$	$5/2, 1/4$

The sender observes θ and chooses between H and L. The receiver chooses between h and l after observing the sender's action choice. We leave the receiver's prior belief unspecified as it plays no role. One can check that according to the orders $\theta_1 \succ \theta_0$, $H \succ L$ and $h \succ l$, the generalized Spence-Mirrlees condition is satisfied. In fact, these payoffs even satisfy a stronger notion of complementarity, that is, both u_1 and u_2 are strictly supermodular functions of the triple (θ, a_1, a_2).[10] Intuitively, there are complementarities between players' actions as well as between the state and the action profile.

However, the above signalling game admits the following non-monotone equilibrium. The sender plays L if her type is θ_1 and plays H if her type is θ_0. The receiver, who could perfectly

[10] Let X be a lattice. A function $f : X \to \mathbb{R}$ is *strictly supermodular* if $f(x \vee x') + f(x \wedge x') \geq f(x) + f(x')$ for every $x, x' \in X$, and the inequality is strict if $\{x, x'\} \neq \{x \vee x', x \wedge x'\}$.

learn the state from the sender's action, plays l after observing H and plays h after observing L. Clearly, the sender's strategy is non-monotone and no player has any incentive to deviate.[11]

In fact, since players have strict incentives and there are no off-path beliefs, the above strategy profile and its induced belief system also form a sequential equilibrium (Kreps and Wilson, 1982). Moreover, it cannot be refined away using the selection criteria proposed in Kohlberg and Mertens (1986), Cho and Kreps (1987), and Banks and Sobel (1987). Since players' incentives are strict, this equilibrium is also robust against perturbations on the game's payoff matrices.

We argue that this non-monotone equilibrium is driven by three features of the game: sequential moves, non-separable payoff (of the sender) and interdependent values. When players move sequentially, every a_1 induces a distribution over a_2. As a result, the sender is effectively choosing a distribution over *action profiles* instead of just her own action. Because u_1 is *non-separable* with respect to θ and a_2, her preferences over the receiver's actions also vary with the state. Therefore, her state contingent action choice depends not only on her comparative advantage in a_1 but also on her preferences over a_2. Since the receiver's best response to a_1 depends on his belief about the state (i.e. values are *interdependent*), choosing h after observing L and choosing l after observing H can be rationalized despite there are complementarities between players' actions in u_1 and u_2. In our non-monotone equilibrium, the receiver believes that the state is θ_1 after observing L and the state is θ_0 after observing H, which provides the sender an incentive to use non-monotone strategies and makes the receiver's belief self-fulfilling.

While sequential moves and interdependent values are standard in signalling games, non-separability of the sender's payoff distinguishes our model from the classic education signalling game (Spence, 1973) and the beer-quiche game (Cho and Kreps, 1987). In these examples, the sender's valuations of money and fighting do not depend on her type. Nevertheless, non-separable payoffs arise naturally in many other economic applications. For example, consider a firm (receiver) offering a worker (sender) a job after observing her education. The worker's preferences over jobs depend on her type (for example, her taste and talent) no matter whether the jobs are horizontally differentiated (Roy, 1951) or vertically differentiated (Gibbons and Waldman, 1999; Waldman, 1984).[12] Non-separability also occurs in many applications in industrial organization, some of which will be discussed in Section 1.5.

It is also worth to point out that the existence of non-monotone equilibrium in Example 1 does not contradict the well-known results in Van Zandt and Vives (2007) on *simultaneous move* Bayesian supermodular games. This is because once we maintain the pre-specified orders over players' actions, a signalling game with supermodular payoffs (Example 1.1) is not supermodular in its normal form.[13]

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[11]In the context of this game, the non-monotone equilibrium is Pareto dominated by a monotone equilibrium. However, this is not a general feature and the Pareto dominance criteria cannot always refine away all non-monotone equilibria. In Example 1.3 (Section 1.4.3) when the prior probability of state θ_0 is sufficiently high, our non-monotone equilibrium Pareto dominates the alternative unique monotone equilibrium, in which both types play H and player 2 plays l regardless of what the sender does.

[12]When jobs are horizontally differentiated, different types of workers prefer different kinds of jobs, as in Roy (1951)'s hunting-fishing example. When jobs are vertically differentiated, the worker's gain from a job position depends on her talent due to the piece-rate incentive schemes, the prospects of promotion, etc.

[13]Echenique (2004a) shows that every game that has at least two pure Nash equilibria is supermodular if the analyst can flexibly choose the order over players' strategies. However, the results that are established under an

1.4 Sufficient Conditions for Monotone Equilibrium

In this section, we introduce sufficient conditions that guarantee the monotonicity of all equilibria. At the heart of our analysis is the following *monotone-supermodular* condition on the sender's payoff:

Definition 1.3 (Monotone-Supermodular Condition)**.** *The sender's payoff is monotone-supermodular if (i) u_1 is strictly decreasing in a_1 and strictly increasing in a_2 , and (ii) u_1 exhibits strictly increasing differences in (θ, a_1) and increasing differences in (θ, a_2).*

Compared to the more demanding requirement that both u_1 and u_2 are strictly supermodular functions of (θ, a_1, a_2), our monotone-supermodular condition does not require any complementarities within players' actions, nor does it impose any restrictions on the receiver's payoff function. Nevertheless, it incurs two important requirements in addition to the generalized Spence-Mirrlees condition. First, the sender's payoff exhibits increasing differences between the state and the receiver's action. This includes the separable payoff (i.e., there exist $f : A_1 \times A_2 \to \mathbb{R}$ and $c : \Theta \times A_1 \to \mathbb{R}$ such that $u_1(\theta, a_1, a_2) = f(a_1, a_2) + c(\theta, a_1)$) as a special case. It also fits into many other applications where payoffs are non-separable by nature. For example, in warranty provision games, the firm's per unit profit (sales price minus the expected refund paid to the consumers) increases with its product quality, and therefore, its total profit exhibits increasing differences between its quality and the quantity sold. In education signalling games in which a firm assigns workers to various job positions after observing their years of education, more talented workers receive higher benefits from higher level jobs due to the piece-rate incentive schemes and better prospects of promotion.

Second, the sender's payoff is monotone with respect to her own action and the receiver's action in appropriate directions.[14] This is natural in many applications. For example, it is costly for the firm to provide lengthier warranties and higher refunds, but it can benefit when consumers increase their purchasing quantities. Similarly, workers face higher opportunity costs to receive more education but they can benefit from more decent job positions.

In the next two subsections (1.4.1 and 1.4.2), we state results that establish the monotonicity of all equilibria in signalling games where the sender's payoff is monotone-supermodular. The role of the monotone-supermodular condition in our results will be discussed in subsection 1.4.3. We will elaborate in Section 1.5 how the assumptions of our results fit into the classic applications of signalling games in industrial organization and labor economics, and outline their implications in these settings.

1.4.1 Binary Action Games

In this subsection, we study games in which the receiver's action choice is binary, i.e. $|A_2| = 2$. This holds the warranty provision game when the consumer has unit demand, i.e. $a_2 \in \{0, 1\}$.

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arbitrary order cannot imply the monotonicity of the sender's action with respect to her type.

[14] Our results also hold when the sender's payoff is increasing in her own action and decreasing in the receiver's action. However, non-monotone equilibria can exist when the sender's payoff is strictly increasing (or strictly decreasing) in both players' actions (see Example 1.3 in subsection 1.4.3)

Our first result below states that for these games, monotone-supermodularity alone is sufficient to guarantee the monotoncity of all equilibria.

Theorem 1.1. *If $|A_2| = 2$ and the sender's payoff is monotone-supermodular, then every equilibrium is monotone.*

Proof. Let $A_2 \equiv \{\overline{a}_2, \underline{a}_2\}$ with $\overline{a}_2 \succ \underline{a}_2$. Suppose towards a contradiction that in some equilibrium σ, there exist $\theta \succ \theta'$ and $a_1 \succ a_1'$ such that $\sigma_1^{\theta}(a_1') > 0$ and $\sigma_1^{\theta'}(a_1) > 0$. Let $\alpha_2 \equiv \sigma_2(a_1)$ and $\alpha_2' \equiv \sigma_2(a_1')$ be the mixed actions played by the receiver after observing a_1 and a_1', respectively. Since type θ prefers (a_1', α_2') to (a_1, α_2) and type θ' prefers (a_1, α_2) to (a_1', α_2'), we have:

$$u_1(\theta, a_1', \alpha_2') \geq u_1(\theta, a_1, \alpha_2) \tag{1.3}$$

and

$$u_1(\theta', a_1, \alpha_2) \geq u_1(\theta', a_1', \alpha_2'). \tag{1.4}$$

These together imply that:

$$u_1(\theta, a_1', \alpha_2') - u_1(\theta, a_1, \alpha_2) \geq 0 \geq u_1(\theta', a_1', \alpha_2') - u_1(\theta', a_1, \alpha_2). \tag{1.5}$$

Because u_1 is strictly decreasing in a_1, (1.4) also implies that $u_1(\theta', a_1', \alpha_2) > u_1(\theta', a_1, \alpha_2) \geq u_1(\theta', a_1', \alpha_2')$. This further implies that α_2 must attach a higher probability to $\overline{a}_2$ compared to α_2', as the sender's payoff is strictly increasing in a_2. Therefore, we have $\theta \succ \theta'$, $a_1 \succ a_1'$ and α_2 dominates α_2' in the sense of first-order stochastic dominance (FOSD). Since u_1 has strictly increasing differences in (θ, a_1) and increasing differences in (θ, a_2), we have:

$$u_1(\theta, a_1', \alpha_2') - u_1(\theta, a_1, \alpha_2) < u_1(\theta', a_1', \alpha_2') - u_1(\theta', a_1, \alpha_2), \tag{1.6}$$

which contradicts (1.5). $\square$

The intuition of Theorem 1.1 is as follows. When $|A_2| = 2$, every pair of distributions over the receiver's action can be ranked according to FOSD. Since playing a higher action is more costly for the sender, she only has an incentive to do so when it induces a more favorable response from the receiver. This implies that the ranking over the sender's equilibrium actions must coincide with the ranking over the receiver's (possibly mixed) actions that they induce. Because a high type sender has a stronger preference towards higher action profiles, she will never play a strictly lower action than a low type.

Since the above proof makes no reference to the receiver's incentives, our monotonicity property also applies to every ex ante rationalizable strategy (Bernheim, 1984; Pearce, 1984). In fact, only monotone strategies can survive the first round of elimination. The irrelevance of the receiver's incentives also makes it clear that our result immediately extends to cases where the receiver has private information about his preferences, the sender is signalling to a population of receivers with heterogeneous preferences, etc.

Theorem 1.1 can also be generalized to signalling games with infinite A_1 and Θ with two caveats.[15] First, when the type space is infinite, Nash equilibrium needs to be defined at the interim stage after the sender observes her type. This is to ensure that the sender will play a best reply at every state. Second, when A_1 is infinite, some of the actions in the support of a sender's strategy can be suboptimal. Hence, the monotonicity condition in Definition 1.2 is too demanding. Nevertheless, we show in Appendix A.4 that the sender's equilibrium strategy is *almost surely monotone* in the following sense: For every $\theta \succ \theta'$ and $a_1 \in \text{supp}(\sigma_1^\theta)$, the probability that type θ' plays an action strictly higher than a_1 equals zero.

Theorem 1.1 also has the following implication on *repeated* signalling games where the state is perfectly persistent, the sender's stage game payoff is monotone-supermodular and the receiver's action choice is binary. For every pair of states $\theta \succ \theta'$ and every Nash equilibrium (σ_1, σ_2) of the repeated signalling game, if playing the highest action in every period is the sender's best reply against σ_2 in state θ', then according to σ_1, she will play the highest action with probability 1 at every on-path history in state θ.[16] As shown in Pei (2018), this is an intermediate step towards establishing the commitment payoff theorem and the uniqueness of the sender's on-path equilibrium behavior in reputation games.

1.4.2 Games with $|A_2| \geq 3$

In this subsection, we generalize our findings in binary action games to ones in which the receiver has more than two actions. To illustrate the difficulties, we first present a counterexample showing that the sender's payoff being monotone-supermodular is no longer sufficient to guarantee the monotonicity of all equilibria.

Example 1.2. Consider the following $2 \times 2 \times 3$ game:

$\theta = \theta_1$	h	m	l
H	$2-\epsilon, 1$	$1-2\epsilon, 0$	$-3\epsilon, -2$
L	$2, 0$	$1, 1$	$0, 0$

$\theta = \theta_0$	h	m	l
H	$2-3\epsilon, 0$	$1-3\epsilon, 0$	$4\epsilon, 0$
L	$2-\epsilon, 0$	$1, 2$	$8\epsilon, 3$

Suppose $\epsilon \in (0, 1/8)$ and apply the rankings $\theta_1 \succ \theta_0$, $H \succ L$ and $h \succ m \succ l$. One can verify that the sender's payoff is monotone-supermodular. However, consider the following strategy profile: The sender plays L if $\theta = \theta_1$, and plays H if $\theta = \theta_0$. The receiver plays m after observing L, and plays h and l with equal probabilities after observing H. One can check that the sender's strategy is non-monotone although this strategy profile and its induced belief constitute a sequential equilibrium.[17]

[15] The existence of equilibrium is guaranteed by the monotonicity condition (see footnote 9).

[16] However, the monotonicity of equilibria in the stage game does not imply the monotonicity of the sender's behavior strategy at every on-path history in the repeated signalling game. Therefore, our result cannot guarantee that the receiver will always positively update his belief about the sender's type after observing the sender playing her highest action.

[17] This counterexample is not driven by the receiver's non-generic payoff. Even when the receiver has strict preferences over A_2 conditional on $(\theta, a_1) = (\theta_0, H)$, there still exists a non-monotone partial pooling equilibrium in which type θ_1 mixes between H and L, and type θ_0 always plays H.

Example 1.2 highlights the following issue: When $|A_2| \geq 3$, the *distributions* over the receiver's actions cannot be completely ranked via FOSD. In particular, u_1 being monotone-supermodular does *not* imply the following:

- For every $\alpha_2, \alpha_2' \in \Delta(A_2)$, if α_2 is preferred to α_2' for some $\theta \in \Theta$ when she plays $a_1 \in A_1$, then the sender's expected payoff difference between α_2 and α_2' is increasing in her type conditional on a_1.

We proceed along two directions to address this challenge, leading to two monotonicity results. First, we introduce a property on the receiver's payoff under which every pair of his (pure or mixed) *best replies* can be ranked via FOSD. This, together with the monotone-supermodular condition on the sender's payoff, implies the monotonicity of all equilibria (Theorem 1.2). Second, we identify a class of utility functions u_1 which can endogenously generate a complete order on $\Delta(A_2)$. When the sender's payoff function belongs to this class and is monotone-supermodular, every equilibrium is monotone irrespective of the receiver's payoff (Theorem 1.3).

1.4.2.1 Quasiconcavity-Preserving Property

For this part, we assume that $A_2 \equiv \{a_2^1, ..., a_2^n\}$ is completely ordered with $a_2^1 \prec a_2^2 \prec ... \prec a_2^n$.[18] For every $(\theta, a_1) \in \Theta \times A_1$ and $i \in I \equiv \{1, 2, ..., n-1\}$, let

$$\gamma_\theta^{a_1}(i) \equiv u_2(\theta, a_1, a_2^i) - u_2(\theta, a_1, a_2^{i+1}) \tag{1.7}$$

be the receiver's payoff gain by decreasing his action locally, and let

$$\Gamma_{\tilde{\pi}}^{a_1}(i) \equiv \int \gamma_\theta^{a_1}(i) d\tilde{\pi} \tag{1.8}$$

be his expected payoff gain under belief $\tilde{\pi} \in \Delta(\Theta)$. We now recall the definition of *strict single-crossing* functions in Milgrom and Shannon (1994):

Definition 1.4. *Function $\gamma : I \to \mathbb{R}$ satisfies the strict single-crossing property (SSCP) if for every $i \in I$, $\gamma(i) \geq 0$ implies that $\gamma(j) > 0$ for every $j \in I$ with $j > i$.*

If $\gamma_\theta^{a_1}(\cdot)$ satisfies SSCP for every $(\theta, a_1) \in \Theta \times A_1$, then $u_2(\theta, a_1, \cdot)$ is strictly quasi-concave in a_2. In that case, the receiver has at most two pure best replies to every (θ, a_1), which must be adjacent elements in A_2. This further implies that every pair of his (pure or mixed) best replies to a degenerate distribution on $\Theta \times A_1$ can be ranked according to FOSD. However, some of the sender's actions may induce non-degenerate beliefs in some equilibria, so the issue of aggregating the single-crossing property arises. This motivates us to introduce a *quasiconcavity-preserving property* (QPP) on the receiver's payoff.

[18] Our monotonicity result in this subsection (Theorem 1.2) can also be extended to settings where A_2 is a multi-dimensional convex set.

Definition 1.5 (Quasiconcavity-Preserving Property)**.** *The receiver's payoff satisfies QPP if* $\Gamma_{\tilde{\pi}}^{a_1} : I \to \mathbb{R}$ *satisfies SSCP for every* $(\tilde{\pi}, a_1) \in \Delta(\Theta) \times A_1$*.*[19]

A sufficient condition for QPP is $\gamma_\theta^{a_1}(\cdot)$ being strictly increasing for every $(\theta, a_1) \in \Theta \times A_1$. This fits into the warranty provision game when the consumer faces decreasing marginal returns with respect to quantity. Intuitively, u_2 is *strictly concave* in a_2 when $\gamma_\theta^{a_1}(\cdot)$ is strictly increasing. The latter implies QPP as strict concavity is preserved under positive linear aggregation. Nevertheless, strict concavity is by no means necessary for QPP. In Appendix A.2, we provide a full characterization of QPP by relating it to a strict version of the signed-ratio monotonicity condition introduced in Quah and Strulovici (2012).

Under QPP, the receiver's (pure or mixed) best replies to every $(\tilde{\pi}, a_1) \in \Delta(\Theta) \times A_1$ can be ranked according to FOSD. This leads to our second result:

Theorem 1.2. *If (i) the order on* A_2 *is complete, (ii) the sender's payoff is monotone-supermodular, and (iii) the receiver's payoff satisfies QPP, then every equilibrium is monotone.*

The proof follows along the same line as that of Theorem 1.1, which we omit to avoid repetition. Note that despite the extra condition on the receiver's payoff function, Theorem 1.2 only requires him to play a best reply against some $\tilde{\pi} \in \Delta(\Theta)$ after every $a_1 \in A_1$ on the equilibrium path. Therefore, the above monotonicity result does not depend on the receiver's belief updating process and applies to all outcomes under weaker solution concepts such as $S^\infty W$ (Dekel and Fudenberg, 1990) and iterative conditional dominance (Shimoji and Watson, 1998), which are variants of rationalizability that can rule out the receiver's suboptimal plays at off-path information sets.[20] Moreover, when applying the elimination procedure for $S^\infty W$, all non-monotone strategies will be deleted after one round of elimination of weakly dominated strategy followed by another of round elimination of strictly dominated strategy. When applying iterative conditional dominance, all surviving strategies are monotone after two rounds of elimination.

1.4.2.2 Increasing Absolute Differences over Distributions

In what follows, we take an alternative approach by introducing a condition on the sender's payoff that can guarantee the monotonicity of equilibria irrespective of the receiver's payoff. Unlike the previous subsection, we allow the order on A_2 to be incomplete. As illustrated in Example 1.2, the main obstacle against equilibrium monotonicity is the lack of a complete order on $\Delta(A_2)$. We introduce the following *increasing absolute differences over distributions* condition (IADD) on the sender's payoff under which a complete order on $\Delta(A_2)$ can be constructed endogenously.

Definition 1.6 (Increasing Absolute Differences over Distributions)**.** *The sender's payoff satisfies IADD if for every* $a_1 \in A_1$ *and every* $\alpha_2, \alpha_2' \in \Delta(A_2)$*, we have* $u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_1, \alpha_2')$

[19] A more general version of the QPP property when A_2 is any subset of $\mathbb{R}$ is introduced and characterized by Choi and Smith (2017). In the case where A_2 is finite, our condition is equivalent to a strict version of theirs.

[20] $S^\infty W$ is the solution concept when applying one round elimination of weakly dominated strategies followed by iterative elimination of strictly dominated strategies. Dekel and Fudenberg (1990) show that it characterizes the set of rationalizable strategies when players entertain small amount of uncertainty about their opponents' payoffs. Shimoji and Watson (1998) show that iterative conditional dominance generalizes rationalizability in normal form games to extensive form games.

being either increasing in θ and non-negative for all $\theta \in \Theta$, or decreasing in θ and non-positive for all $\theta \in \Theta$.

To make sense of the terminology, note that IADD implies that the absolute value of the difference $u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_2, \alpha_2')$ is increasing in θ.[21] Intuitively, if u_1 satisfies IADD, then for every $a_1 \in A_1$, there exists a complete ordinal preference on $\Delta(A_2)$ (denoted by $\succsim_{a_1}$) that is shared among all types of senders. In addition, this ordinal ranking coincides with the one based on the intensity of preferences. In other words, if $\alpha_2 \succsim_{a_1} \alpha_2'$, then the difference in the sender's payoffs between (a_1, α_2) and (a_1, α_2') must be increasing in θ. This leads to our last monotonicity result:

Theorem 1.3. *If u_1 is monotone-supermodular and satisfies IADD, then every equilibrium is monotone.*

Proof. Suppose towards a contradiction that in some equilibrium (σ_1, σ_2), there exist $\theta \succ \theta'$ and $a_1 \succ a_1'$ such that $\sigma_1^{\theta}(a_1') > 0$ and $\sigma_1^{\theta'}(a_1) > 0$. Let $\alpha_2 \equiv \sigma_2(a_1)$, $\alpha_2' \equiv \sigma_2(a_1')$ with $\alpha_2, \alpha_2' \in \Delta(A_2)$. Since type θ prefers (a_1', α_2') to (a_1, α_2), and type θ' prefers (a_1, α_2) to (a_1', α_2'), we have:

$$u_1(\theta, a_1', \alpha_2') \geq u_1(\theta, a_1, \alpha_2) \tag{1.9}$$

and

$$u_1(\theta', a_1, \alpha_2) \geq u_1(\theta', a_1', \alpha_2'). \tag{1.10}$$

Since u_1 is strictly decreasing in a_1, we have $u_1(\theta', a_1', \alpha_2) > u_1(\theta', a_1, \alpha_2)$. Inequality (1.10) then implies that $u_1(\theta', a_1', \alpha_2) > u_1(\theta', a_1', \alpha_2')$. Applying (1.9) and (1.10) we have:

$$u_1(\theta, a_1', \alpha_2') - u_1(\theta, a_1, \alpha_2) \geq u_1(\theta', a_1', \alpha_2') - u_1(\theta', a_1, \alpha_2). \tag{1.11}$$

Meanwhile, note that

$$u_1(\cdot, a_1', \alpha_2') - u_1(\cdot, a_1, \alpha_2) = u_1(\cdot, a_1', \alpha_2') - u_1(\cdot, a_1', \alpha_2) + u_1(\cdot, a_1', \alpha_2) - u_1(\cdot, a_1, \alpha_2).$$

Since u_1 exhibits strictly increasing differences between θ and a_1, we have:

$$u_1(\theta, a_1', \alpha_2) - u_1(\theta, a_1, \alpha_2) < u_1(\theta', a_1', \alpha_2) - u_1(\theta', a_1, \alpha_2). \tag{1.12}$$

In addition, IADD and $u_1(\theta', a_1', \alpha_2) - u_1(\theta', a_1', \alpha_2') > 0$ imply that:

$$u_1(\theta, a_1', \alpha_2') - u_1(\theta, a_1', \alpha_2) \leq u_1(\theta', a_1', \alpha_2') - u_1(\theta', a_1', \alpha_2). \tag{1.13}$$

Summing up (1.12) and (1.13), we obtain a contradiction against (1.11). Therefore, every equilibrium must be monotone. □

As a remark, since the order on $\Delta(A_2)$ can be constructed endogenously under IADD,

[21] IADD is also necessary for $\left|u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_2, \alpha_2')\right|$ to be increasing in θ when Θ is a continuum and u_1 is a continuous function of θ.

our result does not rely on the pre-specified order on A_2, nor does it rely on the monotone-supermodularity condition on u_1 with respect to a_2. In fact, it is clear from the above proof that once u_1 satisfies IADD, all equilibria are monotone if u_1 is strictly decreasing in a_1 and exhibits strictly increasing differences in (θ, a_1).

Furthermore, since the proof of Theorem 1.3 makes no reference to the receiver's rationality and incentives, it possesses the same robustness properties as Theorem 1.1. That is, all ex ante rationalizable strategies of the sender must also be monotone. This monotonicity result continues to hold when the receiver has private information about his payoff, when the sender is signalling to a population of receivers with heterogeneous preferences, etc. In addition, Theorem 1.3 immediately extends to cases where A_2 is infinite as the cardinality of A_2 plays no role in the above proof. Finally, extensions of Theorem 1.3 to cases where Θ and A_1 are infinite are subject to the same cautions mentioned in subsection 1.4.1 and Appendix A.4, with the main issue being that the support of the sender's strategy may include suboptimal actions that are played with zero probability.

In order to apply Theorem 1.3, it is necessary to verify whether IADD is satisfied. To facilitate this process, we fully characterize the functional form restrictions of IADD in the following proposition:

Proposition 1.1. *u_1 satisfies IADD if and only if there exist functions $f : A_1 \times A_2 \to \mathbb{R}$, $v : \Theta \times A_1 \to \mathbb{R}_+$ and $c : \Theta \times A_1 \to \mathbb{R}$ with $v(\theta, a_1)$ increasing in θ, such that:*

$$u_1(\theta, a_1, a_2) = f(a_1, a_2)v(\theta, a_1) + c(\theta, a_1). \tag{1.14}$$

Proof. See Appendix A.1. □

Remark on IADD. IADD enables us to construct endogenous orders on $\Delta(A_2)$. Nevertheless, as shown in Kartik et al. (2018) and Kushnir and Liu (2018), there are other conditions on players' payoff functions under which we could obtain a complete order over distributions. These include the *single-crossing expectational differences* (SCED) and the *monotone expectational differences* (MED) in Kartik et al. (2018), and the *increasing differences over distributions* (IDD) in Kushnir and Liu (2018).

When applied to the same probability space, IADD is more demanding than MED and SCED. This is because, for example, IADD on $\Delta(A_2)$ requires that (i) $u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_2, \alpha_2')$ does not change sign when we vary θ and (ii) its absolute value is increasing in θ. These together imply that the expected difference $u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_2, \alpha_2')$ is monotone in θ.[22]

However, as shown in subsection 1.4.3, neither MED, SCED nor IDD on $\Delta(A_2)$ is sufficient for our purpose. It is also important to note that IADD on $\Delta(A_2)$ neither implies nor is implied by MED or SCED on $\Delta(A_1 \times A_2)$. We will further elaborate on this in the context of education

[22] In our signalling game context, IDD on $\Delta(A_2)$ would require that for every $a_1 \in A_1$ and every $\alpha_2, \alpha_2' \in \Delta(A_2)$, the expected payoff differences $u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_1, \alpha_2')$ is either strictly increasing, strictly decreasing, or constant in θ. In contrast, IADD only implies that these differences are either increasing or decreasing. Thus, in general Kushnir and Liu (2018)'s IDD is only implied by the strict version of our IADD (i.e. for every $a_1 \in A_1$ and $\alpha_2, \alpha_2' \in \Delta(A_2)$, $|u_2(\theta, a_1, \alpha_2) - u_2(\theta, a_1, \alpha_2')|$ is either constant of strictly increasing in θ).

signalling (section 1.5.2), which helps clarify the novel implications of Theorem 1.3 compared to a related result in Kartik et al. (2018).

1.4.3 Discussion

In this subsection, we argue that the monotone-supermodular condition on the sender's payoff plays an indispensable role in our analysis. In particular, we will show that neither the monotoncitiy nor the supermodularity part of the condition can be replaced by other appealing alternatives.

1.4.3.1 Alternative Monotonicity Conditions

One may conjecture that the existence of non-monotone equilibria (e.g. the one in Example 1.1) is driven by the state dependence of the sender's *ordinal* preferences over a_2, or whether we could modify the monotonicity assumption on the sender's payoff by letting it to be strictly increasing (or strictly decreasing) in both a_1 and a_2. However, the following counterexample suggests that these conjectures are not true.

Example 1.3. Consider the following $2 \times 2 \times 2$ game:

$\theta = \theta_1$	h	l
H	$2, 2$	$0, 0$
L	$1, 1$	$-1/2, 0$

$\theta = \theta_0$	h	l
H	$1/4, -1/2$	$1/8, 0$
L	$0, -1$	$-1/16, 1/4$

One can verify that according to the order $\theta_1 \succ \theta_0$, $H \succ L$ and $h \succ l$, the sender's payoff satisfies the generalized Spence-Mirrlees condition. Moreover, as in Example 1.1, both u_1 and u_2 are supermodular functions of the triple (θ, a_1, a_2). Different from Example 1.1, the sender's ordinal preferences over a_1 and a_2 are state independent. In particular, the sender's payoff is strictly increasing in both a_1 and a_2. However, her cardinal preferences over the receiver's actions still depend on the state. As a result, there exists a non-monotone equilibrium in which type θ_1 plays L, type θ_0 plays H, and the receiver plays h after observing L and plays l after observing H. One can also construct similar counterexamples in which the sender's payoff exhibits strictly increasing differences in (θ, a_1), increasing differences in (θ, a_2) but is strictly decreasing in both a_1 and a_2.

1.4.3.2 Single-Crossing Differences vs. Increasing Differences

In this part, we show that our strictly increasing difference condition on u_1 cannot be replaced with the *strict single-crossing difference property* in Milgrom and Shannon (1994), which is well-known in the monotone comparative statics literature.

Definition 1.7. *The sender's payoff has strict single-crossing differences (SSCD) if for every* $\theta \succ \theta'$ *and every* $(a_1, a_2) \succ (a_1', a_2')$, $u_1(\theta', a_1, a_2) - u_1(\theta', a_1', a_2') \geq 0$ *implies that* $u_1(\theta, a_1, a_2) - u_1(\theta, a_1', a_2') > 0$.

By definition, SSCD is a weaker property than strictly increasing differences. The following example shows that SSCD is not sufficient for guaranteeing the monotonicity of equilibria in signalling games, even when the sender's payoff satisfies our monotonicity condition.

Example 1.4. Consider the following $2 \times 2 \times 2$ game:

$\theta = \theta_1$	h	l
H	$1, 2$	$-3, 0$
L	$3, 1$	$-1, 0$

$\theta = \theta_0$	h	l
H	$1, 0$	$-2, 0$
L	$2, -1$	$-1, 0$

Consider the orders $\theta_1 \succ \theta_0$, $H \succ L$ and $h \succ l$. One can check that, first, u_1 is strictly increasing in a_2 and is strictly decreasing in a_1. Second, u_1 has SSCD although it fails to have increasing differences. Let $\alpha_2 \equiv \frac{2}{3}h + \frac{1}{3}l$ and $\alpha_2' \equiv \frac{1}{3}h + \frac{2}{3}l$ be two mixed actions of the receiver, we have:

$$u_1(\theta_0, h, \alpha_2) - u_1(\theta_0, l, \alpha_2') = 0 > -\frac{2}{3} = u_1(\theta_1, h, \alpha_2) - u_1(\theta_1, l, \alpha_2'). \tag{1.15}$$

When the receiver's prior belief attaches probability $1/3$ to state θ_1, one can proceed to construct the following non-monotone equilibrium: Type θ_1 plays L for sure, type θ_0 plays H and L each with probability $1/2$, the receiver plays α_2 after observing H and α_2' after observing L.

In the above example, the receiver's best reply to the sender's action is mixed. SSCD only requires that $u_1(\theta, a_1, a_2) - u_1(\theta, a_1', a_2')$ has the strict single-crossing property for every pair of *pure* action profiles that can be ranked. This does not imply that $u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_1', \alpha_2')$ is also strict single-crossing for every $(a_1, \alpha_2), (a_1', \alpha_2') \in A_1 \times \Delta(A_2)$ with $a_1 \succ a_1'$ and α_2 FOSDs α_2'.[23] This leaves open the possibility of (1.15), which leads to the existence of non-monotone equilibria.

1.5 Applications

In this section, we revisit two classic applications of signalling games in industrial organization and labor economics. We apply our sufficient conditions to establish the monotonicity of all equilibria in these games and discuss the relationships between our results and the existing ones in the literature.

1.5.1 Advertising and Warranty Provision

Consider a firm (sender) selling products to a consumer (receiver). Let $\theta \in \Theta \subset \mathbb{R}$ be the product's quality, which is the firm's private information. For simplicity, we assume that the per unit sales price is exogenous, which is normalized to 1. Every product sold has a positive probability of breakdown, which depends on its quality. The firm chooses a 3-dimensional action: $a_1 \equiv (a_1^{ad}, a_1^{len}, a_1^{re}) \in A_1 \subset \mathbb{R}_+ \times \mathbb{R}_+ \times [0, 1]$, where a_1^{ad} is the intensity of advertising, a_1^{len} is the length of warranty, and a_1^{re} is the (per unit) refund the firm commits to pay if the product breaks

[23] In fact, since $|A_1| = |A_2| = 2$ in this example, the payoff function u_1 also has SCED on both $\Delta(A_1)$ and $\Delta(A_2)$ (Kartik et al., 2018). However, it does not have SCED on the larger space $\Delta(A_1 \times A_2)$.

down during the length of the warranty. The consumer chooses how many units to purchase after observing a_1, which is denoted by $a_2 \in A_2 \subset \mathbb{N}$.

Our monotone-supermodular condition requires that (i) u_1 is strictly decreasing in the triple $(a_1^{ad}, a_1^{len}, a_1^{re})$ and is strictly increasing in a_2, and (ii) u_1 has strictly increasing differences in (θ, a_1^{ad}), (θ, a_1^{len}) and (θ, a_1^{re}), and increasing differences in (θ, a_2). The first part of the monotonicity requirement is most straightforward, as advertising, providing lengthier warranty and more refund are all costly for the firm. Monotonicity also requires that, keeping other factors fixed, the firm's profit is higher when the consumer purchases larger quantities.[24]

Next, we justify the supermodular part. First, there are complementarities between θ and a_1^{ad} when the cost of promoting a good product is lower than the cost of promoting a bad one. This can be driven by regulation policies, reputation concerns, umbrella branding (Wernerfelt, 1988), etc. Second, there are complementarities between θ and a_1^{re} when higher quality product has lower probability of breakdown, so therefore, committing to a higher per unit refund is less costly. Similarly, the firm's per unit profit (defined as sales price minus expected refund payment) is strictly increasing in the product's quality, and therefore, u_1 has strictly increasing differences in (θ, a_2). Finally, there are complementarities between θ and a_1^{len} when breakdown arrives according to a time homogeneous Poisson process with intensity strictly decreasing in the product's quality (Gal-Or, 1989).

As the firm's payoff is monotone-supermodular, it will use a monotone strategy in every equilibrium when the consumer has unit demand (Theorem 1.1), when the consumer faces decreasing marginal returns to quantities (Theorem 1.2), or when its payoff takes the following functional form (Theorem 1.3):

$$u_1(\theta, a_1, a_2) = \Big(1 - \underbrace{g(\theta, a_1^{len})}_{\substack{\text{prob. of breakdown} \\ \text{within } a_1^{len} \text{ periods}}} \cdot \overbrace{a_1^{re}}^{\text{per unit refund}} + f(\theta)\Big)\, a_2 - \underbrace{c(\theta, a_1^{ad})}_{\text{cost of advertising}}, \tag{1.16}$$

where

(i) $g : \Theta \times \mathbb{R}_+ \to [0, 1]$ is strictly decreasing in θ, strictly increasing in a_1^{len} and exhibits strictly decreasing differences in (θ, a_1^{len}),

(ii) $f : \Theta \to \mathbb{R}_+$ is strictly increasing, which captures the firm's benefit from initial sales beyond the sales price in a reduced form,[25] and

(iii) $c : \Theta \times \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing in a_1^{ad} and exhibits strictly decreasing differences in (θ, a_1^{ad}).

1.5.2 Education Signalling with Vertically Differentiate Jobs

Consider the following variant of the Spence (1973) education signalling model. Let $\theta \in \Theta$ be the talent of the worker, $a_1 \in A_1$ be the education the worker receives, and $a_2 \in A_2$ be the job

[24] This rules out cases in which the low quality seller will lose money when offering the equilibrium warranty/refund policy of the high quality seller. Nevertheless, it still fits into a number of cases of economic interest.

[25] This is relevant when the product is a newly introduced experience good Milgrom and Roberts (1986a); Nelson (1974). Nevertheless, the absence of $f(\theta)$ will not affect the applicability of our monotonicity result.

offered by the employer after he observes a_1.

The sender's payoff being monotone-supermodular implies that (i) u_1 is strictly decreasing in a_1 and strictly increasing in a_2, (ii) u_1 exhibits strictly increasing differences between θ and (a_1, a_2). The monotonicity assumption requires that receiving education is costly, and the jobs are *vertically* differentiated so that every worker prefers a higher level job.[26] For the supermodularity assumption, first, u_1 exhibits strictly increasing differences in (θ, a_1) when receiving education is less costly for more talented workers (Spence, 1973). Second, u_1 exhibits strictly increasing differences in (θ, a_2) when the returns from a higher level job (relative to a lower level one) increases with the worker's talent, which is a well-established fact in the personnel economics literature.[27]

When the worker's payoff is monotone-supermodular, more talented workers receive more education in every equilibrium when there are only two jobs (Theorem 1.1). If the employer's payoff function is strictly concave in a_2,[28] then Theorem 1.2 guarantees the monotonicity of all equilibria even when there are three or more jobs. Alternatively, suppose the worker's payoff function takes the following form:

$$u_1(\theta, a_1, a_2) = \underbrace{f(\theta, a_1)g(a_2)}_{\text{worker's return from job assignment}} - \underbrace{c(\theta, a_1)}_{\text{cost of education}}, \tag{1.17}$$

where $f : \Theta \times A_1 \to \mathbb{R}_+$ is strictly increasing in θ, and $g : A_2 \to \mathbb{R}$ and $c : \Theta \times A_1 \to \mathbb{R}$ are functions compatible with u_1 being strictly decreasing in a_1 and exhibiting strictly increasing differences in (θ, a_1). Intuitively, this means that the net (material) return from education is always negative, but it is strictly increasing with respect to the sender's type. Compared to the separable payoff function studied in Spence (1973), (1.17) allows the worker's return from a job offer to depend non-trivially on her type and education. Despite the non-separability of the sender's payoff function, Theorem 1.3 implies that every equilibrium is monotone regardless of how the employer evaluates various matches between jobs, talent and education.

Kartik et al. (2018) study a similar application, with $a_2 \in \mathbb{R}_+$ being the wage offered by the firm. They show that if the worker's payoff function has SCED over $\Delta(A_1 \times A_2)$, then every equilibrium of this game is monotone. According to their characterization result, u_1 has SCED over $\Delta(A_1 \times A_2)$ if and only if it takes the following functional form:

$$u_1(\theta, a_1, a_2) = g_1(a_1, a_2)f_1(\theta) + g_2(a_1, a_2)f_2(\theta) + h(\theta), \tag{1.18}$$

where both f_1 and f_2 are single-crossing functions that satisfy a ratio-ordered condition.[29] Their

[26]If jobs are instead horizontally differentiated (Roy, 1951), then the resulting payoff structure resembles Example 1.1, in which we have shown that non-monotone equilibria can exist.

[27]See for example, Waldman (1984) and Gibbons and Waldman (1999). Different types of workers receive different returns to a higher level job can be due to, for example, that the talent of a worker affects her prospects of promotion, the expected compensation she receives under piece-rate incentive schemes, etc.

[28]Concavity of u_2 is satisfied if for every $(\theta, a_1) \in \Theta \times A_1$, there exists an ideal job assignment $a_2^*(\theta, a_1)$ that maximizes $u_2(\theta, a_1, \cdot)$, and the employer incurs a quadratic loss when there is a mismatch between talent and jobs.

[29]According to Kartik et al. (2018), for two single-crossing functions $f_1, f_2 : \Theta \to \mathbb{R}$, f_1 ratio dominates f_2 if (i) $\forall \theta \succeq \theta'$, $f_1(\theta')f_2(\theta) \leq f_1(\theta)f_2(\theta')$, and (ii) $\forall \theta \succeq \hat{\theta} \succeq \theta'$, $f_1(\theta')f_2(\theta) = f_1(\theta)f_2(\theta')$ if and only if $f_1(\theta')f_2(\hat{\theta}) = f_1(\hat{\theta})f_2(\theta')$ and $f_1(\hat{\theta})f_2(\theta) = f_1(\theta)f_2(\hat{\theta})$. Functions f_1 and f_2 are ratio-ordered if either f_1 ratio

results provide insights on cheap talk games and education signalling games when the receiver's payoffs are unknown to the sender. However, their results are not applicable to education signalling games where the worker's payoff is given by (1.17) and $c(\theta, a_1)$ cannot be written as the product of two functions $c_1(\theta)$ and $c_2(a_1)$.[30] Our Theorem 1.3 accommodates these cases and implies the monotonicity of all equilibria.

1.6 Conclusion

This paper makes two main contributions. First, we show that equilibrium monotonicity does not follow from the Spence-Mirrlees condition nor is it implied by the complementarities in players' payoff functions. Our counterexamples are robust against equilibrium refinements and highlight the problems that can arise when the sender's returns from the receiver's action depend on her type. Second, we provide sufficient conditions under which all Nash equilibria are monotone. These conditions are easy to verify and fit into a number of applications, including advertising, warranty provision, education and job assignment, etc. In these scenarios, our results imply that it is without loss of generality to focus on monotone equilibria.

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dominates f_2 or f_2 ratio dominates f_1.

[30] The cost function is not multiplicative separable in applications when $c(\theta, a_1) = k(\theta)a_1 + t(a_1)$, with $t(a_1)$ being a fixed cost, interpreted as the cost of tuition, and $k(\theta)a_1$ being a variable cost which depends on the worker's talent.

2 Confusion, Indecisiveness and Polarization[1]

Joint with Andreas Hefti and Armin Schmutzler

2.1 Introduction

Some of the most important consumption decisions are inherently complex. For instance, when faced with the choice between two different cars, smartphones or insurance contracts, consumers often have a hard time figuring out which alternative they prefer. Assessing the difference in the monetary value of two goods is an even more challenging task. For complex consumption decisions, it therefore appears reasonable to assume that consumers' judgments are noisy at best.

The extent of such noise in consumer decisions is typically not an entirely exogenous characteristic of the goods under consideration. On the one hand, firms can engage in measures to educate consumers. They can describe the products' properties in a transparent fashion and discuss the exact needs of consumers with them. On the other hand, firms can also deliberately confuse consumers. For instance, insurance companies may write contracts in such a way that comparison becomes difficult. Smartphone manufacturers may add features with unclear value to their products. More generally, when advertising differentiated products, firms may emphasize irrelevant product details rather than those characteristics that really matter for consumer valuations.

This paper asks whether firms want to *educate* consumers or whether instead they want to engage in *obfuscation* activities to confuse consumers. The special case of homogeneous goods might suggest that firms' incentives are clear-cut. Oligopolistic producers of such goods suffer from the temptation to undercut each others' prices, resulting in a zero-profit equilibrium under well-known conditions.[2] To alleviate this problem, firms may seek to reduce the undercutting temptation. With homogeneous goods, obfuscation may allow competitors to escape the "Bertrand trap" by reducing market transparency. The literature has made this point in several variants, but the bottom line is that producers of homogeneous goods can obtain positive profits by confusing consumers, even when this would otherwise be impossible.[3]

While the case of homogeneous goods is an important theoretical benchmark, in many industries firms offer differentiated products to cater to the needs of heterogeneous consumers.

[1] This paper should be cited as Hefti, A., S. Liu and A. Schmutzler (2019): "Confusion, Indecisiveness and Polarization," Mimeo.

[2] Such an equilibrium arises, e.g., if the following conditions hold simultaneously: static interaction, identical and constant marginal costs, no capacity constraints; see, e.g., Tirole (1988).

[3] For instance, firms can benefit by using hidden fees (Gabaix and Laibson, 2006; Heidhues et al., 2016), spurious differentiation resulting from the credulity of consumers (Spiegler, 2006), complex price formats (Carlin, 2009; Chioveanu and Zhou, 2013; Piccione and Spiegler, 2012), intransparent webpages (Ellison and Ellison, 2009), or more generally from increasing consumer search costs (Ellison and Wolitzky, 2012).

In such environments, the role of obfuscation is more subtle. On the one hand, the scope for confusion is larger than with homogeneous goods. For example, there can be many ways to present the differences between products, and the dimensions that firms emphasize are likely to influence the perceived valuations. On the other hand, the incentives to confuse consumers are less obvious. Firms usually obtain positive profits in differentiated markets even without obfuscation. It therefore is possible in principle that, by blurring the perception of consumers, obfuscation reduces rather than increases profits. It could thus potentially be in the interest of firms to educate consumers.

We seek to identify conditions under which consumer confusion arises in markets with differentiated products. To this end, we study a duopoly framework, where the population of consumers is characterized by a distribution of valuations for the two goods with an arbitrary correlation structure, encompassing a wide range of discrete choice models. The two firms first decide on their marketing activities. Thereafter they compete in prices. Finally consumers choose which product to buy. Firms can choose their marketing activities from an exogenously given set of options. The activities jointly determine the noise in consumer perceptions, thereby resulting in a distribution of perceived valuations in the consumer population that may differ from the true valuation distribution.[4] We abstract from any cost heterogeneity between different activities. As we are asking whether and how firms want to influence the noise in consumer decisions, we assume that the stochastic perturbations do not bias valuations systematically. More precisely, we assume that marketing does not affect the expected valuation *differences*.[5] In this sense, firms cannot systematically fool consumers.

Our main result establishes that both confusion and education can be equilibrium phenomena, with the outcome depending on the true valuation distribution. Consumer confusion arises if the distribution of perceived valuation differences and the distribution of true valuation differences do not coincide. Any marketing profile that induces confusion is called *obfuscating*, and any profile inducing (or restoring) the true distribution is called *educating*. Our analysis identifies simple properties of the true preference distribution determining whether firms will engage in obfuscation activities. We distinguish between *indecisive* preferences, for which, roughly speaking, indifferent consumers are relatively common and *polarized* preferences, for which indifferent consumers are relatively rare. For instance, in the standard textbook Hotelling model, the former (latter) case arises when the density of the consumer distribution has a maximum (minimum) in the middle of the interval.[6] To illustrate the relevant properties of the taste distribution, consider for instance the hospitality industry. It is hard to imagine that a guest will be indifferent when faced with the choice between a “family” hotel and a “business” hotel – instead, most consumers will clearly prefer one alternative over the other, resulting in a polarized distribution. However, if the comparison is between two business hotels, the situation may be better described by a substantial amount of indecisive consumers.

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[4] As we will discuss in Section 2.4.1, our analysis is thus related to Johnson and Myatt (2006) who consider a monopolist’s incentive to influence valuation distributions

[5] From a theoretical perspective, unbiasedness can be seen as playing a similar disciplining role in our (non-Bayesian) analysis as the assumption of “conformity with the prior” in models of persuasion (Kamenica and Gentzkow, 2011) or costly information acquisition (Caplin and Dean, 2015)

[6] For definiteness, think of a situation with firms located at the ends of a compact interval that is symmetric around the mid-point; moreover, take transportation costs to be linear in distance.

Our first result assumes that firms' obfuscation possibilities are constrained by the degree of true taste differentiation.[7] In this case, we find that consumer confusion arises in equilibrium if preferences are indecisive, whereas it does not arise if preferences are polarized. Our second result requires more structure on the set of feasible marketing activities. If firms' marketing activities can be partially ordered by the amount of confusion generated (e.g., by a mean-preserving spread of the noise distribution), then with indecisive tastes, the unique equilibrium features maximal consumer confusion. By contrast, full education arises as the unique equilibrium outcome for polarized tastes.

For the third result, we allow that the marketing tools can be so powerful that even the most loyal consumer of a firm can be confused enough to perceive the other firm's good as better, which seems plausible if there is only very little dispersion in the true tastes. When such "massive" confusion is possible, a U-shaped relation between confusion and firm payoffs results if tastes are polarized. We show that confusion can then arise in equilibrium despite polarized tastes. In particular, in the limit case of homogeneous goods, true taste differentiation is negligible, so that obfuscation is an equilibrium phenomenon independently of the shape of the taste distribution.

Our results shed lights on the advertising literature which has discussed firms' incentives to engage in informative advertising. Interpreting a reduction in the noise of relative valuations as informative advertising, we see that the preference distribution determines whether such advertising arises as an equilibrium phenomenon.

The welfare analysis of obfuscation differs from the case with homogeneous goods, for which, in the absence of binding outside options, the main effect is redistribution from consumers to firms. With differentiated goods, whenever firms choose to obfuscate, this not only increases prices, but it also leads to a mismatch between consumers and products. In addition, with a strictly binding outside option, obfuscation can make some consumers inefficiently opt out of the market.

The general logic of our model applies beyond the oligopoly setting, for instance, to competition between candidates for voters. Candidates can choose how much information to provide to voters about their platforms. In addition, they can engage in other measures to convince voters, such as promises and campaigning efforts. Promises are costly only if the candidate wins the election; efforts are costly even when she does not. In the former case, our oligopoly model applies directly. The latter case requires modifications of the setting, as unconditional efforts lead to a contest structure. In both situations, we find that candidates will want to engage in obfuscation activities only if the voter preference distribution displays indecisiveness. Intuitively, with indecisive preferences obfuscation distorts the preference distribution so that it becomes more polarized. This reduces the necessary efforts of the candidates when competing for voters.

The paper is organized as follows. Section 2.2 introduces the model, Section 2.3 presents the main results. We discuss several aspects of our framework in Section 2.4. Section 2.5 contains the application to political economy. The related literature is discussed in Section 2.6, and Section 2.7 concludes. All proofs are relegated to Appendix B.

[7] More precisely, no matter which marketing activities the firms choose and what the realization of the noise term is, for each firm there exist consumers who will not buy from the competitor at equal prices.

2.2 The Model

Consider a duopoly where each firm $i = 1, 2$ produces a good at zero marginal cost. There is a unit mass of consumers. Each consumer has a *true* valuation $v_i \in \mathbb{R}$ for the good produced by firm i. The true valuations (v_1, v_2) of the consumers are drawn according to a joint distribution function $F_0 : \mathbb{R}^2 \to [0, 1]$. The firms play a two-stage complete information game. In the first stage, they simultaneously choose marketing activities $a_i \in A$. The marketing profile $\mathbf{a} = (a_1, a_2) \in \mathcal{A} \equiv A \times A$ determines a distribution function $F_{\mathbf{a}} : \mathbb{R}^2 \to [0, 1]$, from which the *perceived* valuations $(\tilde{v}_1, \tilde{v}_2)$ of the consumers are drawn. In the second stage, firms observe the chosen marketing profile $\mathbf{a}$ and infer the resulting distribution $F_{\mathbf{a}}$, after which they simultaneously choose prices $p_i \in \mathbb{R}_+$. The chosen prices (p_1, p_2) and the realizations of the perceived valuations $(\tilde{v}_1, \tilde{v}_2)$ then determine the consumption choices of the consumers, and thus also the profits of the firms. Throughout the main analysis, we assume there is no outside option (or it is not binding if there is one).[8] Accordingly, a consumer will acquire the alternative with the highest perceived net utility $\tilde{u}_i = \tilde{v}_i - p_i$, $i \in \{1, 2\}$. Thus, the distribution of perceived valuation differences $\tilde{v}_\Delta \equiv \tilde{v}_2 - \tilde{v}_1$ suffices for studying the model.

For every $\mathbf{a} \in \mathcal{A}$, the marketing activities affect the perceived valuation differences according to

$$\tilde{v}_\Delta = v_\Delta + \varepsilon_{\mathbf{a}}, \tag{2.1}$$

where the random variables $\varepsilon_{\mathbf{a}}$ and $v_\Delta = v_2 - v_1$ are independently distributed. Let $G_0 : \mathbb{R} \to [0, 1]$ and $\Gamma_{\mathbf{a}} : \mathbb{R} \to [0, 1]$ be the distribution functions of v_Δ and $\varepsilon_{\mathbf{a}}$, respectively. With (2.1), the distribution function of $\tilde{v}_\Delta$, which we denote by $G_{\mathbf{a}}$, is then induced by G_0 and $\Gamma_{\mathbf{a}}$ according to the convolution

$$G_{\mathbf{a}}(x) = \int_{-\infty}^{+\infty} G_0(x - \varepsilon) d\Gamma_{\mathbf{a}}(\varepsilon), \ \forall x \in \mathbb{R}.$$

For given $(p_1, p_2) \in \mathbb{R}_+^2$ and $\mathbf{a} \in \mathcal{A}$, since $\Pr(\tilde{v}_\Delta \leq p_2 - p_1) = G_{\mathbf{a}}(p_2 - p_1)$, the expected demands of firms 1 and 2 are $D_1(p_1, p_2; \mathbf{a}) = G_{\mathbf{a}}(p_2 - p_1)$ and $D_2(p_1, p_2; \mathbf{a}) \equiv 1 - G_{\mathbf{a}}(p_2 - p_1)$, respectively. Further, the expected profits of the firms, which they aim to maximize, are $\Pi_i(p_1, p_2; \mathbf{a}) = p_i D_i(p_1, p_2; \mathbf{a})$, $\forall i = 1, 2$.

Main assumptions We say that a random variable X is symmetric at zero if its distribution function $G : \mathbb{R} \to [0, 1]$ satisfies either (i) $G(x) = 1 - G(-x)$ $\forall x \in \mathbb{R}$, or (ii) $G(x) = 1$ if $x \geq 0$ and $G(x) = 0$ otherwise. Further, if case (ii) holds, we say that X is degenerate at zero, and we denote such a random variable by O. If X admits a density function $g : \mathbb{R} \to \mathbb{R}_+$, we also use $supp(g)$ to denote its support. We are now ready to state the main assumptions on the variables of interests, v_Δ and $\varepsilon_{\mathbf{a}}$.

Assumption 2.1 (Distributional assumptions)**.** *The following conditions are satisfied:*

(A2.1.1) v_Δ *is symmetric at zero and has a density* g_0.

[8] This assumption simplifies the analysis but does not alter the qualitative nature of our main results. See Section 2.3.5 for an extension with outside options.

(A2.1.2) G_0 *is log-concave on supp*(g_0).

(A2.1.3) g_0 *is continuous at zero and* $g_0(0) > 0$.

(A2.1.4) $\forall \mathbf{a} \in \mathcal{A}$, $\varepsilon_{\mathbf{a}}$ *is symmetric at zero.*

(A2.1.5) If $\varepsilon_{\mathbf{a}} \neq O$, $\varepsilon_{\mathbf{a}}$ *has a density* $\gamma_{\mathbf{a}}$ *that is log-concave on supp*$(\gamma_{\mathbf{a}})$.

The log-concavity conditions (A2.1.2) and (A2.1.5) assure that the first-order conditions for profit maximization in the pricing stage are sufficient. Jointly with the technical condition (A2.1.3), this is useful for establishing equilibrium existence. (A2.1.1) and (A2.1.4) are of economic importance. They state that no firm has a pre-existing systematic advantage over the other, nor can it gain such an advantage by marketing activities. We discuss the economic rationale for (A2.1.4) in Section 2.4. Further, the condition in (A2.1.1) that v_Δ has a density g_0 implies that true consumer valuations are heterogeneous, and we can view the (Lebesgue) measure of *supp*(g_0) as a measure of the degree of taste (or product) differentiation.[9]

Assumption 2.1 guarantees that a density function $g_{\mathbf{a}}$ of $\tilde{v}_\Delta$ exists in the pricing stage, given by

$$g_{\mathbf{a}}(x) = \int_{-\infty}^{\infty} g_0(x - \varepsilon) d\Gamma_{\mathbf{a}}(\varepsilon), \ \forall x \in \mathbb{R}. \tag{2.2}$$

In particular, $g_{\mathbf{a}}(x) = g_0(x)$ $\forall x \in \mathbb{R}$ if $\varepsilon_{\mathbf{a}} = O$.

2.3 Main Results

In this section, we derive the subgame perfect equilibrium (SPE) of the game. Section 2.3.1 contains the main results requiring only little structure on $\mathcal{A}$ and the marketing technology. In Section 2.3.2, we derive stronger results by imposing a partial order on $\mathcal{A}$. In Section 2.3.3, we discuss the case where consumer confusion can be arbitrarily large relative to the true taste differentiation. Finally, we provide welfare results in Section 2.3.4.

2.3.1 Endogenous Confusion and Education

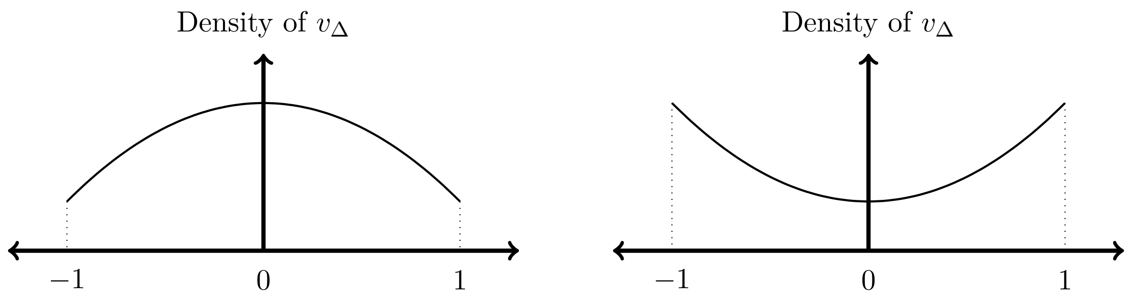
Our first result characterizes the equilibria in the second-stage pricing subgames.

Lemma 2.1. *Under Assumption 2.1 there is a unique symmetric pure-strategy equilibrium in every pricing subgame, where both firms choose the price* $p^*_{\mathbf{a}} = \frac{1}{2g_a(0)}$, $\forall \mathbf{a} \in \mathcal{A}$.

[9] It is straightforward to derive G_0 from the more primitive joint distribution F_0. For example, if F_0 has a density function f_0, G_0 can be expressed as

$$G_0(v_\Delta) = \int_{-\infty}^{+\infty} \int_{-\infty}^{v_\Delta} f_0(v, v + x) dx dv, \ \forall v_\Delta \in \mathbb{R}.$$

It is then easy to derive sufficient conditions on F_0 under which the symmetry condition (A2.1.1) holds. For example, it holds if F_0 itself is symmetric, i.e., $F_0(x, y) = F_0(y, x)$ $\forall x, y \in \mathbb{R}$. Likewise, log-concavity (A2.1.2) can be checked by standard calculus tools if F_0 has a differentiable density function; a sufficient condition is that v_1 and v_2 are independent and each is drawn from a log-concave distribution function. Alternatively, (A2.1.2) holds if F_0 is an (arbitrary) multivariate normal distribution.



(a) An indecisive distribution

(b) A polarized distribution

Figure 2.1: Examples of preference distributions, $supp(g_0) = [-1, 1]$.

As perceived valuation differences are dispersed with a zero-symmetric log-concave density $g_{\mathbf{a}}$ for any $\mathbf{a} \in \mathcal{A}$, an equilibrium existence in the pricing subgames exists. Crucially, Lemma 2.1 shows that the equilibrium price $p^*_{\mathbf{a}}$ is determined by and decreasing in $g_{\mathbf{a}}(0)$, the measure of perceptually indifferent consumers. Since higher prices correspond to higher revenues, firms prefer marketing profiles that reduce the measure of perceptually indifferent consumers. Intuitively, such marginal consumers react most sensitively to price changes. Thus, if $g_{\mathbf{a}}(0)$ is low, there are many infra-marginal customers whom the firms want to exploit, and high equilibrium prices become sustainable.

Building on the above insight, we now proceed to show that the distribution of true valuations determines whether consumer confusion arises in equilibrium. The conclusion will depend on differences in the shape of the distribution of v_Δ as depicted in the two parts of Figure 1. The following definition makes the relevant concepts precise.

Definition 2.1. *Let $\delta > 0$ be such that $[-\delta, \delta] \subset supp(g_0)$.*

(i) ***(Indecisiveness)*** *True preferences are*

(a) weakly δ-indecisive if $g_0(0) > g_0(x)\ \forall x \in [-\delta, 0) \cup (0, \delta]$,

(b) δ-indecisive if g_0 is strictly increasing (decreasing) on $[-\delta, 0]$ (on $[0, \delta]$), and

(c) strongly δ-indecisive if g_0 is strictly concave on $[-\delta, \delta]$.

(ii) ***(Polarization)*** *True preferences are*

(a) weakly δ-polarized if $g_0(0) < g_0(x)\ \forall x \in [-\delta, 0) \cup (0, \delta]$,

(b) δ-polarized if g_0 is strictly decreasing (increasing) on $[-\delta, 0]$ (on $[0, \delta]$), and

(c) strongly δ-polarized if g_0 is strictly convex on $[-\delta, \delta]$.

The consumer tastes represented in Figure 2.1 feature strongly δ-indecisive and strongly δ-polarized preferences, respectively, where $[-\delta, \delta] = supp(g_0)$. Given the zero-symmetry of v_Δ, strong δ-indecisiveness implies δ-indecisiveness, which in turn implies weak δ-indecisiveness. The difference between the latter two concepts is the monotonicity requirement in the definition of δ-indecisiveness. For δ-indecisive preferences, less pronounced valuation differences occur more frequently than more pronounced ones, while weakly δ-indecisive preferences only require that indifference ($v_\Delta = 0$) occurs more often than all other alternatives on $[-\delta, \delta]$. The relation between different concepts of polarization is similar. Our most general result (Theorem 2.1) only requires the weak notions of indecisiveness of polarization. The more restrictive concepts are useful for obtaining stronger results on equilibrium uniqueness and monotonicity (Theorem 2.2), which will be presented in the next subsection.

For expositional simplicity, in the rest of the paper we indicate marketing activities by real numbers, so that $A \subset \mathbb{R}$ and $\mathcal{A} \subset \mathbb{R}^2$. We also use the convention that $\varepsilon_{\mathbf{0}} = O$. Thus, if the marketing profile $\mathbf{a} = \mathbf{0}$ is available to and chosen by the firms, consumers will be fully educated (or the market will be transparent) in the sense that their perceived valuation differences will coincide with their true ones. We impose the following minimal structure on the set of marketing profiles $\mathcal{A}$.

Assumption 2.2. *The set $\mathcal{A}$ satisfies the following two conditions:*

(A2.2.1) $\mathbf{0} \in \mathcal{A}$.

(A2.2.2) $\forall i = 1, 2$, $j \neq i$ *and* $\forall a_j \in A$, $\exists\, a_i \in A$, *such that* $\varepsilon_{(a_i, a_j)} \neq O$.

(A2.2.1) assures that full consumer education is among the feasible options. (A2.2.2) states that each firm can induce some consumer confusion unilaterally. Thus, Assumption 2.2 implies that, while a transparent market is a possible outcome, it cannot be enforced unilaterally. We are now ready to state our main result.

Theorem 2.1. *Suppose that Assumptions 2.1 and 2.2 hold.*

(i) If there exists $\delta > 0$ with supp$(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ and the true preferences are weakly δ-polarized, then an SPE without consumer confusion exists.

(ii) If there exists $\delta > 0$ with supp$(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ and the true preferences are weakly δ-indecisive, then no SPE without consumer confusion exists.

Even with a small degree of taste differentiation, full consumer education can be sustained as an equilibrium outcome if preferences are polarized. Conversely, if preferences are indecisive, consumer confusion necessarily arises in any equilibrium even with an arbitrarily large degree of taste differentiation. As an illustration, suppose that *supp*$(\gamma_{\mathbf{a}}) \subset [-1, 1]$ $\forall \mathbf{a} \in \mathcal{A}$. The distributions g_0 and g_0' in Figure 2.2 (a) differ in their degrees of taste differentiation, but in both cases an SPE without consumer confusion, or an *education equilibrium*, exists. By contrast, g_0 and g_0' in Figure 2.2(b) have the same degree of taste differentiation, but an education equilibrium only exists in the latter case, as g_0 is polarized while g_0' is indecisive.

The rationale for Theorem 2.1 is as follows. If firms choose an obfuscating marketing profile (i.e., $\varepsilon_{\mathbf{a}} \neq O$), some truly indifferent consumers perceive one good as strictly superior, while some consumers who strictly prefer one good over the other become indifferent. By Lemma 1, firms benefit from consumer confusion if the former effect dominates the latter. With polarized tastes, confusion breaks pre-existing allegiance with a firm, as more consumers are pushed towards

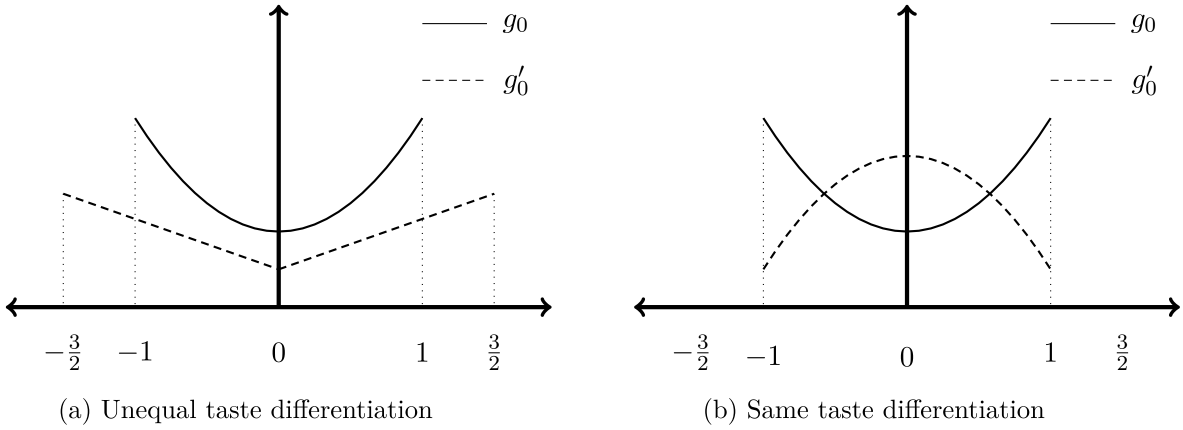


Figure 2.2: Taste differentiation and preference distribution

indifference than vice versa. Therefore, both firms have the incentive to avoid the intensified price competition caused by an obfuscated market. Because fully educating the consumers is feasible ($\mathbf{0} \in \mathcal{A}$), it must be part of an SPE. By contrast, confusion is beneficial for firms if true preferences are indecisive. In such a situation, confusion reduces the measure of marginal consumers, as more truly indifferent consumers end up perceiving one of the products as superior than vice versa. As any individual firm can always force some confusion on the market (condition (A2.2.2)), full education cannot be supported as an equilibrium outcome.

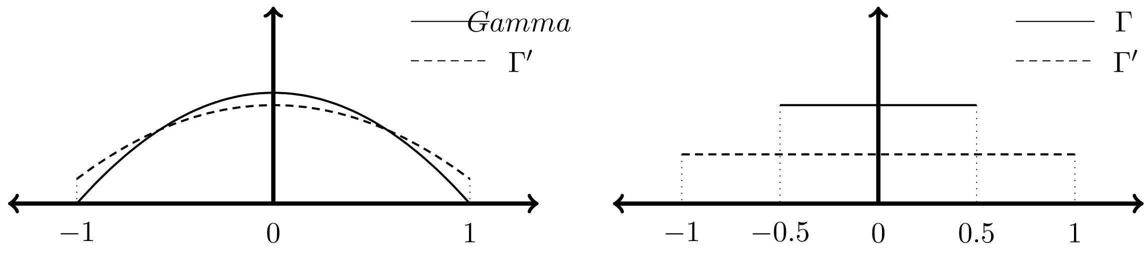
If, contradicting condition (A2.2.2), consumer education could be enforced unilaterally, then part (i) of Theorem 2.1 could even be strengthened in that any SPE involves full education. Further, there could be SPE without consumer confusion despite indecisive preferences. Specifically, if, similar to Heidhues et al. (2016), each firm can perfectly educate all consumers by choosing some marketing activity $a^e \in A$ i.e., $\varepsilon_{\mathbf{a}} = O$ if $a^e \in \{a_1, a_2\}$, then full education (with both firms choosing a^e) is always an equilibrium outcome because neither firm can unilaterally affect the distribution of perceived valuation differences. Such an education equilibrium, however, is strictly dominated by any possible SPE with consumer confusion.

2.3.2 Maximal Confusion and Education

On intuitive grounds, one should expect that firms seek to confuse consumers as much (as little) as possible if preferences are indecisive (polarized). In this section, we study this idea more formally.

We will assume that the noise distributions $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ can be partially ordered in one of two ways. The first order appeals to the familiar notion of a mean-preserving spread (MPS).[10] The second order is induced by a property which we call *sidewise single-crossing* (SSC): Let Γ, Γ' be two zero-symmetric distribution functions with supports $[-\omega, \omega]$ and $[-\omega', \omega']$, respectively, Γ is either degenerate at zero or has a density function γ, and Γ' has a density function γ'. We say

[10]Formally, for two random variables X and Y, Y is a MPS of X if Y has the same distribution as $X + \eta$, where $\eta \neq O$ and $E[\eta|X] = 0$. Intuitively, Y is a noisy version of X. Rothschild and Stiglitz (1970) show that if the involved distribution functions have a uniformly bounded support, then the MPS ordering between distributions is equivalent to the order induced by second-order stochastic dominance. Müller (1998) shows how to extend the equivalence result to the case of an unbounded support.



(a) $\Gamma' \succ_{SSC} \Gamma$, same support. (b) $\Gamma' \succ_{SSC} \Gamma$, variable support.

Figure 2.3: Examples of SSC orderings

that Γ' is *more dispersed* than Γ in the sense of sidewise single-crossing, denoted by $\Gamma' \succ_{SSC} \Gamma$, if either (i) Γ is degenerate at zero, or (ii) $\omega' \geq \omega$ and $\forall e, e' \in [0, \omega']$ with $e' > e$,

$$\gamma(e) - \gamma'(e) \geq 0 \Longrightarrow \gamma'(e') - \gamma(e') > 0. \tag{2.3}$$

In words, (2.3) requires that the two densities intersect only once in $[-\omega', 0]$ and $[0, \omega']$, respectively; see Figure 2.3 for illustrations.

Assumption 2.3. *$A \subset \mathbb{R}_+$ is compact, and $\varepsilon_{\mathbf{a}} = O$ if and only if $\mathbf{a} = \mathbf{0}$. Moreover, one of the following conditions holds:*

(A2.3.1) $\forall \mathbf{a}, \mathbf{a}' \in \mathbf{A}$ *with* $\mathbf{a} \neq \mathbf{a}'$ *and* $\mathbf{a} \leq \mathbf{a}'$, $\Gamma_{\mathbf{a}'}$ *is an MPS of* $\Gamma_{\mathbf{a}}$.

(A2.3.2) $\forall \mathbf{a}, \mathbf{a}' \in \mathbf{A}$ *with* $\mathbf{a} \neq \mathbf{a}'$ *and* $\mathbf{a} \leq \mathbf{a}'$, $\Gamma_{\mathbf{a}'} \succ_{SSC} \Gamma_{\mathbf{a}}$.

Provided that $\{0\} \subsetneq A$, Assumptions 2.3 implies Assumption 2.2.[11] Moreover, consumer confusion is maximal (minimal) in the MPS if both firms choose $\bar{a} \equiv \max A$ ($\underline{a} \equiv \min A$). Our second main result provides conditions under which such maximal/minimal consumer confusion is the unique equilibrium outcome.

Theorem 2.2. *Suppose that Assumption 2.1 holds.*

(i) If there exists $\delta > 0$ such that supp($\gamma_{\mathbf{a}}$) $\subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and either

(a) true preferences are strongly δ-indecisive and (A2.3.1) is satisfied, or

(b) true preferences are δ-indecisive and (A2.3.2) is satisfied,

then there exists a unique SPE. In equilibrium, consumer confusion is maximal.

(ii) If there exists $\delta > 0$ such that supp($\gamma_{\mathbf{a}}$) $\subset [-\delta, \delta], \forall \mathbf{a} \in \mathcal{A}$ and either

(a) true preferences are strongly δ-polarized and (A2.3.1) is satisfied, or

(b) true preferences are δ-polarized and (A2.3.2) is satisfied,

then there exists a unique SPE. In equilibrium, consumer confusion is minimal.

[11] This follows because i) $\varepsilon_{\mathbf{0}} = O$ and ii) $\varepsilon_{\mathbf{a}} \neq O$ whenever some firm chooses $a > 0$.

The conditions (a) and (b) in parts (i) and (ii) of the theorem cannot be ranked according to their generality. If the distributions $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ are ordered by the SSC criterion, they are also ordered by the MPS criterion. The converse statement does not hold. However, we only need indecisive or polarized tastes in the SSC case, while their strong counterparts are required in the MPS case.[12]

Several examples of marketing technologies illustrate Assumptions 2.2 and 2.3. Suppose that for every $\mathbf{a} \neq \mathbf{0}$, $\varepsilon_{\mathbf{a}}$ follows the uniform distribution with support $supp(\gamma_{\mathbf{a}}) = [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, $\omega_{\mathbf{a}} > 0$, so that an increase of $\omega_{\mathbf{a}}$ means more consumer confusion in the sense of a larger range of possible opinions. Consider the special cases (i) $\omega_{(a_1,a_2)} = a_1 + a_2$, (ii) $\omega_{(a_1,a_2)} = \max\{a_1, a_2\}$, and (iii) $\omega_{(a_1,a_2)} = \min\{a_1, a_2\}$. In case (i), individual marketing activities have an independent incremental effect on the overall level of confusion, while with (ii) the level of confusion depends only on the firm that engages most in obfuscation. If $0 \in A$, then case (i) is consistent with both Assumptions 2.2 and 2.3. Case (ii) is consistent with Assumption 2.2, but violates Assumption 2.3. In case (iii), the firm with the lower level of obfuscation determines the prevailing level of confusion. Thus, neither assumption holds.

2.3.3 Massive Confusion

Theorems 1.1 and 2.2 apply to situations where the scope for consumer confusion is constrained by the degree of the existing taste differentiation, i.e., $supp(\gamma_{\mathbf{a}}) \subset supp(g_0)$ $\forall \mathbf{a} \in \mathcal{A}$. In other words, we have implicitly assumed that obfuscation can never convert consumers with the most extreme true valuations in favor of one firm to the other. Our next result shows that if such "massive" reversals in consumer opinions are possible, then firms may choose to obfuscate the market even when preferences are polarized.

Theorem 2.3. *Suppose that conditions (A2.1.1) - (A2.1.3) hold, A is compact, and $\forall \mathbf{a} \in \mathcal{A}$, $\varepsilon_{\mathbf{a}}$ is uniformly distributed on $[-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$, where $\omega_{\mathbf{a}} \geq 0$. Then, an SPE with maximal confusion ($\omega_{\mathbf{a}^*} = \bar{\omega} \equiv \max_{\mathbf{a}\in\mathcal{A}} \omega_{\mathbf{a}}$) exists if either (i) true preferences are indecisive on $supp(g_0)$, or (ii) true preferences are polarized on $supp(g_0)$ and $\bar{\omega}$ is sufficiently large.*

Thus, under the simplifying assumption of uniform noise distributions, we obtain clear results for the case of massive confusion: For indecisive consumers, the result that firms want to obfuscate as much as possible generalizes, independently of $\bar{\omega}$, the maximal degree of possible confusion. Even with polarized consumers, maximal obfuscation arises when it is possible to confuse consumers sufficiently. Intuitively, when firms can induce arbitrarily large differences in perceived valuations, the mass of indifferent consumers will eventually become negligible, regardless of the true valuation distribution. As a result, firms always benefit from confusing consumers if the scope for confusion is sufficiently large. This discussion suggests a reinterpretation of the idea that obfuscation always arises in equilibrium with homogeneous good: In a situation where true valuation differences are all zero, *any* confusion is massive, so firms benefit from introducing it.

[12] The proof of Theorem 2.2 shows that Assumption 2.3 is sufficient but not necessary for our results. For example, for the proof of the MPS case to go through, it suffices to assume that i) $\varepsilon_{\bar{a},a_j}$ is a MPS of ε_{a_i,a_j} $\forall a_i, a_j \in A$ with $a_i < \bar{a}$ and ii) ε_{a_i,a_j} is a MPS of $\varepsilon_{\underline{a},a_j}$ $\forall a_i, a_j \in A$ with $a_i > \underline{a}$. Therefore, the order on the distributions $\{\Gamma_{\mathbf{a}}\}_{\mathbf{a}\in\mathcal{A}}$ can be even more incomplete than what Assumption 2.3 requires.

2.3.4 Welfare

In the homogeneous goods case, consumer confusion increases prices and therefore benefits firms at the expense of consumers. In our setting, consumer confusion could, in principle, reduce prices. However, whenever this would be the case (when preferences are polarized), firms avoid obfuscating the market according to Theorems 2.1 and 2.2. In addition to the price effect, confusion also reduces consumer surplus because some consumers buy the wrong good. This not only has redistributive effects, but also reduces total surplus.

Taking the equilibrium marketing profile $\mathbf{a}^*$ of the firms in the first stage as given, we can compute the total expected welfare loss from mismatch as follows:

$$\begin{aligned} L &= \int_0^{+\infty} x\Gamma_{\mathbf{a}^*}(-x)g_0(x)dx + \int_{-\infty}^{0} (-x)(1-\Gamma_{\mathbf{a}^*}(x))g_0(-x)dx \\ &= 2\int_0^{+\infty} x\Gamma_{\mathbf{a}^*}(-x)g_0(x)dx, \end{aligned} \tag{2.4}$$

where the second equality uses the symmetry of $\Gamma_{\mathbf{a}}$ and g_0. To understand the measure in (2.4), note that if a consumer buys from the wrong firm, the welfare loss is her true valuation difference $|v_2 - v_1|$. Without loss of generality, suppose that $x = v_2 - v_1 > 0$. Then, $g_0(x)$ captures the likelihood of type x and $\Gamma_{\mathbf{a}^*}(-x)$ is the probability that type x buys from the wrong firm. Clearly, the welfare loss is zero (i.e., $L = 0$) if and only if $\varepsilon_{\mathbf{a}^*}$ is degenerate at zero. In what follows, we shall focus on the case where $\varepsilon_{\mathbf{a}^*}$ is uniformly distributed. The next result shows that more confusion always leads to larger welfare loss, regardless of the shape of the true preference distribution.

Proposition 2.1. *Suppose that $\varepsilon_{\mathbf{a}^*}$ is uniformly distributed on $[-\omega_{\mathbf{a}^*}, \omega_{\mathbf{a}^*}]$, where $\omega_{\mathbf{a}^*} > 0$. The expected welfare loss from mismatch is strictly increasing in $\omega_{\mathbf{a}^*}$.*

Intuitively, increasing obfuscation increases the chances that consumers buy from the wrong firm. A more subtle question is how the size of the welfare loss depends on the distribution of true preferences and, in particular, on whether it is indecisive or polarized. Equation 2.4 reflects two competing intuitions. On the one hand, when confusion arises, chances are high that almost indifferent consumers will buy the wrong product, and there are many such consumers when the preference distribution is indecisive, contrary to when it is polarized. On the other hand, when almost indifferent consumers buy from the wrong firm, the welfare loss is smaller than when those with strong preferences do so. Thus, the net effect of preference polarization/indecisiveness on welfare loss is not obvious in general. In the remainder of this section, we resolve this issue for a concrete example.

Competition on the line We suppose that each consumer is characterized by a parameter θ, which is drawn from a commonly known distribution H with support $\Theta = [-\lambda, \lambda]$, where $\lambda > 0$. The true valuation of a type θ consumer for product $i \in \{1, 2\}$ is $v_i^\theta = \mu - (x_i - \theta)^2$, where $\mu > 0$, and $x_1 = -\lambda$, $x_2 = \lambda$ are the locations of the firms. We assume that H has the symmetric density $h(\theta) = \alpha\theta^2 + \beta$ on Θ, where $\alpha \in \left[-\frac{3}{4\lambda^3}, \frac{3}{2\lambda^3}\right]$ and $\beta = \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}$. H translates into a distribution G of valuation differences, where $G(x) = H(\frac{x}{4\lambda})$, $\forall x \in \mathbb{R}$. If $\alpha > 0$, the true preferences are strongly polarized on the support of G, $[-4\lambda^2, 4\lambda^2]$. Conversely, if $\alpha < 0$, the true preferences are strongly indecisive on $[-4\lambda^2, 4\lambda^2]$. Naturally, in this setting the parameter α is a measure of preference polarization/indecisiveness, while λ is a measure of product/taste differentiation.

We first show that the above canonical model can be covered by our analysis. The following proposition is a direct application of Theorems 2.1 and 2.2.

Proposition 2.2. *Consider the model with competition on the line. Suppose that* $\alpha \leq \hat{\alpha} \equiv (6-3\sqrt{3})/4\lambda^3$*, (A2.1.4) and (A2.1.5) hold, and supp*$(\gamma_{\mathbf{a}}) \subset [-4\lambda^2, 4\lambda^2]$ $\forall \mathbf{a} \in \mathcal{A}$*.*

(i) If Assumption 2.2 also holds, then there exists (does not exist) an SPE without consumer confusion if $\alpha > 0$ *(*$\alpha < 0$*).*

(ii) If Assumption 2.3 also holds and $\alpha \neq 0$*, then there exists a unique SPE. This SPE features minimal (maximal) consumer confusion if* $\alpha > 0$ *(*$\alpha < 0$*).*

The proof of Proposition 2.2 shows that the location distribution H, and thus also the distribution G, is log-concave for $\alpha \leq \hat{\alpha}$. This allows us to apply Theorems 2.1 and 2.2 The result confirms the main insight of our paper: with differentiated goods, the firms' incentives to obfuscate the market crucially depends on the shape of the distribution of consumer preferences and, in particular, whether it is polarized or indecisive.

Under the additional assumption that the noise resulting from the equilibrium marketing profile $\mathbf{a}^*$ is uniformly distributed, our last result in this section describes how the expected welfare loss depends on α, the measure of polarization.

Proposition 2.3. *Consider the model with competition on the line. Suppose that* $\varepsilon_{\mathbf{a}^*}$ *is uniformly distributed on* $[-\omega_{\mathbf{a}^*}, \omega_{\mathbf{a}^*}]$*, where* $\omega_{\mathbf{a}^*} > 0$*. If* $\omega_{\mathbf{a}^*} < \hat{\omega} \equiv 64\lambda^2/15$*, then the expected welfare loss is strictly decreasing in* α*. If* $\omega_{\mathbf{a}^*} > \hat{\omega}$*, then the expected welfare loss is strictly increasing in* α*.*

Thus, if the maximal confusion is small (large) relative to product differentiation, the expected welfare loss always decreases (increases) as preferences become more polarized. A complete welfare analysis must also identify the circumstances under which consumer confusion arises in equilibrium. Under the conditions of Proposition 2.2, confusion only arises for indecisive consumers ($\alpha < 0$). There is no welfare loss with polarized consumers, as the firms do not obfuscate the market. When the maximal degree of confusion is sufficiently large, however, firms no longer want to maintain a transparent market even with polarized consumers (Theorem 2.3), so that Proposition 2.3 also applies.

2.3.5 Outside Options

In this section, we illustrate how the presence of binding outside options affects the analysis. For simplicity, assume that the perceived valuations are given by

$$\tilde{v}_1 = \frac{m}{2} + \frac{v}{2} + \frac{\varepsilon}{2}, \quad \tilde{v}_2 = \frac{m}{2} - \frac{v}{2} - \frac{\varepsilon}{2},$$

where $m > 0$ is some constant, and $v \in [-1, 1]$ is drawn from some distribution G_0.[13] We now suppose that the consumers have a reservation value $u_0 = 0$, so that a consumer will only buy from firm i if $\tilde{v}_i - p_i \geq \max\{\tilde{v}_j - p_j, 0\}$. We use $p_{\mathbf{a}}^M$ to denote the solution to the monopoly problem

$$\max_{p \geq 0} \; \Pi^M(p) \equiv p\left(1 - G_{\mathbf{a}}(2p - 1)\right).$$

Note that a unique solution of this maximization problem exists if $G_{\mathbf{a}}$ is log-concave on $supp(g_{\mathbf{a}})$. The following extension of Lemma 2.1 characterizes the equilibria in the pricing stage.

Proposition 2.4. *Suppose that Assumption 2.1 holds. In the game with outside options, there exists a unique symmetric pure-strategy equilibrium in every pricing subgame, and each firm chooses the price*

$$p_{\mathbf{a}}^* = \begin{cases} \frac{1}{2g_{\mathbf{a}}(0)} & \text{if } g_{\mathbf{a}}(0) > \frac{1}{m}, \\ \frac{m}{2} & \text{if } g_{\mathbf{a}}(0) \in \left[\frac{1}{2m}, \frac{1}{m}\right], \\ p_{\mathbf{a}}^M & \text{otherwise.} \end{cases}$$

Thus, when the concentration of consumers around indifference is sufficiently high, everything is as in the equilibrium without outside options. Competition for the indifferent consumers keeps prices so low that everybody is served. Below a certain threshold concentration of indifferent consumers, firms charge lower prices than without outside options ($p = m/2$), but the market remains completely covered. Finally, for a very low concentration of indifferent consumers, firms give up on these consumers and charge higher (monopoly) prices. Thus, with a binding outside option and monopolistic pricing there is another source of inefficiency associated with confusion: some consumers may not purchase at all though a transaction would be socially desirable.

To see that inefficient opt-outs can indeed arise an SPE outcome, consider a simple example where $m = 2$, and G_0 has a density function g_0 with $g_0(x) = x+1$ if $-1 \leq x \leq 0$, $g_0(x) = -x+1$ if $0 \leq x \leq 1$, and $g_0(x) = 0$ otherwise (thus true preferences are strongly indecisive). Suppose also that the set of marketing activities is given by $A = \{0, 1\}$, and $\varepsilon_{\mathbf{a}}$ is uniformly distributed on $[-2(a_1 + a_2), 2(a_1 + a_2)]$. It is easy to verify that in this game there is a unique SPE where both firms choose to obfuscate the market in the first stage (i.e., $\mathbf{a}^* = (1, 1)$), and then set the monopoly price $p_{a^*}^M = 1.5$ in the second stage. In this example, the presence of consumer outside options does not alter the firms' equilibrium choices of marketing activities, but it does discipline the firms by forcing than to set a lower price (since $p_{\mathbf{a}^*}^M < \frac{1}{2g_{\mathbf{a}^*}(0)} = 4$). In equilibrium, $\frac{1}{4}$ of the consumers stay out of the market even though $\max\{v_1, v_2\} \geq 0 \; \forall v \in [-1, 1]$.

2.4 Discussion

In this section, we discuss several aspects of our framework. In Section 2.4.1, we link our approach to the treatment of demand rotations, as introduced by Johnson and Myatt (2006).

[13] As in the Hotelling example discussed in Section 2.3.4, we therefore assume that valuations are negatively correlated; however, we now assume that the support is a straight line in $\mathbb{R}^2$.

Section 2.4.2 discusses the firms' marketing activities. Section 2.4.3 deals with default activities and marketing costs.

2.4.1 Demand Rotations

At an abstract level, our paper asks whether competing firms benefit from changes in demand that increase the (perceived) valuations of some consumers and decrease those of others. Johnson and Myatt (2006) explore a related question for a monopolist. They suppose the firm can engage in measures related to advertising, marketing or product design that result in a rotation of the entire demand function (or, equivalently, the valuation distribution). A clockwise rotation increases the distribution function below a threshold value, but reduces it above the threshold. In particular, a rotation around the point corresponding to the median valuation takes away mass from this point. In our setting, a clockwise rotation of the distribution G_0 of valuation differences around the point of indifference ($v_\Delta = 0$) would therefore always be desirable for firms. In fact, it would be sufficient for rotation to occur locally. However, a key observation of our paper is that obfuscation, i.e., adding noise to the distribution as in (2.1), does not necessarily lead to a desired local rotation. When the preferences satisfy weak polarization, the mass at zero increases under obfuscation, which is not consistent with a clockwise rotation of the distribution. Therefore, while firms would want to induce a rotation of distribution, they do not want to engage in obfuscation in this case.

In Johnson and Myatt (2006), the monopolist does not even necessarily want to induce a clockwise rotation of the valuation distribution. Doing so is only desirable if the monopolist follows a *niche strategy* according to which he only serves consumers with valuations above the rotation point. When consumers are sufficiently homogeneous, the monopolist will instead follow a *mass market* strategy, that is, he also serves consumers with valuations below the rotation point. In this case, a rotation is undesirable because it necessitates a price reduction. The difference in the role of rotations in Johnson and Myatt (2006) and our paper reflects the fact that in a duopoly setting the distribution of valuation differences is relevant, rather than the valuations themselves.

Contrary to Johnson and Myatt (2006), our focus is on demand changes resulting from valuation distortions rather than changes in true valuations. As far as the positive analysis (Theorems 1.1 - 2.3) is concerned, this difference is immaterial: All our results can, for instance, be applied to changes in product design that change the true valuations from v_i to $\tilde{v}_i$, as long as they are in the form of (2.1). The welfare analysis of Section 2.3.4, however, refers only to the case that there is a conflict between true and perceived valuations, so that confused consumers take wrong decisions.

2.4.2 Marketing Activities

We now explain in more detail what kind of marketing activities are consistent with our approach. Our analysis requires symmetry of the distributions $\Gamma_{\mathbf{a}}$ capturing the noise in valuation differences. This assumption is consistent with activities that increase the relative valuations for some consumers, but decrease them for others. These activities fall into two broad categories depending on whether they only affect the valuation for a firm's own good or also the valuation for the competitor's product.

2.4.2.1 Confusion about a firm's own good

The key restrictions of our framework captured in Condition (2.1) and Assumption 2.1 are expressed in terms of distributions of valuation differences. As we show in more detail in the appendix, these assumptions can be generated from a random utility model where the valuation for each firm's good consists of a noise term reflecting its marketing activities. In particular, we consider a specification where the degree of noise reflects product complexity that results in information overload.

A large literature in marketing has documented confusion resulting from information overload (Kasabov, 2015; Walsh et al., 2007). Overload confusion arises because information recipients fail to properly match their responses to stimuli once the overall stimulus volume passes a certain threshold (Miller, 1956). For instance, such stimuli can come from marketing messages (Jacoby, 1977). Overload confusion occurs once consumers are "confronted with more product information and alternatives than they can process in order to get to know, to compare and to comprehend alternatives" (Walsh et al., 2007). The general consequences of information overload, surveyed by Eppler and Mengis (2004) across fields as diverse as accounting, organizational science, marketing and consumer research, are unsystematic decision mistakes, a decreased decision accuracy, a lack of critical evaluation, a "failure to develop correct interpretations of various facets of a product or service" (Turnbull et al., 2000), and ambiguous perceptions by consumers (Solomon, 2014). Overload confusion is directly linked to the notion of product complexity in marketing. There is a strong consensus in over 40 years of research on product complexity that the number of attributes, functions or labels of a product contribute to the total complexity carried on each product, and as such to the information volume to which the consumer is exposed (Mützel and Kilian, 2016).

Product labels constitute a specific example of product attributes that may cause confusion.[14] In the case of labels, the evidence indicates that not only the sheer number of labels, but also their contents, are a source of consumer confusion. As an illustration, a two-year study by the British Food Advisory Committee concluded that labels like "fresh", "original" or "pure", which are frequently used to describe food products, result in consumer confusion. To quote the principal policy advisor at the British Consumer Association: "Labels are all too often more of a marketing gimmick than a way of providing meaningful information to help consumers make. Confusion in the comparison of products due to food labels has also been experimentally verified by Leek et al. (2015). Likewise, the "Nutrition facts label" introduced in the 1980's in the US, originally intended to allow consumers to make better informed food choices, has turned out to be a source of consumer confusion. Moreover, overload confusion has been associated with product packaging, arising for instance from small fonts or a dense writing style (Mitchell

[14] Langer et al. (2007) find that the number of eco-labels on yogurts is positively correlated with consumer confusion, and Kuester and Buys (2009) find that increasing the number of describing attributes, increases consumer confusion in case of jams. Complexity confusion has been found in computers, mobile phones, automobiles, digital cameras, buildings or insurance policies. See Kasabov, 2015; Mützel and Kilian, 2016; Walsh et al., 2007 for surveys on complexity confusion.

and Papavassiliou, 1999). Further, lengthy and complicated contracts involving “fine print” can cause overload confusion. A typical example where complex contracts have caused consumer confusion is the mobile phone market (Turnbull et al., 2000). In Appendix B.10, we also provide a formulation that accounts for the possibility that certain combinations of features have stronger effects on confusion than others.

2.4.2.2 Confusion about the market

The second broad category of marketing instruments consists of activities that not only affect a firm’s own perceived valuation, but also the perceived valuation for the competitor’s good. This is consistent with the view that complexity is a synthetic phenomenon of all marketing messages interacting with each other, leading to a market level or “category complexity” Mützel and Kilian (2016). This means that the chosen marketing activities (labels, ads, design aspects, etc.) jointly shape consumer confusion. For instance, if one food brand uses the label “original” while another brand uses “authentic”, the comparison of the two labels by consumers may cause some confusion, which possibly could have been avoided if both firms were to coordinate on the same label. In such cases, the noise in the valuation of one good is not independent of the noise in the valuation of the other good. Then, it is more convenient to depict the confusion effects directly in the valuation differences – which is perfectly in line with Condition (2.1).

Market confusion is also addressed in the work of Carlin (2009), Piccione and Spiegler (2012) and Chioveanu and Zhou (2013). These authors study homogeneous goods models where the mutual choice of a “frame”, i.e., a way to present the price of the product, by two firms determines whether or not a consumer can draw a product comparison. If such a comparison is made, the consumer (correctly) chooses the cheaper product; if no comparison is made, the consumer picks at random. With the framing interpretation, it is again not reasonable to impose a notion of i.i.d. valuation shocks; instead we think of the realized frame as a consequence of an interaction of the chosen marketing activities. For example, if a firm chooses to highlight a specific set of attributes, the (comparative) evaluation of these attributes may depend on what attributes the competitor has highlighted. Alternatively, disclosing some information about features of the own product may affect the perception of the competitor, who tries to hide those features.

Spiegler (2014) considers a generalization of Piccione and Spiegler (2012), where two firms simultaneously choose their marketing messages, which jointly determine the distribution of the frames a single consumer could adopt. The adopted frame, in turn, determines the market shares (choice probabilities) of each firm. Our model can be easily interpreted through the lens of his setting by assuming that there is a set of possible frames. Any realized frame in this set corresponds to a particular (deterministic) way how a given consumer evaluates the two products, which can shift consumer perception in favor of one or the other alternative. The precise frame adopted by the consumer after being exposed to marketing profile $\mathbf{a}$ is unknown to the firms, but they know the probability distribution over the frames induced by $\mathbf{a}$. Then $\varepsilon_{\mathbf{a}}$ is the random variable that describes the effects of the various frames on the likelihood of a given consumer to choose a certain option (given prices p_1, p_2). Our assumptions require that, whatever marketing activities are chosen, the probabilities to obtain a favorable frame are symmetric from each firm's perspective.[15]

2.4.3 Default Strategies and Marketing Costs

So far, we have implicitly assumed that marketing strategies differ only with respect to the amount of noise they induce. We now address two further, closely related sources of heterogeneity. First, there could be a default marketing strategy $a \in A$ which corresponds to "inactivity". Second, there could be heterogeneous marketing costs. As a normalization, it is useful to assume a firm can implement the default marketing activity with no cost. In principle, the default could be $a = 0$, and the interpretation is that if a firm does "nothing", perceived and true valuations coincide. It would then be natural to further assume that the cost function is increasing in obfuscation for an order satisfying (A2.3.1) or (A2.3.2). Alternatively, there could just as well be some pre-existing confusion, which will remain unresolved unless the firms engage in costly education activities. In this case, the default satisfies $a \neq 0$. Assuming that larger deviations from the default are more costly would be consistent with a cost function that is decreasing in obfuscation (if the default corresponds to maximal confusion) or non-monotone (if the default corresponds to an interior level of confusion).

In the presence of default activities and pre-existing confusion, Theorems 2.2 apply directly if marketing costs are negligible: With indecisive tastes, firms still choose marketing activities that induce maximal confusion; with polarized tastes, they strive to eliminate any confusion. With marketing costs, the analysis would change in two related ways. First, extreme marketing profiles as predicted by Theorem 2.2 would no longer arise if they are too costly for the firms. Second, there would be a potential conflict of interest between the firms who would have to coordinate on who engages in more costly marketing activities. As a result, even if obfuscation (or education) can increase joint profits, it may not necessarily arise in equilibrium.

2.5 Competition for Voters

The ideas of our model can be applied to other types of competitive interactions. We illustrate this for the case of competition for voters. This involves extending our formal apparatus to deal with contests.

We consider a two-stage model where candidates $i \in \{1, 2\}$ compete for voter k by first choosing the clarity of their platform and then engaging in costly activities to persuade voters. Paralleling the set-up of Section 2.2, we assume there is a population of voters that can be described by a joint valuation distribution $F_0(v_1, v_2)$ of idiosyncratic "baseline valuations" for the politicians. However, we assume that politicians can influence the perceived valuations of

[15] In Piccione and Spiegler (2012) and Spiegler (2014), a property called Weighted Regularity (WR) plays a critical role for the equilibrium analysis. WR means that each firm can always choose her marketing messages, possibly by randomization, such that it can unilaterally enforce a certain *fixed* probability distribution over the frames the consumer can adopt, independent of the other firm's marketing choices. WR may or may not be satisfied under the assumptions imposed on our model: Suppose $A = \{0, 1\}$. Let $\mu_{\mathbf{a}}$ denote the probability measure induced by $\mathbf{a} \in \{0, 1\}^2$. If $\mu_{(1,0)} = \mu_{(0,1)} = \mu_{(1,1)}$ and $\mu_{(0,0)} \neq \mu_{(1,1)}$, meaning that there can only be two effective "confusion states", then WR must hold. By contrast, if $\mu_{(1,0)} = \mu_{(0,1)}$ but $\mu_{(1,1)} \neq \mu_{(0,0)} \neq \mu_{(1,0)}$, such that there are three confusion states, e.g., because marketing activities are ranked in the sense of second-order stochastic dominance, then WR generally fails. Put differently, WR is not critical for our equilibrium analysis.

voters in two ways. In the first stage, they choose to present their political platforms more or less clearly: A candidate can use mixed messages that can be interpreted positively or negatively by any recipient, leading to a noisy picture of the candidate, whereby some voters get a too positive impression of the candidate's value for them and others get a too negative impression. We model this by assuming that candidate i can choose from a set of platform descriptions A_i which are characterized by additive noise distributions $\Gamma_{\mathbf{a}}$ distorting the underlying baseline valuations and generating perceived baseline valuations $\tilde{v}_i$. We maintain the same assumptions on G_0 and $\Gamma_{\mathbf{a}}$ as imposed in Assumption 2.1 for the oligopoly model.

However, we now also assume that, once the perceived baseline valuations are determined, the candidates can engage in costly effort $s_i \in \mathbb{R}_+$ to convince voters to vote for them, with an effort cost function $c : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying $c' > 0, c'' > 0$, and $c(0) = c'(0) = 0$. As a result of exerting effort, the perceived utility of a voter from candidate i becomes

$$\tilde{u}_i = \tilde{v}_i + s_i. \tag{2.5}$$

The effort cost s_i allows for two different interpretations. First, it is well known that, e.g., a strong media presence of a candidate can persuade voters to favor the candidate. In this interpretation, a persuasion effort of s_i is associated with advertising expenditure $c(s_i)$ which, like in an all-pay auction, the politician needs to pay, no matter whether she is successful or not. Second, s_i could correspond to a commitment to a policy which the candidate needs to fulfil should she be elected. This commitment comes at a cost $c(s_i)$ which only needs to be paid in case of success: We think of this cost as reflecting the reduction in credibility coming from an inability to live up to the expectations generated by the commitments. The politician could be concerned about this loss of credibility because of intrinsic feelings of shame from being perceived as dishonest or because the inability to deliver will impede the election chances in future elections.

In both cases, the probability that candidate i wins is

$$\Pr(\tilde{v}^i_\Delta \le s^i_\Delta + \varepsilon_{\mathbf{a}}) = \int G_0(s_\Delta + e)\gamma_{\mathbf{a}}(e)de, \tag{2.6}$$

where $\tilde{v}^i_\Delta \equiv \tilde{v}_j - \tilde{v}_i$ and $s^i_\Delta = s_i - s_j$ is

We normalize the payoff of the candidate from winning to 1, which is without loss of generality for our purpose. Candidate i's payoff in the advertising case is

$$\Pi_i(s_1, s_2; \mathbf{a}) = \Pr(\tilde{v}^i_\Delta \le s^i_\Delta + \varepsilon_{\mathbf{a}}) - c(s_i)$$

and in the commitment case

$$\Pi_i(s_1, s_2; \mathbf{a}) = \Pr(\tilde{v}^i_\Delta \le s^i_\Delta + \varepsilon_{\mathbf{a}}))(1 - c(s_i)).$$

The commitment case is essentially isomorphic to the model of Section 2.2. In particular, a higher effort in the second stage has an analogous effect as a lower price in the previous model: It increases the chances of winning, but it is costly only in case of success.[16] By contrast, the

[16]Similarly a price reduction in the original model increases demand, but results in a cost only in those cases

second stage of the advertising example is a contest: The candidate incurs the advertising cost no matter whether she wins the election or not.

In spite of these differences, the intuition from Theorem 2.1 applies in both cases. As stated in Proposition B.1 in the appendix, voter preferences fully determine whether or not candidates engage in (small) obfuscation: Candidates will only use obfuscation when preferences are indecisive rather than polarized. In particular, ex ante indecisive preferences provide a breeding ground for voter confusion. With ex-ante indecisive preferences, both candidates are forced to choose high levels of commitments in order to win the campaign if there is no confusion, because small differences in commitments have a strong effect on the voter shares. With obfuscated campaigns, the perception of commitments (of their differences) becomes noisy, which reduces the measure of perceptually indecisive consumers and thereby allows candidates to reduce their commitments.

It is useful to think more about the nature of the "baseline preferences" and the sources of indecisiveness. In reality, voters form judgments about policy platforms under potentially large uncertainty about their consequences. Even if candidates describe their position in the clearest possible way, this uncertainty will not be resolved. For instance, the effects of a politician's approach to climate policy are subject to scientific uncertainty as well as uncertainty about the strategies of other countries. We should therefore think of the baseline valuations as reflecting the valuation that a voter puts on a candidate when being clearly informed about the platform – but this valuation leaves large scope for uncertainty about the true effects of different policies if implemented. One reason for indecisiveness may then be that this uncertainty is so large that, absent obfuscation strategies, many voters will be unsure whom to vote for. This does not reflect indifference about policy platforms, but uncertainty about the relation between candidate platforms and outcomes. In such a situation, our model then predicts polarizing obfuscation strategies as a means to overcome voter indifference.[17]

2.6 Relation to the Literature

Our paper belongs to the literature on competition with boundedly rational consumers.[18] The most distinctive feature of our contribution is that we treat the case of heterogeneous preferences, where the incentives for confusing consumers are less immediate than for homogeneous goods.

Building on the intuition of Scitovsky (1950), several authors have argued that producers of homogeneous goods can escape the Bertrand trap if they manage to mislead, deceive or confuse consumers. For example, firms may present prices in such a way that comparison becomes difficult (Carlin, 2009; Chioveanu and Zhou, 2013; Piccione and Spiegler, 2012). Hence,

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where a consumer buys the good.

[17]In principle, this idea is applicable to the oligopoly context as well. For instance, even when producers of health food describe the contents in a perfectly transparent fashion, there can be irreducible uncertainty about the long-term health effects, reflecting a lack of knowledge at the firm level. Then consumers may be indecisive because even with the best possible information they simply cannot tell which good serves them better.

[18]A peripherally related literature going back to Grossman and Hart (1980), Grossman (1981), Milgrom (1981) and Milgrom and Roberts (1986b) asks under which conditions firms want to disclose product information to consumers who otherwise only have stochastic information about quality, assuming (unlike we do) that the consumers are sufficiently sophisticated to use Bayesian updating to interpret the disclosure decisions of firms.

consumers may sometimes choose from high-price firms even if better offers are available. Firms exploit this by playing mixed-strategy equilibria in which expected profits are positive. Several authors have shown how firms can benefit if consumers perceive homogeneous goods as better than they really are. For instance, in Gabaix and Laibson (2006) firms hide add-on costs of products, so that naïve consumers ignore these costs in their purchasing decisions. The authors show how firms and sophisticated consumers can benefit from the presence of naïve consumers. Heidhues et al. (2016) establish that fooling naïve consumers with hidden fees becomes particularly profitable for socially wasteful products. On a related note, Spiegler (2006) shows how firms producing valueless services can benefit if consumers apply simple sampling procedures to evaluate the services. In our framework, obfuscation does not necessarily bias consumer valuation upwards. Thus, firms cannot be systematically fooled.[19] Nevertheless, firms may still benefit from consumer confusion. In fact, with indecisive preferences, firms may want to obfuscate the market even if it causes a decrease of expected (absolute) consumer valuations.[20]

More broadly, our paper sheds light on the advertising literature. First, this literature deals with the role of advertising that is informative about "the existence, prices and characteristics of goods" (Belleflamme and Peitz, 2010). We can think of our model as capturing the choice between relatively informative advertising (little or no obfuscation) and uninformative advertising (more obfuscation); thus informative advertising reduces the noise in perception of the relative product valuations. Our analysis thus shows that information about product values may or may not increase prices, contrary to the effects of information about product existence, which typically reduces prices by increasing competition (Bagwell, 2007).

The paper can also be put into the perspective of persuasive advertising. This strand of the advertising literature considers advertising activities that firms use to increase the willingness to pay for their product or, similarly, shift the taste distributions in their favor (see, e.g., Dixit and Norman, 1978; Von der Fehr and Stevik, 1998). The literature argues that games of persuasive advertising have the structure of a prisoners' dilemma: Firms engage in costly advertising races, which, in equilibrium, do not affect prices and gross profits. Contrary to the bulk of this literature, the marketing activities in our model can be interpreted as activities that persuade some consumers at the cost of alienating others.[21] At first glance, it would appear that such activities are less attractive to firms than persuasive advertising, since they do not shift demand systematically in the direction of a firm. Our equilibrium analysis shows, however, that this is not the case: Firms either refrain from such advertising measures (with polarized preferences) or they use the measures to soften competition – under no circumstances do they end up in the a standard prisoners' dilemma.

Many papers in the literature have worked with the assumption that consumers differ with respect to their degrees of sophistication, distinguishing between a group of perfectly rational consumers and a group of boundedly rational or cognitively constrained ones. The relative size of these groups is usually a central parameter in this literature (e.g., Gabaix and Laibson, 2006;

[19]Spiegler (2017) asks when decision-makers can be fooled if they hold a misspecified causal model. He defines "fooling" in terms of the expected outcome or prediction from the mispecified model relative to the correct model.

[20]While we assume that obfuscation leaves expected valuation differences unaffected, there is no such statement for expected valuations.

[21]As an example, consider the cold-calls of tele-marketing agents (see Schumacher and Thysen, 2017).

Heidhues et al., 2016). In our setting, one can think of the noise distribution as capturing the degree of sophistication in the population in a continuous rather than in a discrete fashion: For completely sophisticated consumers, the perceived valuation differences are equal to zero; the more naive a consumer is, the greater the difference between perceived and actual valuations. Our assumptions on the noise distribution can thus be interpreted as assumptions on the distribution of consumer sophistication.

2.7 Conclusion

In this paper, we analyze firm-driven consumer confusion when consumers differ in their opinions about the values of existing alternatives. We find that the overall dispersion of consumer tastes has decisive implications for the propensity of firms to confuse or educate consumers and for the subsequent price competition. If undecided consumers are relatively common, firms benefits from obfuscating the market, because it results in more consumers conceiving one product as better than the other. This allows the firms to increase its price without losing too much demand. Consumers can be harmed in up to three ways by such obfuscation: they may purchase less-matched product, pay higher prices, or forego purchases that would be socially efficient. With a polarized dispersion of tastes, we obtain the opposite result that consumer confusion is harmful for firms. In this case, firms seek to increase market transparency by educating confused consumers, e.g., by providing qualified information and guidance that helps consumers understand the offered alternatives in the market.

Finally, no matter what the shape of the true preference distribution is, obfuscation tends to be beneficial for firms and harmful for consumers if confusion becomes so strong that it dominates true preference differences.

In sum, our results suggest that research on market transparency and confusion cannot be agnostic about the nature of the existing dispersion of opinions, as this interacts non-trivially with the firms' incentives to influence the effective perception of consumers.

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3 Voting with Public Information[1]

3.1 Introduction

A common argument for voting mechanisms is that they help aggregate the information that agents in a committee privately hold, and thus lead to better decisions compared to the case of a single decision-maker. Indeed, in a setting of collective decision-making where agents have purely common interests, the celebrated Condorcet Jury Theorem (CJT) suggests that the simple majority rule can lead to the first-best outcome if agents truthfully convey their private information through their votes (Condorcet, [1785], 1994). However, Kawamura and Vlaseros (2017) (henceforth KV) make the interesting observation that, as long as there exists a public signal that can be commonly observed by all agents and that is superior to each of their private signals, a vote-your-private-information strategy profile will not constitute an equilibrium under the simple majority rule, even though this would have been the case if the public signal were absent. What's worse, the presence of public information opens the possibility for agents to coordinate on an equilibrium in which everyone just votes according to whatever the public signal suggests. Clearly, in such an equilibrium, the private information of the committee members is completely disregarded. This can be very inefficient since public information is rarely perfect and the total private information possessed by the committee is often more valuable in determining the optimal collective decision. Experimentally, KV find that a large proportion of subjects in the laboratory behave quite consistently with what the inefficient equilibrium would predict. Consequently, the outcome of the collective decision almost always coincides with that in the inefficient equilibrium.

This observation is highly relevant, because it should be clear that the access to both private and public information for the voters is the rule rather than the exception: in business, members of the board of directors receive (or even ask) advice from the advisory board of the company; in a court, an expert witness states his/her testimony in front of all members of the jury; the Central Committee of the Communist Party of China, which has only seven members, often invites renowned scholars in the relevant fields to give short presentations when important decisions that affect the well-being of more than 1.3 billion people are needed to be made. If in the end only the public information counts, why should we bother to use the voting mechanism in the first place? This issue is even more alarming if we take into account that in reality, the party that provides the relevant public information is often strategic and self-interested as well.

With these practical concerns in mind, we first take KV's observation one step further in this paper. We study the effect of public information in a richer setting where agents have both common and conflicting interests: while agents share the common goal of making a collective

[1] A version of this paper is published as Liu, S. (2019): "Voting with Public Information," *Games and Economic Behavior*, 113, 694-719.

decision that will match the state, they may have different payoffs from the different types of decision errors that could occur. We show that the presence of public information can have a profound impact on the agents' voting behavior. In particular, it significantly limits the existence of the *informative voting equilibrium*, in which every agent simply casts her vote in accordance with her private information: If the public information is superior to each agent's private information and the voting threshold is fixed (which is the case for the simple majority rule), the informative voting equilibrium does not exist for *any* preference profile of the agents.[2] To make things worse, the presence of public information introduces the intuitive but inefficient *obedient voting equilibrium*, which robustly exists under different voting rules. In the obedient voting equilibrium, agents always support the alternative suggested by the public information and, hence, the public information is the only determinant of the final decision outcome. We later show that a self-interested party who controls the provision of public information may exploit its influential effect by strategically disclosing (withholding) good (bad) news about his favored alternative.

The inefficient outcome of the obedient voting equilibrium echos the common concern that public information, especially expert opinions, may have excessive influence on decision making.[3] In theory, if agents are sophisticated enough to coordinate on equilibria that entail mixed and/or asymmetric strategy profiles, then the committee's decision may still incorporate both the private and the public information. However, as we argue in Section 3.4.1, the concern of public information being detrimental should be far from being resolved by this theoretical possibility. In particular, by extending the baseline model and considering more generally how the provision of public information introduces correlation between the signals privately observed by the agents, we are able to show that informational inefficiency can persist even in large elections, no matter how sophisticated the equilibria played by the agents are.

We then study the design of optimal voting mechanisms in environments with public information. We first introduce a class of more flexible voting rules that we call the *contingent k-voting rules*. Under a contingent k-voting rule, the number of votes required for the committee to select an alternative will depend on the content of the public information: For example, if a job candidate is supported by an exceptionally strong recommendation letter, the committee may consider requiring less votes to approve the hire of this candidate. We show that, if agents' preferences satisfy a (mild) no-indifference condition, then for any *ex post incentive compatible* direct mechanism that is optimal there exists an equivalent contingent k-voting rule. Specifically, by sustaining informative voting as an equilibrium (or *implementing* informative voting), the equivalent contingent k-voting rule achieves the same informational efficiency as the optimal ex post incentive compatible mechanisms. Therefore, in the search for optimal mechanisms it is without loss to focus on contingent k-voting rules that can implement informative voting.

A contingent k-voting rule incorporates the public information by letting its voting threshold

[2] Even if the public information is less accurate than the private information, the set of preference profiles that allow for informative voting under *some* voting rule with a fixed threshold is strictly smaller than it would be in the absence of the public information. For example, if the public information is just slightly less precise than each agent's private information, under the simple majority rule the informative voting equilibrium exists only if all agents are sufficiently unbiased *ex ante* (see Corollary 3.2).

[3] For instance, because of the concern that their testimonies will have too much influence upon the jury, in the US court rules are set to prevent expert witnesses from "usurping the province of the jury" (Tanay, 2010).

be contingent on the realization of the public signal. It also incorporates the private information of the agents if it is *responsive*, which requires that the agents' votes can always make a difference on the final decision, regardless of the realization of the public signal. We show that it is often optimal to use a responsive contingent k-voting rule to implement informative voting. Moreover, the informative voting equilibria sustained by the responsive contingent k-voting rules are asymptotically efficient, in the sense that the ex ante probability of the collective decision being matched to the state becomes arbitrary close to 1 as the size of the committee increases. In other words, we obtain a version of the CJT in a voting environment with both private and public information.

Within a setting where agents have purely common interests, which is mostly studied in the literature, we demonstrate that the first-best informational efficiency can always be achieved by using a specific contingent k-voting rule, the *contingent majority rule*, under which the informative voting equilibrium is guaranteed to exist. In particular, we show that given all the information that is available to the committee, the probability of the collective decision being matched to the state is maximized in the informative voting equilibrium under the contingent majority rule. In other words, the contingent majority rule aggregates *both* the private *and* the public information efficiently.

To strengthen the applicability of our results, we further introduce a simple two-stage voting mechanism that can *equivalently implement* the informative voting equilibrium under the contingent k-voting rules. In the first stage of this voting mechanism, agents vote to select the voting threshold that will be used. In the second stage, they proceed to vote about which collective decision to take by using the voting rule that they agreed on. We argue that this two-stage voting mechanism is practically appealing because its procedure is deterministic and independent of the informational details of the environment.

Finally, we show, perhaps to one's surprise, that using voting procedures that incorporate the public information can actually have additional advantages when there is a concern for strategic disclosure of public information. Intuitively, the use of the contingent k-voting rules or the above two-stage voting mechanism makes it possible for the agents to rationally commit to informative voting, independent of the disclosure policy of the public information. Thus, even a self-interested party may find it optimal to always publicly communicate the information it receives to the agents, given that its message will not directly affect the agents' voting behavior but will indirectly increase the accuracy of the collective decision.

The paper proceeds as follows. Section 3.2 reviews the related literature. Section 3.3 presents the model. In Section 3.4 we show how the presence of public information can lead to inefficient information aggregation. We study in Section 3.5 the design of optimal voting mechanisms with public information. In Section 3.6, we analyze settings where the provision of public information is strategically determined by a self-interested information controller. Finally, Section 3.7 concludes. All proofs are contained in Appendix C.

3.2 Related Literature

There is an extensive literature on strategic voting starting with the seminal paper of Austen-Smith and Banks (1996). Many of the papers in this literature study how informational efficiency of various voting mechanisms is affected by the agents' strategic behavior (see, e.g., Feddersen and Pesendorfer (1997) and Duggan and Martinelli (2001) on simultaneous voting rules, and Dekel and Piccione (2000) on sequential voting rules). Among all of them, the most closely related paper besides KV is actually Austen-Smith and Banks (1996). Specifically, they notice that whenever the voters do not have an extremely biased prior, the informative voting equilibrium will exist under some simultaneous voting rule with a fixed voting threshold value (p. 38, Lemma 2). However, our paper shows that if we explicitly take into account how agents' prior is shaped by public information, then the simultaneous voting rules commonly used in practice may no longer suffice to incentivize agents to truthfully reveal their private information via their votes. As another connection to our paper, Section 2 of Austen-Smith and Banks (1996) extends their analysis to a case where agents have access to both private and (exogenous) public information. They conclude that in such a setting, sincere voting, which is equivalent to obedient voting in our model whenever the public information is more precise than each agent's private information, cannot be both informative and rational (p. 42, Theorem 3). In contrast, we address the related but distinct question of whether informative voting can be rational under some simultaneous voting rule when it is not required to be sincere. Our model and focus are also quite different from the few other papers that study the effect of public information in a voting environment (e.g., Gersbach, 2000; Tanner, 2014; Taylor and Yildirim, 2010).

Several papers study the effect of pre-voting deliberation (e.g., Austen-Smith and Feddersen, 2006; Coughlan, 2000; Gerardi and Yariv, 2007). In these models, agents can communicate their private information before the vote takes place, thus public information *endogenously* arises. Our model differs from them in two main aspects. First, in the models with deliberation, conflicts between an agent's private information and the public information usually do not matter because the former has already been incorporated in the latter. In our model, however, such conflicts have a direct and profound effect on agents' provision of private information, which can lead to a severe loss of informational efficiency. Second, unlike in the obedient voting equilibrium in the current paper, in these models it is actually socially efficient for the agents to always follow the public information, conditional on their private information being credibly revealed in the deliberation stage.[4]

Finally, there is a third strand of literature on committee design and optimal voting rules with strategic agents.[5] For example, Persico (2004) studies the optimal size and threshold value for simultaneous voting rules when agents' private information is endogenous. Subsequently, Gershkov and Szentes (2009) show that when information is costly, the optimal direct mechanism can actually be implemented by a random, sequential reporting/voting scheme, which suggests in general that the use of more flexible voting rules can be welfare-enhancing. This insight is also shared by Gersbach (2004, 2009, 2017), who shows that allowing the voting rule to depend on the

[4] Buechel and Mechtenberg (2018) is a recent exception that shows that pre-voting communication can actually impede efficient information aggregation within a common-interest setting. They consider a network model in which agents are heterogeneously informed, and each informed agent can privately make a voting recommendation to the uninformed agents that are connected to her. They show that if the network structure is too centralized around a few informed agents, majority voting may lead to inefficient information aggregation. Compared to their paper, we focus on the public communication between a (strategic or non-strategic) information controller and a group of homogeneously informed agents.

[5] See Nitzan and Paroush (1982) and Ben-Yashar and Nitzan (2014) for the design of optimal collective decision rules with non-strategic agents.

proposal to be determined may yield efficient outcomes for classic social choice problems such as provision of public projects and division of limited resources among agents. More recently, Gershkov et al. (2017) show that in an environment where agents have single-crossing preferences, a successive voting rule with a descending threshold achieves the highest utilitarian efficiency among all anonymous, unanimous and dominant strategy incentive-compatible mechanisms. Our paper contributes to this literature by showing that when relevant public information is salient in the strategic environment being considered, the voting rules should also be more carefully and flexibly designed in order to achieve a more efficient outcome.

3.3 The Model

3.3.1 Players, actions and payoffs

Consider a committee of n members (agents) indexed by $i \in \mathcal{I} \equiv \{1, ..., n\}$. We assume n is odd and $n \geq 3$. Agents need to make a collective decision $d \in \mathcal{D} \equiv \{0, 1\}$ over a binary set of alternatives. For concreteness, one could think of a setting in which a board of directors is choosing between two business proposals.

Each agent can cast a vote to support one of the alternatives. We denote $v_i = 1$ if agent i votes in favor of the decision $d = 1$, and $v_i = 0$ otherwise. A voting profile of the agents is denoted by $v = (v_1, ..., v_n) \in \mathcal{V} \equiv \{0, 1\}^n$. For the moment, we restrict our attention to a class of collective decision rules $g^k : \mathcal{V} \to \mathcal{D}$ called k-voting rules, which are arguably most commonly used in practice. Formally, if we set the alternative associated with $d = 0$ as the default option, under the voting rule g^k the alternative associated with $d = 1$ will be chosen if and only if there are at least $k \in \{1, .., n\}$ votes in favor of it:

$$g^k(v) = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} v_i \geq k, \\ 0 & \text{otherwise.} \end{cases}$$

Each k-voting rule is uniquely characterized by its threshold value k. In particular, the simple majority rule is given by $k = (n + 1)/2$.

The state of the world θ is drawn from a binary set $\Theta \equiv \{0, 1\}$ with equal probability.[6] In the context of the board of directors and business proposals, one could think of θ as the uncertain (relative) quality of the two proposals, where $\theta = 1$ means the proposal associated with $d = 1$ is of higher prospective revenue, while the other is better if $\theta = 0$. We assume agent i's utility function $u_i : \mathcal{D} \times \Theta \to \mathbb{R}$ takes the following form (see also Coughlan, 2000; Iaryczower and

[6] The assumptions that the prior probability of θ is uniform and that the accuracy of the agents' private signals is state-independent (see Section 3.3.2) are mainly made for the convenience of exposition. Most of our analysis can be straightforwardly extended beyond the current setting. See, for example, how we prove Proposition 3.1 in Section 3.4 more generally in Appendix C without the above two assumptions.

Shum, 2012; Kojima and Takagi, 2010):

$$u_i(d,\theta) = \begin{cases} 0 & \text{if } d = \theta, \\ -q_i & \text{if } d = 1, \theta = 0, \\ -(1-q_i) & \text{if } d = 0, \theta = 1, \end{cases}$$

where $q_i \in [0,1]$. In words, we assume the agents in the committee have a common interest in matching the collective decision to the state (i.e., choosing the proposal of higher quality), and we normalize the payoff of successfully choosing $d = \theta$ to zero. However, we allow the agents' payoffs to differ when committing different types of decision errors. We also allow these differences to be heterogeneous across agents. Each agent's utility function is uniquely characterized by the parameter q_i, and the preference profile $\mathbf{q} = (q_i)_{i \in \mathcal{I}}$ is common knowledge among the agents. We interpret q_i as a measure of how biased agent i is towards the default option *ex ante*: If $q_i = 1/2$, agent i is unbiased and indifferent between the two alternatives; if $q_i < 1/2$, agent i is inclined to choose $d = 1$; similarly, $q_i > 1/2$ implies that agent i would prefer $d = 0$ if there is no further information to be revealed. In addition, if $q_i \neq q_j$, the two agents i and j may strictly prefer different alternatives even when they have exactly the same information. Hence, we interpret $q_i \neq q_j$ as a potential conflict of interest between the two agents. We refer to the case where $q_i = 1/2\ \forall i \in \mathcal{I}$ as the setting where agents have purely common interests.

Note that, given the above specification of payoffs, if agent i assigns a posterior probability $\pi \in [0,1]$ to the event $\theta = 1$, she would prefer $d = 1$ over $d = 0$ if and only if $\pi \geq q_i$, that is, whenever the evidence of the state being 1 is sufficiently strong.

3.3.2 Information structure and timing

Before casting their votes, each agent privately receives an i.i.d. signal $s_i \in S_i \equiv \{0,1\}$, which is drawn according to the conditional probability distribution $\Pr(s_i = 1|\theta = 1) = \Pr(s_i = 0|\theta = 0) = \alpha \in (1/2, 1)$. We denote $s = (s_1, ..., s_n) \in S \equiv \Pi_{i=1}^n S_i$ as the agents' (private) signal profile. In addition to their private signals, all agents commonly observe a public signal $s_p \in S_p \equiv \{0,1\}$, which is independently drawn from the conditional probability distribution $\Pr(s_p = 1|\theta = 1) = \Pr(s_p = 0|\theta = 0) = \beta \in [1/2, 1)$. We choose to model public information as an additional conditionally independent signal mainly because it has a clear interpretation, especially when considering committees of moderate sizes: In the context of the board of directors and business proposals, for example, one can think of the public signal as the opinion expressed by the advisory board to all directors before the vote takes place. If $\beta > \alpha$, we can further interpret the public signal as the advice provided to the committee by some external *expert*. In addition, this modeling assumption allows us to conveniently extend our analysis to settings where the disclosure of public information is strategically determined by a biased party (see Section 3.6). We will discuss an alternative way to model public information when considering large elections in Section 3.4.1.

For later use, we define a measure of (relative) informativeness of the public signal:

$$r \equiv \frac{\ln \beta - \ln(1-\beta)}{\ln \alpha - \ln(1-\alpha)}. \tag{3.1}$$

For given α and β the value of r is uniquely determined, and we will say that the public signal is *r-times as informative* as a private signal. For example, if $\alpha = 0.6$, then $\beta = 0.55, 0.69, 0.77$ correspond to the cases where the public signal is 0.5-, 2- and 3-times as informative as a private signal, respectively. Intuitively, the measure r tells us how many private signals of opposite realization would counter-balance the informational effect of the public signal.

The timing of the voting game is as follows. First, nature draws θ. After that, each agent observes her private signal and, in addition, the public signal. Agents then cast their votes, and the collective decision d is determined according to the voting profile and the voting rule. Finally, the state is revealed and agents collect their payoffs.

3.3.3 Strategies and equilibrium

In the voting game, a strategy of agent i is a mapping $\sigma_i : S_i \times S_p \longrightarrow [0, 1]$, where $\sigma_i(s_i, s_p)$ denotes the probability that agent i will vote $v_i = 1$ when observing (s_i, s_p). We will frequently refer to the following two types of (pure) voting strategies (see also KV):

Definition 3.1. *A strategy is* ***informative*** *if* $\sigma_i(s_i, s_p) = s_i$, $\forall s_i \in S_i, s_p \in S_p$.

Definition 3.2. *A strategy is* ***obedient*** *if* $\sigma_i(s_i, s_p) = s_p$, $\forall s_i \in S_i, s_p \in S_p$.

The informative strategy is interesting because it is simple and allows the agent to fully convey her private information via her vote. In addition, as we will show in Section 5, the outcomes of the optimal voting mechanisms with public information can be indirectly implemented by voting procedures that incentivize the agents to play the informative strategy. The obedient strategy is interesting because it is also simple and it can be very appealing in a context where the public signal is considered as a recommendation from someone supposed to be an expert of the issue. The downside of this “follow-the-expert" strategy is that it entirely disregards the agent's private information, which is also informative about the state.[7]

We call a Bayes-Nash equilibrium in which all agents play the informative strategy an *informative voting equilibrium* (IVE). Similarly, a Bayes-Nash equilibrium in which all agents play the obedient strategy will be called an *obedient voting equilibrium* (OVE). For a given preference profile $\mathbf{q}$, if there exists a k-voting rule under which the IVE exists, we say that such a preference profile allows for the existence of the informative voting equilibrium or simply allows for informative voting.

In the absence of public information, if $q_i \in [1-\alpha, \alpha]$ $\forall i \in \mathcal{I}$, it is easy to check that under the simple majority rule the IVE exists and the CJT holds. If all agents are highly biased towards one of the alternatives, we may still be able to sustain informative voting as an equilibrium by using a threshold value different from $(n+1)/2$. For example, if $q_i \in [\alpha, \alpha^3/(\alpha^3+(1-\alpha)^3)]$ $\forall i \in \mathcal{I}$, one can show that the IVE still exists in a voting game with the super-majority rule $k = (n + 3)/2$, and the CJT continues to hold as n becomes sufficiently large (Laslier and Weibull, 2013). In fact, in all the above-mentioned cases the informative voting strategy profile also constitutes an *ex post Nash equilibrium* (Crémer and McLean, 1985), since no agent would ever have a strict

[7]Nevertheless, provided it exists, the equilibrium in which all agents play the obedient strategy maximizes the predicted accuracy of the collective decision among all symmetric equilibria in which the agents use a private-information-independent voting strategy (i.e., $\sigma_i(0, s_p) = \sigma_i(1, s_p)$ $\forall s_p \in S_p$).

incentive to revise her vote even if she could observe the whole voting profile. However, as shown in the next section, the set of preferences that allow for informative voting may shrink drastically in the presence of public information.

3.4 Inefficient Information Aggregation

To see how the presence of a public signal could affect the equilibrium outcome of the voting game, we first provide a necessary and sufficient condition for the existence of the informative voting equilibrium under any given k-voting rule:

Proposition 3.1. *Given a k-voting rule, the informative voting equilibrium exists if and only if*

$$\forall i \in \mathcal{I},\ q_i \in \left[\frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2}\frac{1-\beta}{\beta}}, \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k-n}\frac{\beta}{1-\beta}} \right]. \tag{3.2}$$

In Appendix C, we prove a more general version of Proposition 3.1 which allows the prior probability of the state to be non-uniform and the accuracy of the private signals to be state-dependent. By doing so, we generalize a similar result obtained by Wit (1998) for common-interest voting games with majority rule.

To understand Proposition 3.1, first note that under a given k-voting rule, an agent is pivotal only when there are exactly $k-1$ other agents who vote in favor of the decision $d=1$, while the remaining $n-k$ agents choose to support the decision $d=0$. Second, if agent i prefers to vote according to her private signal even when it conflicts with the public signal, she will also prefer to do so when the two signals agree. Assuming all other agents $j \neq i$ follow the informative voting strategy, for a given k-voting rule, the left (right) endpoint of the interval in (3.2) is the posterior probability that a Bayesian agent i will assign to the event $\theta = 1$ conditional on $s_i = 0, s_p = 1$ ($s_i = 1, s_p = 0$) and being pivotal. Since a rational agent cares only about the cases in which she is decisive about the final voting outcome, we can conclude that all q_i lying between the above two posterior probabilities is a necessary and sufficient condition for the existence of the informative voting equilibrium under the given k-voting rule.

KV observe that if the public signal is more accurate than each of the private signals ($\beta > \alpha$), informative voting for agents who have purely common interests cannot constitute an equilibrium under the majority rule. The next two corollaries, which follow Proposition 3.1 immediately, generalize this important observation to arbitrary precision of the public signal, the whole class of k-voting rules, and a much larger set of preferences.

Corollary 3.1. *Suppose $\beta > \alpha$. For any threshold value k and any preference profile $(q_i)_{i\in\mathcal{I}}$, the informative voting equilibrium does not exist.*

Corollary 3.2. *Suppose $\beta \leq \alpha$. The informative voting equilibrium does not exist under any k-voting rule if there exist $i, j \in \{1, .., n\}$ such that $q_i < 1/\left(1+\frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}\right)$ and $q_j > 1/\left(1+\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)$.*

In words, Corollary 3.1 confirms that whenever the public signal is strictly more precise than each of the private signals, it is impossible to obtain the informative voting equilibrium under any k-voting rule.[8] Meanwhile, Corollary 3.2 implies that even if the public signal is less accurate, it is still hard to guarantee the existence of the informative voting equilibrium as long as there are two or more agents who are sufficiently biased toward different alternatives ex ante. Note that the required bias becomes arbitrarily small when β is close to α.

The intuition behind both corollaries can be understood via the following simple example of three agents with heterogeneous preferences, such that $q_1 = 1 - \alpha$, $q_2 = 1/2$ and $q_3 = \alpha$. Assume that the collective decision is made according to the majority rule ($k = 2$). In the absence of public information, one can check that informative voting constitutes an equilibrium, even though the first and third agents are biased toward different alternatives ex ante. Suppose now agents also observe a public signal that is more informative than each of their private signals. If the unbiased agent 2 assumes that the other two agents will vote informatively, she could infer that the only situation in which she is pivotal is when agent 1 and 3 receive conflicting signals, but this implies that the others' private signals are collectively uninformative about the state. Hence, in this case, agent 2 would make her voting decision by comparing the observed public signal and her own private signal, and simply follows the public one because of its higher precision. Conversely, suppose the public signal is less informative than the private signals. While it is now rational for the unbiased agent 2 to vote informatively (assuming the other two agents do so as well), this is not the case for the two biased agents. For example, agent 1 will still strictly prefer to choose $v_1 = 1$ if $s_1 = 0$ and $s_p = 1$, even when she assumes that the other two agents are voting informatively. This is because the public signal, albeit less informative, is still in favor of her preferred alternative. Moreover, this problem cannot be resolved by using the unilateral ($k = 1$) or unanimity rule ($k = 3$) instead. For example, suppose all three agents are unbiased and the public signal is just slightly more informative than the private signals. While adopting the unanimity rule can successfully encourage agents to vote informatively whenever $s_p = 0$, it provides even stronger incentives for the agents to disregard their private information whenever $s_p = 1$.

Figure 3.1 interprets the above results graphically. Suppose for a given k-voting rule, an agent i with q_i will find it optimal to play the informative voting strategy when assuming that all other agents $j \neq i$ are voting informatively. Let $Q^{\alpha,k}(\beta) \subseteq [0, 1]$ denote the set of all such q_i, for given k, α and β. For a preference profile $\mathbf{q}$, the informative voting equilibrium exists under a given k-voting rule if and only if $q_i \in Q^{\alpha,k}(\beta)$, $\forall i \in \mathcal{I}$. For fixed parameter values $n = 3$ and $\alpha = 0.75$, the top, middle, and bottom part of the gray area in Figure 3.1 corresponds to the graph of $Q^{\alpha,3}(\beta)$, $Q^{\alpha,2}(\beta)$ and $Q^{\alpha,1}(\beta)$, respectively.[9] As the precision of the public signal increases, the size of each $Q^{\alpha,k}(\beta)$ decreases. In particular, when $\beta > \alpha$, $Q^{\alpha,k}(\beta) = \emptyset, \forall k = 1, 2, 3$.

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[8] In general, in this case the strategy profile that the agents would vote for some alternative if and only if it is supported by both private and public signals (while the other alternative is always chosen whenever the two signals disagree) does *not* constitute an equilibrium either. This would be the case, for example, if $\forall i \in \mathcal{I}, q_i \in [1 - \alpha, \alpha]$ and the simple majority rule is used. This is because that whenever an alternative is supported by the more precise public signal, then conditional on all other agents would vote for that alternative if and only if it is also supported by their own private signals, an agents whose private signal disagrees with the public signal would then have the incentive to deviate from the proposed strategy.

[9] For every $\alpha \in (1/2, 1]$, $Q^{\alpha,k}(1/2)$ corresponds to the set of preferences that allow for informative voting under the given k-voting rule when the public signal is absent.

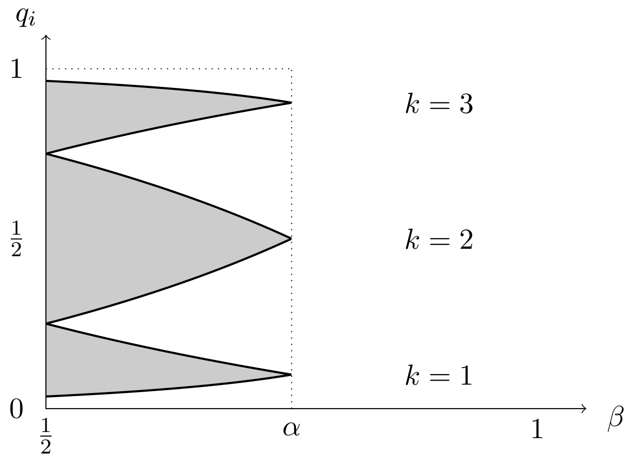

Figure 3.1: The graphs of the correspondences $Q^{\alpha,k}(\beta)$ given $n = 3, \alpha = 0.75$.

Besides shrinking the set of preference profiles that allow for informative voting, the presence of the public signal also opens the possibility for the agents to coordinate on the obedient voting equilibrium. In fact, when $1 < k < n$, the OVE always exists.[10] Clearly, the OVE can be highly inefficient, especially when the public signal is less accurate or just moderately more accurate than each of the private signals.[11] As a numerical example, suppose that $n = 7$, $\alpha = 0.6$ and the simple majority rule is used. By introducing a public signal that is twice as informative as each agent's private signal (i.e., $\beta = 0.69$), the probability of reaching a correct decision can actually decrease (from 0.71 to 0.69) if the agents are induced to switch to the OVE from the IVE. In contrast, if instead we enlarge the size of the committee by two, then the predicted accuracy will increase to 0.73 provided that the agents continue to coordinate on the IVE.[12]

3.4.1 Discussion

Informative voting and information aggregation. The result regarding the (non-)existence of the informative voting equilibrium is interesting for the following reasons. First, the IVE has the desirable property that it can aggregate potentially a large amount of private information by asking the agents to play a simple and intuitive strategy that requires little coordination. Hence, it may seem reasonable to expect that, provided it exists, an equilibrium in informative voting strategies is more likely to be played by the agents than other possibly more sophisticated and/or less efficient equilibria.

Second, if the IVE does not exist, then in many settings obedient voting may already be the most appealing equilibrium prediction of the game. In particular, if there is minimal heterogeneity in preferences (i.e., $q_i \neq q_j$ for some $i, j \in \mathcal{I}$), the OVE is already the most efficient

[10] For $k = 1$, the OVE exists if $\forall i \in \mathcal{I}$, $q_i \geq 1/\left(1 + \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)$. For $k = n$, the OVE exists if $\forall i \in \mathcal{I}$, $q_i \leq 1/\left(1 + \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}\right)$.

[11] For such inefficient use of private information to arise as an equilibrium outcome, it suffices to have more than $n_0 \equiv \max\{k+1, n-k+1\}$ agents who follow the obedient voting strategy.

[12] Suppose that the IVE exists given the preference profile of the original seven-member committee. Then, the IVE also exists after the size of the committee is increased if the preferences of the new members do not exaggerate the initial maximal degree of conflict of interest in the committee (see Proposition 3.4).

equilibrium among the ones that are *symmetric* (with respect to both the agents and the realization of the public signal). The symmetry requirement is important because even in the simplest voting games, there are lots of (and even a continuum of) asymmetric equilibria (see, e.g., the example with three homogeneous agents in p. 417, Guarnaschelli et al., 2000). Thus, in the absence of an effective coordination device, it is rather demanding to require all agents to correctly anticipate which asymmetric equilibrium will be selected. A possible ramification of miscoordination is that the committee may get stuck in the inefficient OVE *outcome* even if some agents make the attempt to deviate from it (e.g., because they may be confused about who should vote informatively and who should follow the public signal).

In fact, KV present strong experimental evidence supporting the above arguments. In particular, for the important benchmark case where all agents are unbiased, they show that if the public signal is *less* accurate than the private signals (thus both the IVE and the OVE, and possibly many other equilibria, exist under the majority rule), most subjects vote for their private signals over 90% of the time when the public and the private signals disagree. In contrast, when the public signal is *more* accurate than the private signals and thus the IVE does not exist, a large proportion of voters tend to follow the public signal instead of their private signals much more frequently than non-OVE equilibria would predict.[13] Consequently, the collective decisions coincided with what the public signal suggested over 97% of the time, which is consistent with the prediction of the OVE.

As pointed out by Feddersen and Pesendorfer (1997), from a game-theoretic point of view the non-existence of the IVE *per se* does not necessarily imply a failure of information aggregation. Indeed, unless we restrict ourselves to symmetric equilibria, in general it is very difficult to rule out the existence of equilibria which may efficiently incorporate both public and private information in a more sophisticated way than the OVE (e.g., by asking agents to play idiosyncratic mixed strategies). It is also possible that these equilibria can lead to asymptotic information aggregation as in Feddersen and Pesendorfer (1997).[14] However, in our view, an important lesson from KV's experimental results is that the non-existence of the IVE *together with* the presence of an "expert opinion" type of public signal can indeed lead to a substantial efficiency loss. A possible explanation is that a fraction $\rho \in (0, 1]$ of agents are somewhat naive, in the sense that they always vote *sincerely* (i.e., they follow the strategy $\sigma_i(s_i, s_p) = 1$ if $\mathbb{E}[\theta|s_i, s_p] \geq q_i$ and $\sigma_i(s_i, s_p) = 0$ otherwise). Assume that the fraction of naive/sincere agents is sufficiently large ($\rho > \alpha - \frac{1}{2}$). It is straightforward to show that whenever the public signal is more precise than the private signals (i.e., it is of the "expert opinion" type), there will a non-vanishing probability for the committee to reach the wrong decision, no matter which k-voting rule is used and what strategies are played by the sophisticated/fully strategic agents.[15]

[13]In their setting, in addition to the obedient voting equilibrium KV also identify a symmetric equilibrium in which the agents play mixed strategies whenever the public signal disagrees with their private ones, and an asymmetric equilibrium in which only a small subset of the agents vote obediently.

[14]Note that we cannot use the general results from Feddersen and Pesendorfer (1997) to conclude that information will be perfectly aggregated as the size of committee increases either. This is because Feddersen and Pesendorfer (1997) assume that all observed signals are conditionally independent between agents.

[15]In contrast, when $\beta \leq \alpha$ holds, efficient information aggregation is possible even if *all* agents are naive ($\rho = 1$). This will be the case, for example, if the agents are unbiased and the simple majority rule is used.

Correlated signals and large elections. So far we have modeled public information as an additional conditionally independent signal that is perfectly observed by all agents. This is equivalent to the assumption that each agent privately observes a *correlated* signal $\hat{s}_i = s_i + \eta s_p$ with $\eta > 1$, as the agents can perfectly back out the signal profile (s_i, s_p) from the realization of $\hat{s}_i$. This assumption fits into applications with committees of moderate size (e.g. boards of directors, hiring committees, juries), since typically in these scenarios not only the public information itself but also its source is clear (e.g., the expert invited to the board meeting, the reference letters submitted to the hiring committees, the witnesses testify in the court). However, this assumption may not capture very well what happen in large elections (i.e., large size committees), where information often comes from multiple sources and is transmitted in a more decentralized way. In that case, an agent may find it difficult to tell for sure what is publicly known from what is her private knowledge. To capture such a situation, we can instead assume that each agent's private signal is given by $\hat{s}_i = s_i + s_p$ (i.e., $\eta = 1$): in this case, an agent becomes uncertain about what is publicly known when she observes $\hat{s}_i = 1$.[16] Despite this uncertainty, assuming $\beta \geq \alpha$, from an individual agent's point of view the correlated signal $\hat{s}_i$ is actually more precise than the independent signal s_i, since the expected conditional variances satisfy $\mathbb{E}[\mathrm{Var}(\theta|\hat{s}_i)] < \mathbb{E}[\mathrm{Var}(\theta|s_i)]$.

In the final part of this section, we will investigate the above more general model with correlated signals and show that how making agents' observations correlated by providing public information can have profound ramifications for information aggregation. Let us fix an arbitrary sequence of preference profiles $\{\mathbf{q}^n = (q_1, ..., q_n)\}_{n\in\mathbb{N}}$. We say that $\{\sigma^{k_n}\}_{n\in\mathbb{N}}$ is a sequence of equilibria *induced by* a sequence of k-voting rules $\{g^{k_n}\}_{n\in\mathbb{N}}$ if $\forall\ n \in \mathbb{N}$, σ^{k_n} constitutes an equilibrium under the voting rule g^{k_n}. We say that a sequence of k-voting rules $\{g^{k_n}\}_{n\in\mathbb{N}}$ *aggregates information asymptotically* if (i) it admits a subsequence $\{g^{k_{n(\tau)}}\}_{\tau\in\mathbb{N}}$ such that $\lim_{\tau\to\infty} k_{n(\tau)}/n(\tau) = \kappa \in [0,1]$, and (ii) $\{g^{k_{n(\tau)}}\}_{\tau\in\mathbb{N}}$ induces a sequence of equilibria in which the probability of reaching a correct decision goes to one. The following result shows that, in general, with correlated signals the probability of reaching the wrong decision may not vanish even in large elections.

Proposition 3.2. *Suppose that each agent only observes a correlated signal $\hat{s}_i = s_i + s_p$. For any sequence of preference profiles, there exists no sequence of k-voting rules that aggregates information asymptotically.*

To sum up, unlike endowing voters with better private information, introducing public information may actually worsen the quality of the collective decision. This is similar to one of the most striking findings in the global games literature, namely the heterogeneous effect of public and private information. For instance, in a highly influential paper, Morris and Shin (2002) show that in a setting where agents' actions are strategic complements, additional public information can have negative social value. Although agents in the current setting have no intrinsic motive of coordination, our results suggest similarly that the conventional wisdom that

[16] While the case $\eta = 1$ may seem to be non-generic, this is largely due to the binary structure of the signals in our model. If, for example, both s_i and s_p are continuously distributed over $[0,1]$, then no η can guarantee that an agent will always be able to perfectly back out both s_i and s_p from the signal $\hat{s}_i = s_i + \eta s_p$.

additional information is always beneficial for decision-makers should be carefully examined.[17]

3.5 Optimal Voting Mechanisms

In this section, we study the design of optimal voting mechanisms with public information. We will show that the outcomes of the optimal mechanisms can be indirectly implemented by simple voting procedures that incorporate the public information appropriately. For clarity of exposition, we will maintain the assumption that the public signal is exogenous throughout this section. We will illustrate in Section 3.6 that our new voting procedures have also additional advantages when strategic information disclosure is a non-negligible concern.

By the revelation principle, we consider only direct mechanisms $f : S \times S_p \to [0, 1]$. The interpretation is that for every signal profile $(s, s_p) \in S \times S_p$ the mechanism specifies the probability $f(s, s_p)$ that the alternative associated with $d = 1$ will be chosen. We start by introducing some definitions.

Definition 3.3. *A mechanism f is* ***ex post incentive compatible*** *if $\forall s_p \in S_p$, $\forall s_{-i} \in S_{-i}$, $\forall s_i, s_i' \in S_i$ and $\forall i \in \mathcal{I}$, $\mathbb{E}\left[u_i(f(s_i, s_{-i}, s_p), \theta)|s_i, s_{-i}, s_p\right] \geq \mathbb{E}\left[u_i(f(s_i', s_{-i}, s_p), \theta)|s_i, s_{-i}, s_p\right]$.*

Definition 3.4. *A mechanism f is* ***responsive*** *(to private information) if $\forall s_p \in S_p$, there exist $s, s' \in S$ such that $f(s, s_p) \neq f(s', s_p)$.*

The notion of ex post incentive compatibility (EPIC) requires every agent to prefer truth-telling at every signal profile (s, s_p) if all the other agents also report truthfully. Similar to the role of dominant-strategy incentive compatibility in private-value environments, EPIC guarantees robust behavior of agents in interdependent-value environments (Bergemann and Morris, 2005). Trivially, an EPIC mechanism exists: For example, the mechanism f_o with $f_o(s, s_p) = s_p \forall (s, s_p) \in S \times S_p$ satisfies ex post incentive compatibility. However, f_o is not a responsive mechanism as it makes no use of the agents' reports. By matching the realization of the public signal, it replicates the outcome of the obedient voting equilibrium discussed in the previous section.

We next introduce a new class of voting rules $g^{k_0,k_1} : \mathcal{V} \to \mathcal{D}$ that we call *contingent k-voting rules*, which can be obtained by adjusting the standard k-voting rules in an intuitive way. In particular, the threshold values in such voting rules will be no longer fixed but a function of the realization of the public signal:

$$k_{s_p} = \begin{cases} k_0 & \text{if } s_p = 0, \\ k_1 & \text{if } s_p = 1, \end{cases} \tag{3.3}$$

where $k_0, k_1 \in \{0, 1, ..., n+1\}$. Any standard k-voting rule amounts to a special case of the contingent k-voting rules with $k_0 = k_1 \in \{1, ..., n\}$. We say that a contingent k-voting rule g^{k_0,k_1} is *responsive* if $k_0, k_1 \notin \{0, n+1\}$. We also say that the voting rule g^{k_0,k_1} *implements* informative voting if it can sustain the informative voting strategy profile as a Bayes-Nash equilibrium in

[17] The non-beneficial effect of public information also resembles the finding from the rational herding literature (e.g., Banerjee, 1992; Bikhchandani et al., 1992). In the models studied in this literature, public information arises endogenously as observed actions taken by previous agents. However, agents who arrive in the future need not be able to fully learn about the state from public observables, as herds or information cascades may arise in equilibrium.

the corresponding voting game. Finally, the voting rule g^{k_0,k_1} is said to be *equivalent* to an EPIC mechanism f if, for every realization of the signals (s, s_p), the probability of reaching the correct decision is the same in both the informative voting equilibrium sustained by g^{k_0,k_1} and the truth-telling equilibrium sustained by f.

Given an EPIC mechanism f, the (ex ante) probability of the collective decision being matched to the state can be computed as follows:

$$\Pr(d = \theta \mid f) = \sum_{s,s_p} \Pr(s, s_p, \theta = 1) f(s, s_p) + \sum_{s,s_p} \Pr(s, s_p, \theta = 0)(1 - f(s, s_p)).$$

Let $\mathcal{F}$ be the set of EPIC mechanisms. An EPIC mechanism f^* is optimal if $\Pr(d = \theta \mid f^*) \geq \Pr(d = \theta \mid f)\ \forall f \in \mathcal{F}$. We are interested in finding an optimal EPIC mechanism. Our next result states that if agents' preferences satisfy a (mild) no-indifference condition, then in the search for optimal EPIC mechanisms it is without loss to focus on contingent k-voting rules that can implement informative voting.

Proposition 3.3. *Suppose that $\forall i \in \mathcal{I}$, $\nexists (s, s_p) \in S \times S_p$ such that $q_i = \Pr(\theta = 1 \mid s, s_p)$. For every optimal EPIC mechanism f, there exists a contingent k-voting rule that is equivalent to f. In addition, the equivalent contingent k-voting rule is responsive if and only if f is responsive.*

Intuitively, the condition in Proposition 3.3 requires that if an agent could observe all the signals, she would always strictly prefer one of the two available alternatives. Taking the information structure and the number of agents as given, there are only finitely many $q \in [0, 1]$ such that $q = \Pr(\theta = 1 \mid s, s_p)$ for some $(s, s_p) \in S \times S_p$. Hence, the set of preferences $\mathbf{q} = (q_1, ..., q_n)$ that violate the no-indifference condition is of measure zero relative to the set of all possible preferences $[0, 1]^n$. We will therefore say that a preference profile $\mathbf{q}$ is *generic* if it satisfies the no-indifference condition in Proposition 3.3.

Proposition 3.3 implies that the search for optimal mechanisms with public information can often be reduced to choosing two threshold values $k_0, k_1 \in \{0, 1, ..., n + 1\}$. This is perhaps surprising, because in general an EPIC mechanism can discriminate agents for their possibly asymmetric preferences, but the contingent k-voting rules are in fact anonymous. The driving force behind this result, as we show in a series of lemmas in Appendix C (Lemmas C.1 - C.3), is that the ex post incentive compatibility constraints have already largely restricted the set of agents who can possibly be pivotal. In particular, the preferences of these agents cannot be too heterogeneous. We remark that these lemmas do not rely on the no-indifference condition and may be of interests that are beyond the scope of the current paper.

Assume that agents' preferences are generic and thus focusing on the contingent k-voting rules is without loss. It would be optimal to choose the extreme threshold values $k_0 = n + 1$ and $k_1 = 0$, for example, if the degree of conflicts of interests in the committee is so large that the only available EPIC mechanisms are the non-responsive ones with $f(s, s_p) = f(s', s_p)$ $\forall s, s' \in S$ and $\forall s_p \in S_p$. While these EPIC mechanisms and their equivalent contingent k-voting rules can incorporate the public information, they disregard all the information privately held by the agents. Hence, provided that a responsive EPIC mechanism exists, there can be an efficiency gain by using its equivalent and responsive contingent k-voting rule to further incorporate the agents' private information. It also seems intuitive that such an efficiency gain should be increasing in the size of the committee. Hence, one may conjecture that a responsive contingent k-voting rule is optimal when the size of the committee is sufficiently large. To formalize and prove this conjecture, we introduce the notion of *conflict-preserving expansion*: Let $\mathbf{q}$ be a preference profile with $\bar{q} \equiv \max_{i\in\mathcal{I}} q_i$ and $\underline{q} \equiv \min_{i\in\mathcal{I}} q_i$. We say that a sequence of preference profiles $\{\mathbf{q}^\tau = (\hat{q}_1, ..., \hat{q}_n, ..., \hat{q}_{n+2\tau})\}_{\tau\in\mathbb{N}}$ is a conflict-preserving expansion of $\mathbf{q}$ if $\forall \mathbf{q}^\tau$, $\bar{q}^\tau \equiv \max_{j\in\{1,...,n+2\tau\}} \hat{q}_j \leq \bar{q}$ and $\underline{q}^\tau \equiv \min_{j\in\{1,...,n+2\tau\}} \hat{q}_j \geq \underline{q}$. In words, an expansion of the committee is conflict-preserving if it does not exaggerate the initial maximal degree of conflict of interest. We are now ready to state the main result of this section, which demonstrates the optimality of responsive contingent k-voting rules.

Proposition 3.4. *Suppose that, for a given preference profile $\mathbf{q}$ with $\underline{q}, \bar{q} \in (0,1)$, there exists a responsive contingent k-voting rule g^{k_0,k_1} that implements informative voting. Then, for any conflict-preserving expansion $\{\mathbf{q}^\tau\}_{\tau\in\mathbb{N}}$ of $\mathbf{q}$:*

(i) For each $\mathbf{q}^\tau$, there exists a responsive contingent k-voting rule $g^{k_0^\tau,k_1^\tau}$ that implements informative voting. This contingent k-voting rule is unique if $\bar{q}^\tau \neq \underline{q}^\tau$, and the corresponding threshold values are given by $k_0^\tau = k_0 + \tau$ and $k_1^\tau = k_1 + \tau$.

(ii) If $\mathbf{q}^\tau$ is generic for all $\tau \in \mathbb{N}$, then there exists τ^*, such that $\forall \tau \geq \tau^*$ there exists a responsive contingent k-voting rule that is equivalent to an optimal EPIC mechanism.

(iii) As $\tau \to \infty$, the ex ante probability of the collective decision being matched to the state in the informative voting equilibrium under any responsive contingent k-voting rule becomes arbitrarily close to 1.

We thus obtain a version of the Condorcet Jury Theorem for the contingent k-voting rules in a general voting environment with both private and public information. In particular, Proposition 3.4 implies that the complete-information outcome can be asymptotically achieved if we incorporate the public information into the voting procedure appropriately. In addition, if agents' preferences are generic, then, for finite but large n, no truth-telling equilibrium under any EPIC mechanism may outperform the informative voting equilibrium under a responsive contingent k-voting rule.

Proposition 3.4 shows that it is often desirable to use a responsive contingent k-voting rule to implement informative voting. Our next result characterizes when such a practice would be feasible.

Proposition 3.5. *For a given preference profile $\mathbf{q}$ with $\bar{q}, \underline{q} \in (0,1)$, there exists a responsive contingent k-voting rule that implements informative voting if and only if there exist integers $k_0, k_1 \in \{1, ..., n\}$ such that*

$$k_0 \in K_0 \equiv \left[(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})\right], \text{ and } k_1 \in K_1 \equiv \left[(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})\right],$$

where

$$(\pi_0^0)^{-1}(\underline{q}) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\underline{q}}{\underline{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2 + r\right), \quad (\pi_1^0)^{-1}(\bar{q}) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + r\right),$$

$$(\pi_0^1)^{-1}(\underline{q}) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\underline{q}}{\underline{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2 - r\right), \quad (\pi_1^1)^{-1}(\bar{q}) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n - r\right).$$

Importantly, there are cases where allowing the threshold to be contingent on the realization of the public signal is *necessary* for implementing informative voting. To see this, consider a simple example with $n = 5$ and $q_i = 1/2, \forall i \in \mathcal{I}$. If there is no public signal, the informative voting equilibrium exists under the simple majority rule. Now let us introduce a public signal that is twice as informative as a private signal. By Corollary 3.1, this implies that the informative voting equilibrium no longer exists under any k-voting rule. However, consider the contingent k-voting rule with $k_0 = 4$ and $k_1 = 2$. Suppose all agents $j \neq i$ are voting informatively. If $s_p = 1$, agent i is pivotal only when three of the other agents draw $s_j = 0$ and the remaining one draws $s_j = 1$. Given the above assumption on the informativeness of the public signal, these private signals are collectively uninformative about the state when they are combined with the realization of the public signal. Thus, voting according to her own private signal is a best response for agent i. Similarly, if $s_p = 0$, agent i is pivotal under the contingent k-voting rule only when there are three private signals in favor of $d = 1$ and one in favor of $d = 0$ among all others' private signals. Again, the collective informational effect of all s_j, $j \neq i$, will be exactly counterbalanced by the fact that $s_p = 0$, which makes it optimal for agent i to simply follow her own signal. Intuitively, what we are doing here is to vary the information that agents can infer from pivotality. Under the contingent k-voting rule chosen in the above example, an agent is pivotal when and only when the private signals of the other agents are collectively more against the alternative favored by the public signal. This restores the incentive for agents to vote according to their own signals.

While both $(\pi_0^0)^{-1}$ and $(\pi_0^1)^{-1}$ are strictly increasing in $\bar{q}$, both $(\pi_1^0)^{-1}$ and $(\pi_1^1)^{-1}$ are strictly increasing in $\underline{q}$. Hence, it is possible that both of the intervals K_0 and K_1 contain no integer if $\bar{q}$ is sufficiently larger than $\underline{q}$. Intuitively, if the degree of conflict of interest between the agents is too large, it is very difficult to find a responsive voting rule that ensures the incentive for *all* agents to vote informatively, even if we allow the voting threshold value to be flexibly contingent on the public signal. Nevertheless, we are able to show that for the important limiting cases where agents' preferences are perfectly aligned (e.g., Feddersen and Pesendorfer, 1998; Koriyama and Szentes, 2009; Persico, 2004), there always exists a responsive contingent k-voting rule that implements informative voting, provided that the size of the committee is sufficiently large:

Corollary 3.3. *Suppose that $\forall i \in \mathcal{I}, q_i = q \in (0,1)$. There exists $\bar{n}(q)$, such that for each $n \geq \bar{n}(q)$, there exists a responsive contingent k-voting rule that implements informative voting.*

3.5.1 Contingent majority rule

In this subsection, we provide further analysis of optimal voting mechanisms for the setting where agents have purely common interests, i.e., $q_i = 1/2$ $\forall i \in \mathcal{I}$. This important benchmark setting has been extensively studied in the literature. Especially, KV show that in this setting if the public signal is r-times as informative as a private signal, where $r \leq (n-1)/2$, then under the simple majority rule there exists an asymmetric equilibrium in which $r^* = \mathbb{N} \cap (r-1, r]$

agents obey the public signal, while the remaining $n - r^*$ agents vote according to their private signals. This r^*-asymmetric equilibrium is shown to be more efficient than both the obedient voting equilibrium and the unique symmetric mixed-strategy equilibrium, as well as all other asymmetric pure-strategy equilibria in the same voting game. In the following, we will show in the same setting that one can always construct a responsive contingent k-voting rule that not only implements informative voting, but also leads to strictly higher efficiency than the r^*-asymmetric equilibrium.

Specifically, consider a contingent k-voting rule with the following threshold values:

$$k_{s_p} = \begin{cases} \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+ & \text{if } s_p = 0, \\ \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+ & \text{if } s_p = 1, \end{cases}$$

where $[(r-1)/2]^+$ denotes the smallest integer that is larger or equal to $(r-1)/2$. For convenience, we will call this rule the *contingent majority rule.* Note that the contingent majority rule is responsive whenever $r \leq n$. The following result justifies our focus on this particular contingent k-voting rule:

Corollary 3.4. *Suppose that $r \leq n$. The contingent majority rule implements informative voting if and only if*

$$\forall i \in \mathcal{I},\ q_i \in Q^{\alpha}_{cm}(r) \equiv \left[\frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{|r-2[(r-1)/2]^+|-1}}, \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{-|r-2[(r-1)/2]^+|+1}} \right]. \tag{3.4}$$

Since $|r - 2[(r-1)/2]^+| \in [0, 1]$ for all $r \geq 0$, we always have $1/2 \in Q^{cm}(r)$. Therefore, for the case where all agents are unbiased, one can always use the contingent majority rule to implement informative voting.[18] The next proposition further shows that the informative voting equilibrium under the contingent majority rule achieves the first-best informational efficiency.

Proposition 3.6. *Given all the information that is available to the committee, the probability of the collective decision being matched to the state is maximized in the informative voting equilibrium under the contingent majority rule.*

To gain some intuition, consider a simple example of $n = 5$ and $r = 2$. Assume all agents are unbiased. Imagine that we introduce two additional phantom agents on top of the existing five real agents. These phantom agents are programmed so that they simply vote in line with the public signal. Suppose now the simple majority rule is used to decide which alternative will be chosen. One can easily show that (1) all *real* agents voting informatively constitutes an equilibrium in this game (despite that the public signal observed by the agents is more precise than each of their private ones), (2) the equilibrium outcome is identical to that of the informative voting equilibrium under the contingent majority rule without the phantom agents, and (3) the equilibrium outcome maximizes the probability that the decision will be matched to the state, given all the available information. Intuitively, by allowing the threshold value to be

[18]The contingent majority rule is also the unique responsive contingent k-voting rule that implements informative voting except when r happens to be is an odd integer.

dependent on the public signal and by encouraging agents to vote informatively, the contingent majority rule aggregates both the private and the public information efficiently.

On the contrary, in the r^*-asymmetric equilibrium, inefficiency still prevails because there are r^* agents who always disregard their valuable private information. To see this issue more clearly, consider again the above example. Since in this case we have $r^* = 2 = (n-1)/2$, under the simple majority rule there exists an asymmetric equilibrium in which two agents play the obedient strategy, while the remaining three agents vote informatively. Without loss of generality, assume the first two agents are the obedient voters. Consider the signal profile $s = (1, 1, 0, 1, 1)$ and $s_p = 0$. In equilibrium, such a realization of signals will lead to a collective decision $d = 0$. However, from a benevolent social planner's point of view, given all the available information, the welfare maximizing decision should be $d = 1$. Therefore, the r^*-asymmetric equilibrium is strictly less efficient than the first-best.

3.5.2 Equivalent implementations

The analysis of contingent k-voting rules has highlighted the importance, especially in terms of the potential efficiency gain, of having a more flexible voting procedure that can appropriately incorporate the content of the public information. In practice, however, it might be difficult to implement (or even just prespecify) a voting rule that is contingent on some public information. If it is known that the public information comes from some *non-strategic* expert,[19] an alternative and rather straightforward way to achieve the informative voting equilibrium outcome under a contingent k-voting rule is to also count the expert's opinion as a vote. In addition, the expert's vote should be counted with a larger weight if he is indeed better informed than the agents ($\beta > \alpha$). Similar voting rules are used, for example, in the famous reality television singing competition *The Voice*.[20]

Plainly, the above alternative voting rule would not be feasible if the source of the relevant public information is ambiguous ex ante. We now introduce a simple two-stage voting mechanism that is immune to this concern. The voting rule is as follows. After observing the private and the public signals, the agents first vote to select an integer $k \in \{0, 1, ..., n+1\}$. The integer k^* that receives the most votes will be selected, with ties being broken randomly. In the second stage, the agents vote about which collective decision to take according to the voting rule g^{k^*}, i.e., $d = 1$ if and only if $\sum_{i=1}^n v_i \geq k^*$. Practically, this two-stage voting procedure can be more appealing than the contingent k-voting rules because the procedure itself is deterministic and independent of the informational details of the environment.

Fix a preference profile $\mathbf{q}$, and suppose that there exists a contingent k-voting rule g^{k_0,k_1} that implements informative voting. We say that the above two-stage voting mechanism can *equivalently implement* the informative voting equilibrium under g^{k_0,k_1}, if in the two-stage voting game there exists a Perfect Bayesian Nash equilibrium (PBNE) in which the agents first collectively vote to agree on the threshold value that would have been chosen by g^{k_0,k_1}, and then

[19]See Section 3.6 for an analysis with strategic experts.

[20]In some rounds of the singing competition, both the coaches of the singers (or a jury of music professionals) and the viewers can vote to determine whether a contestant can advance to the next stage. However, the votes of the coaches (or the professionals) typically count more for the final outcome (see `https://en.wikipedia.org/wiki/The_Voice_(franchise)`).

they vote informatively in the second stage.

Proposition 3.7. *Suppose that, for a given preference profile* $\mathbf{q}$*, there exists a contingent* k*-voting rule* g^{k_0,k_1} *that implements informative voting. Then, the two-stage voting mechanism can equivalently implement the informative voting equilibrium under* g^{k_0,k_1}*.*

The intuition behind Proposition 3.7 is simple: Since the voting threshold k^* is determined by a simple plurality rule, no agent could unilaterally change the voting outcome in the first stage if all other agents agree to choose either k_0 or k_1. But then given that the informative voting strategy profile constitutes an ex post Nash equilibrium under g^{k_0,k_1}, no agent would have the incentive to deviate from informative voting in the second stage either, no matter how she updates her beliefs about the state and other agents' private information after observing the voting outcome of the first stage.[21]

We close this section by noting that the use of the plurality rule for determining the voting threshold is not generally necessary for our result. To see this, suppose, for example, that the agents have purely common interests and *either* of the following rules (i) and (ii) is used in the first stage: (i) If all agents agree to use some $k \in \{0, 1, ..., n+1\}$, then we let $k^* = k$. Otherwise, the simple majority rule will be used, i.e., $k^* = (n+1)/2$; (ii) An agent $i \in \mathcal{I}$ is randomly selected, and then she dictates which voting threshold to be used. These alternative two-stage voting mechanisms can also equivalently implement the informative voting equilibrium under the contingent majority rule. The reason is that, according to Proposition 6, the expected social welfare is maximized when the voting threshold values of the contingent majority rule are used. Since each agent's interest is perfectly aligned with the social welfare, any deviation in the first stage will only yield a lower expected payoff to an agent.[22]

3.6 Strategic Information Disclosure

In this section, we drop the assumption that the disclosure of public information is exogenous, and consider it to be strategically determined by a possibly biased *information controller* (e.g., an external expert). As illustrated in Section 3.4, public information can have a huge influence on the committee's decision when the standard simultaneous voting rules are in use. Taking this into account, a biased controller may only publicly reveal his information to the agents when

[21] A legitimate concern that one may have is that the two-stage voting mechanism does not *uniquely* implement the informative voting equilibrium. This problem is not specific to the two-stage voting rule that we propose. For example, even when the public signal is absent and all agents are unbiased, the simple majority rule also cannot achieve the unique implementation of the informative voting equilibrium (it is always an equilibrium that all agents vote for $d = 1$ (or $d = 0$) regardless of their private signals). Nevertheless, given that the informative voting equilibrium is symmetric (thus no sophisticated coordination is required) and can have very desirable efficiency properties (see Propositions 3.4 and 3.6), it seems reasonable to expect that it is more likely to be played by the agents compared to other possibly asymmetric and/or less efficient equilibria in the two-stage voting mechanism.

[22] When agents are unbiased (or, more generally, when condition (3.4) is satisfied), we can consider yet another voting rule: Let $\hat{v}_i = \eta v_i$ if $v_i \neq s_p$, and $\hat{v}_i = \eta' v_i$ if $v_i = s_p$, where $\eta = \frac{n+1}{n+1-2[(r-1)/2]^+}$ and $\eta' = \frac{n+1}{n+1+2[(r-1)/2]^+}$. The decision $d = 1$ is taken if and only if $\sum_{i=1}^n \hat{v}_i \geq (n+1)/2$. It is straightforward to check that an informative voting equilibrium exists under this voting rule, and it is outcome-equivalent to the one under the contingent majority voting rule. Since $\eta \geq \eta'$, this voting rule has the interesting interpretation that it discourages the agents from blindly following the public information, and it does so by assigning a larger weight to the votes that disagree with the public signal.

its content is in support of his favored alternative. For example, an advisory board member who has private interests in the targeted firm may withhold unfavorable information from the directory board when the acquisition decision is being made. In what follows, we will formalize this intuition by extending our baseline model from Section 3.3. In addition, we will show that using a contingent voting rule adapted from the ones constructed in Section 3.5 can mitigate the controller's incentive for strategic disclosure and his influence on the voting outcome, which in turn improves the efficiency of the collective decision.[23]

Suppose now that the signal s_p described in Section 3.3.2 is no longer public by default but can only be observed by an information controller with some probability. Specifically, with probability $\lambda \in (0,1)$, the controller is uninformed and can only send a public message $m = \emptyset$ (remain silent) to the agents. With the complementary probability $1-\lambda$, the controller observes the signal and can decide whether to publicly communicate its content to the agents or not. While we allow the controller to withhold his information, we assume that the signal is hard information and hence cannot be faked. In other words, in the latter case the public message m can only be chosen from the set $\{s_p, \emptyset\}$.

Assume for simplicity that $q_i = 1/2\ \forall i \in \mathcal{I}$ and that the collective decision is made according to the simple majority rule.[24] Note again that in this case, the informative voting equilibrium exists if no additional public information is available to the committee members. Assume also that the controller has the same form of utility function as the agents, and his bias parameter is given by $q_c \in [0,1]$. Let $\hat{\lambda} = \max\{0, (\beta-\alpha)/(\beta-(1-\alpha)\}$. The following proposition establishes that if agents update their beliefs sufficiently little upon observing silence (i.e., λ is large enough), a biased information controller may indeed exploit the publicity of his message and reveal his information selectively.

Proposition 3.8. *Suppose $\lambda \geq \hat{\lambda}$. There exists $\hat{q} \in [1-\beta, 1/2]$ such that if $q_c \leq \hat{q}$ ($q_c \geq 1-\hat{q}$), then there exists a sequential equilibrium in which the controller sends $m = s_p$ if and only if he observes $s_p = 1$ ($s_p = 0$), and the agents vote obediently if $m = s_p$ and informatively if $m = \emptyset$.*

As a numerical example, if $n = 3$, $\alpha = 0.65$ and $\beta = 0.7$, the threshold values are given by $\hat{\lambda} \approx 0.14$ and $\hat{q} \approx 0.48$, respectively. Depending on the relative precision of the signals, the informational efficiency of the committee's decision could be improved if there were more or less information disclosure than that in the equilibrium. For instance, the decision will be more accurate in the above numerical example if the controller always keeps silent and just lets the agents credibly coordinate on informative voting in the voting stage.

Some recent papers look at the question how an information controller can optimally persuade *uninformed* agents by designing the *informational content* of a public signal (e.g., Alonso and Câmara, 2016; Bardhi and Guo, 2018; Wang, 2015). In our model, voters are privately informed and the controller has control over the disclosure of the public signal only. Hence, our environment is notably less favorable for the controller. Nevertheless, Proposition 3.8 suggests that the strategic incentive of the controller and his impact on the committee's decision still

[23]The same result will also hold if we use the two-stage voting mechanism (see Section 3.5.2) instead.

[24]With the general results in Sections 3.4 and 3.5, our analysis in the current section can be straightforwardly extended to settings with general preference profiles and voting rules.

cannot be ignored.[25]

Fortunately, the concern of strategic information disclosure can be mitigated by instead using a contingent voting rule with the following threshold values:

$$k_m = \begin{cases} \frac{n+1}{2} & \text{if } m = \emptyset, \\ \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+ & \text{if } m = 0, \\ \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+ & \text{if } m = 1. \end{cases}$$

Proposition 3.9. *Suppose that $r \leq n$ and the proposed contingent voting rule is used. There exists $q^* \in [0, 1-\beta]$ such that*

1. *If $q_c \in [q^*, 1-q^*]$, there exists a sequential equilibrium in which the controller sends $m = s_p$ whenever he is informed, and the agents always vote informatively.*
2. *If $q_c \leq q^*$ ($q_c \geq 1-q^*$) and $\lambda \geq \hat{\lambda}$, there exists a sequential equilibrium in which the controller sends $m = s_p$ if and only if he observes $s_p = 1$ ($s_p = 0$), and the agents always vote informatively.*

By comparing Propositions 3.8 and 3.9, we can see that the contingent voting rule has two main advantages over the simple majority rule. First, the contingent voting rule incorporates the informational content in the controller's message appropriately and makes it credible for the agents to coordinate on informative voting in the voting stage. Hence, by the same reasoning as in Proposition 3.6, the decision selected by the contingent voting rule is most likely to match the state, given all the information that is available to the committee, independent of the controller's disclosure policy and the relative precision of the signals. Second, under the contingent voting rule the controller also has a higher incentive to share his information unconditionally, since he anticipates that his message will not have a direct impact on the agents' voting behavior and will always help increase the accuracy of the committee's decision. Indeed, for the previous numerical example ($n = 3$, $\alpha = 0.65$ and $\beta = 0.7$), we have $q^* \approx 0.23$, which is substantially smaller than $\hat{q}$.

3.7 Conclusion

This paper makes two main contributions. First, we show in a general setting of collective decision-making that the provision of public information can have a detrimental effect on the efficiency of the committee decision. The inefficient equilibrium outcome is consistent with experimental evidence, and it echos the common concern that expert opinions may have excessive

[25]This result does not necessarily hold, however, if the controller himself is also a member of the committee. This is because other members in the committee may anticipate that the information contained in the controller's message will already be incorporated in his vote. For example, suppose that the controller is agent 1, $q_i = 1/2$ $\forall i = 2, ..., n$, $\beta \geq \alpha$ and the simple majority rule is used. One can check that if $q_c \in [1-\beta, \beta]$ and $r < 2$, then regardless of whether the controller reveals his signal to the other agents or not in the communication stage, it will be incentive compatible for all agents including the controller to vote informatively in the voting stage . This example suggests that the public information emerges from pre-voting deliberations is less likely to threaten the existence of the informative voting equilibrium. We cannot, however, conclude from this that there is no value in pre-voting deliberations, because informative voting per se does not necessarily lead to the efficient outcome under other voting rules (e.g. the unanimity rule).

influence on (both individual and collective) decision-making. We believe these results to be of high policy relevance, especially since the immense influence of public information may be strategically exploited by a biased information controller.

Second, we propose simple voting procedures that can indirectly implement the outcomes of the optimal ex post incentive compatible mechanisms. By appropriately incorporating the public information and providing incentives for the agents to vote informatively, our voting procedures facilitate information aggregation and enhance the accuracy of the collective decision. By reducing the direct effect of public information on the agents' voting behavior, the proposed voting procedures also mitigate the concern of strategic information disclosure.

It should be remarked that our results are not suggesting that experts should be discouraged from providing their expertise to decision makers. For example, besides providing additional information, advice from experts may also help decision makers to better assess the situation based on their private knowledge (i.e., the precision of the private signals α increases). Intuitively, this effect should be beneficial for increasing the probability of reaching the correct decision. Therefore, we would like to highlight that the key message of this paper is that in a voting environment with both private and public information, the voting procedure matters and the optimal voting rule should reflect the content of the public information. For example, if the advisory board indicates that one of the business proposals is more promising than the other, it might be desirable for the board of directors to set up a voting rule that is more in favor of the acceptance of that proposal. The design of optimal mechanisms in more general social choice environments with public information remains an open and important research question.

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4 Designing Organizations in Volatile Markets[1]

Joint with Dimitri Migrow

4.1 Introduction

Many modern organizations operate in multiple markets. The most immediate example, perhaps, is that of multiproduct firms: Apple offers both smartphones and watches, BMW sells both cars and motorcycles, and Google's business is not restricted to running a search engine. In the era of globalization, multinational firms provide another important case in point. According to a recent study by Lincoln and McCallum (2018), the median number of destination countries for U.S. exporting firms was three in 2006. Moreover, for many top U.S. exporters, selling domestically produced goods to foreign consumers only counts as a limited part of their involvement in the world economy. For instance, it is common nowadays for a multinational corporation to own production facilities in several foreign countries.[2]

Motivated by the prevalence and increasing influence of multibusiness firms in the economy, this paper asks the following question: When local markets (defined by products, industries, geographic boundaries, demographics of targeted customers, etc.) feature uncertainty in their relative profitability, how should a multi-divisional organization optimally allocate decision-making authority to its managerial members? In particular, should decision rights be *centralized* to a headquarter manager who can coordinate the activities of different division contingent on the market prospects, or be *decentralized* to division managers who have advantages in collecting costly information about local market conditions?[3]

The uncertainty in relative market profitability is a highly relevant problem for many organizations. A broad set of economic and political conditions, which may be difficult to predict, can affect how rewarding it is to conduct activities in a product market or in a country. Thus, the value of success in a particular local market in terms of overall organizational performance is uncertain from the central management's perspective. For multinational firms, a major source of such uncertainty is the volatility of currency exchange rates.[4] With physically different prod-

[1]This paper should be cited as Liu, S. and D. Migrow (2019): "Designing Organizations in Volatile Markets," Mimeo.

[2]For an overview of the stylized facts about multinational firms documented in the international trade literature, see, e.g., the comprehensive survey by Antràs and Yeaple (2014).

[3]The design of multi-divisional organizations is an active area of research. See, e.g., Athey and Roberts (2001), Alonso et al. (2008, 2015), Dessein et al. (2010), Friebel and Raith (2010), and Rantakari (2008, 2013). Section 4.2 reviews several prominent contributions in this growing literature. We refer interested readers to the excellent survey by Roberts and Saloner (2013).

[4]For example, even without considering the change in tariffs due to the U.S. - China trade war, the recent sharp depreciation of Chinese Yuan already implies a drop in the dollar value per sale that Ford can collect from its joint venture in China (Changan Ford).

ucts, uncertainty regarding the relative profitability or strategic importance of the markets may also be due to general shifts in consumer tastes or changes in market size.[5]

If the organizational activities in different local markets are unrelated, relative changes in market profitability may be of little relevance for central management. However, quite often an organization can benefit from synchronizing its activities across markets (e.g., because of economies of scale), though doing so may imply that these activities are less adapted to the local conditions of each market (e.g., product design is less fitted to the tastes of local consumers). When the prospect of each market cannot be perfectly forecasted, resolving the trade-off between *coordination* and *adaptation* is particularly challenging, because ex ante it is unclear whether the organization should adapt more to one market's local conditions and less to the other's. By centralizing the decision-making process, the headquarter manager can take into account the actual profitability conditions and make contingent decisions that are globally optimal for the organization. Yet, to the extent that local market information is privately acquired and observed by division managers, the flexibility granted by centralization also comes with a downside. That is, it may harm the incentives of the division managers by making them more skeptical about how their acquired information will be used.

The main insight of our paper is that whether the above cost of centralized decision-making may outweigh its benefit depends crucially on how important coordination is compared to adaptation, and in an unexpected way. In particular, due to a reinforcing interaction between the uncertainty in market profitability and the need for coordination, the optimality of a *decentralized* authority structure can be the result of a large coordination motive. In addition, as an important step toward establishing the optimality results, we show that if the information acquired by the division managers is verifiable, complete voluntary disclosure arises as a *unique* equilibrium outcome irrespective of the chosen authority structure.

We formalize our arguments by modeling an organization which needs to adapt and coordinate the strategic decisions of its two divisions. As in Alonso et al. (2008) and Rantakari (2008), a division's performance is determined by how close its action (e.g., the design of a product) is matched to an unobserved *local state*, and how well it is coordinated with the action of the other division. Specifically, any mismatch between division i's action and its local state or division j's action will result a quadratic loss in i's performance. Each division is run by an agent (e.g., a divisional manager, he) who can privately exert effort to acquire a signal about the local state, where more effort results in a better signal. The agents are led by a common and uninformed principal (e.g., a headquarter manager, she). While each agent cares only about the performance of his own division, possibly because of career concerns, the principal cares about the *overall* performance of the organization. The novel feature of our model is that the contribution of each division's success to the overall organizational performance need not be certain. This idea is formally captured by a pair of stochastic weights that the principal's payoff attaches to the divisions' performances. While these weights are observed by all players before

[5] Making precise predictions about these factors can be challenging even for large firms. In 2013, soon after announcing his stepping down as the CEO of Microsoft, Steve Ballmer openly admitted that the company was too slow to recognize the importance of the smartphone market. He blamed this strategic failure on Microsoft's long-time focus on the business of its Windows operating system (see "Microsoft too slow on phones, admits boss Steve Ballmer", *BBC News*, 20 September 2013).

the final actions are taken, they are unknown at the outset of the game and can be arbitrarily correlated. As mentioned, examples of such interrelated uncertainty include the exchange rate volatility incurred by multinational corporations, as well as the constantly changing prospects of different product markets. We refer to these stochastic weights as the *global states* of our model because, unlike the local states, they determine which actions are globally optimal for the organization rather than locally optimal for individual divisions.

We compare two widely-studied authority structures: centralization and decentralization. In both cases the agents first exert efforts to acquire information about the local states, and then they communicate their findings with the player(s) endowed with decision-making authority. Specifically, under centralization, the agents simultaneously report to the principal, who will subsequently dictate the actions of both divisions. Under decentralization, the agents can exchange messages with each other, after which they make independent decisions over the actions of their own divisions.[6] The communication between players is strategic and takes the form of verifiable disclosure, where an informed agent always has the option of certifying the outcome of his information experiment. This includes the persuasion game of Milgrom (1981) and Grossman (1981) and the evidence game of Dye (1985) as special cases. Previous works have argued that the quality of communication is important for explaining the relative performance of different organizational structures (e.g. Alonso et al., 2008; Aoki, 1986; Dessein and Santos, 2006; Rantakari, 2008). Our model predicts that if information is verifiable, the incentive constraints for communication are irrelevant in determining where the authority over decisions should be lodged in the organization. As we prove in Section 4.4, fully revealing communication arises as a unique equilibrium outcome regardless of which authority structure is chosen (Propositions 4.1 - 4.4). Thus, in equilibrium all the obtained information will be truthfully transmitted to the decision-making parties. The full-revelation result is not obvious, because it is known that costly information acquisition and/or uncertain information endowment can prevent complete voluntary disclosure (e.g., Shavell, 1994; Shin, 1994).[7]

While the resulting quality of communication does not differ between centralization and decentralization, the allocation of decision rights does have an impact on the agents' incentives for information gathering. In Section 4.5, we first establish a benchmark result (Theorem 4.1(i)) that if the local markets are always equally profitable, then, regardless of the importance of coordination, a centralized organization always outperforms its decentralized counterpart in motivating information acquisition. This optimality of centralized decision-making shows that the incentive view of delegation in Aghion and Tirole (1997) need not be valid in multi-agent settings with coordination motives. Intuitively, as the principal always deems the two divisions equally important, under centralization she acts as if she were a neutral party who aims to maximize the joint surplus of the agents. In contrast, the decentralized equilibrium outcome

[6]The comparison between centralization and decentralization is only meaningful if contracts are incomplete as in Grossman and Hart (1986) and Hart and Moore (1990), because otherwise any decentralized allocation can be implemented centrally by a suitably designed mechanism. Thus, similar to Alonso et al. (2008) and Rantakari (2008), our analysis applies to situations where the organizational decisions of interests are sufficiently complex (e.g., product design), which renders ex ante contracting infeasible.

[7]Moreover, as shown in Section 4.6.2, if the space of the local states is unbounded and the agents are able to misrepresent their private information at a cost, then in addition to full revelation the equilibria under different authority structures will also feature language inflation (Kartik, 2009; Kartik et al., 2007).

fails to achieve the same efficiency because of the conflicting interests between the agents. This coordination failure lowers the marginal benefit of information and thus discourages information acquisition under decentralization. Hence, given the fully revealing communication equilibrium outcome, the principal is better off retaining the decision rights when doing so can motivate the agents to acquire more information.[8]

Since the local markets are symmetric ex ante, the equal-profitability condition in the benchmark result above is satisfied if and only if the global states are perfectly and positively correlated. This is a knife-edge case, and the picture changes as soon as we move away from it. A key finding of our paper, stated in Theorem 4.1(ii), is that provided there is *any* uncertainty in the relative profitability of the local markets, decentralization will outperform centralization in terms of information gathering if coordination is sufficiently important. To understand the result, note that under centralization the principal would prioritize the adaptation problem of the division that turns out to be more profitable. The information passed on by the less profitable division may thus receive little attention. Not surprisingly, since (i) the agents cannot perfectly forecast the relative profitability of the local markets ex ante and (ii) the loss from misadapting to one's local state is convex, the uncertainty in the ex post value of information tends to discourage information gathering. What is less obvious, perhaps, is the following reinforcing interaction between this negative effect and the need for coordination. As coordination becomes more important, knowledge about local market conditions plays less of a role in the principal's choice of actions. However, since the adaptation problem of the more profitable division is still relatively more important, the decrease of influence in decision-making is more substantial for the less profitable division. Hence, compared to the case where the agents are autonomous, a large coordination motive can be much more harmful for their information-gathering incentives when the principal is in charge.

Next, in Theorem 4.2, we show that if the distribution of the global states is sufficiently volatile, then the agents would also acquire more information under decentralization when coordination is of little importance relative to adaptation. Further, by fully characterizing the cases where the global states are binomially distributed, we demonstrate that with high volatility the comparative advantage of decentralization in motivating information acquisition may even hold regardless of the importance of coordination (Propositions 4.5 and 4.6). Thus, the more volatile the local markets, the larger the motivational benefits of decentralization.

Whether and when the above benefits of decentralization can outweigh the cost of losing control is not obvious, because it is exactly when the local markets are highly volatile that the principal would most appreciate the flexibility granted by centralization. We show in Theorems 4.3 and 4.4 that the answer depends largely on the convexity of the information cost . If the information cost is sufficiently convex, the additional gain in information quality from decentralization is at most minor, so it would not be optimal for the principal to transfer the decision rights to the agents. However, if the cost of information is not too convex, the drop in infor-

[8]The optimality result of centralized decision-making echoes the recent experimental findings by Brandts and Cooper (2018). Assigning the subjects with different managerial roles and endowing them with *exogenous* information, their experimental design simulates how two parallel divisional decisions are made in an organization. They find that the subjects are surprisingly honest in communication and that the organizational performance is higher when decision rights are allocated to a central manager.

mation quality due to centralization is substantial enough to make decentralization optimal. Then, given that strong coordination motives widen the gap in information quality between centralization and decentralization, we arrive at the novel prediction that the importance of coordinating organizational activities can actually strengthen rather than weaken the optimality of decentralized decision-making.

Our results have direct implications on how formal authority over critical decisions should be allocated between organizational agents, which is a design architecture central to the stories of success (or failure) of many modern corporations.[9] More broadly, the results are also related to the core debate in economics on the role of (de)centralized systems in information aggregation and decision-making. Perhaps most famously and influentially, Friedrich von Hayek argued that the problem of rapid adaptation to local changes must be solved by "some form of decentralization" (Hayek, 1945, p. 524), since knowledge about local conditions is dispersed among individual agents rather than existing in concentrated form. However, if adaptation decisions are interdependent and their relative importance is uncertain, efficient use of information may also require some centralized coordination. We show that centralized decision-making need not suffer from the information asymmetry that Hayek criticized, and it is indeed more efficient in adapting to *existing* information. However, decentralization can still be optimal once the *endogeneity* of information is taken into account. This suggests that the fundamental advantage of decentralized systems is in information production.

Our results can shed light on some business cases of multi-divisional corporations. For example, it has been widely discussed that Japan's multinational mobile phone makers perform very poorly in the overseas markets.[10] In particular, many of them, such as NEC and Panasonic, were already struggling in the global competition in the 2000s, which was even before the smartphone era. Their unsatisfactory performance may be explained by the traditional centralized decision-making process of Japanese multinationals (Bartlett and Ghoshal, 2002; Bloom et al., 2012) *and* the large economies of scale of the mobile phone industry. For instance, from their entry to China in 1995 until 2002, NEC and Panasonic "released only a few models that were adapted from their models in Japan" (Marukawa, 2009, p. 428). This practice probably had a lot to do with the fact that many components of the Japanese phones were manufactured domestically rather than overseas in low-wage countries, so the cost-saving benefit from coordinating the choices of handset models across borders could be especially significant. But then, according to our theory it is not surprising that these phone makers were slow in learning some basic differences in the distribution structures and consumer tastes between Japan and China (see, e.g., Marukawa, 2009). Both NEC and Panasonic withdrew their mobile phone business

[9]See, for example, Freeland (2001) on the history of General Motors in the 1920's - 1960's. Narrative evidence supporting the importance of authority allocations for organizational performance includes the reform of decision-making structure that Louis Gerstner implemented soon after he became the CEO of IBM, which is considered to be a key factor that lead to the firm's success in the 1990's. See, for instance, Gerstner's memoir of his tenure in IBM (Gerstner, 2002) as well as the discussion by Malone (2004). Another case in point is the remarkable failure of the merger of Chrysler and Daimler in 1998. As convincingly argued by Garicano and Rayo (2016), a fatal problem of the merged company, DaimlerChrysler AG, was its poorly-designed allocation of authority. For more systematic empirical evidence, see, e.g., Aghion et al. (2017), and Thomas (2011).

[10]See, e.g., online articles "Why Japan's cellphones haven't gone global?", *New York Times*, 19 July 2009, "What happened to Japan's electronic giants?", *BBC News*, 2 April 2013, and "NEC and the sorrow of Japan", *Boy Genius Report*, 17 July 2013.

from China around 2006, and from the entire global market around 2013.

Of course, centralization is not always inferior to decentralization. For example, for a company that sells both smartphones and personal computers (e.g., Apple and Microsoft), there may be very limited benefits from coordinating the design of these two products because they are supposed to serve different consumer needs. In that case, our theory suggests that it is unlikely that empowering the product managers would lead to much more informed decision-making. Thus, it is more important that the allocation of decision rights does not constrain the company's ability to react promptly to the changing prospects of different products (e.g., by allocating resources across divisions). This may help to understand why Steve Ballmer decided to massively reorganize Microsoft in 2013, moving the governance of the company closer to its very centralized peer Apple.[11]

The paper is organized as follows. Section 4.2 discusses the related literature. Section 4.3 introduces the model. Section 4.4 characterizes the equilibria under different organizational structures. Building on the characterization results, Section 4.5 studies the optimal organizational structure. Section 4.6 presents two extensions, one concerning the roles of monetary transfers (Section 4.6.1), while the other considers imperfectly verifiable information (Section 4.6.2). Section 4.7 concludes. All proofs are relegated to Appendix D.

4.2 Related Literature

The organizational problem of coordinated adaptation under dispersed information has a long intellectual history in organizational theory and economics (see, among many others, Barnard, 1938; Cyert and March, 1963; Simon, 1947; Williamson, 1975, 1996). Our paper belongs to a growing strand of this literature, which examines how an organization's decision-making structure can determine its ability to coordinate the activities of its sub-units while remaining responsive to changes in the local environments. Specifically, our model builds on the framework developed by Alonso et al. (2008) and Rantakari (2008), which are among the first papers to model strategic information transmission in the context of designing multi-divisional organizations. They focus on the case where information is "soft", meaning that communication between organizational members takes the form of cheap talk (Crawford and Sobel, 1982). One of their most insightful findings is that as the need for coordination increases, the communication of decision-relevant information under centralization (decentralization) becomes less (more) informative. This implies that the comparative advantage of an authority structure need not be monotone in the importance of coordination (Rantakari, 2008). In addition, if the interests of the local managers are sufficiently aligned, then the optimality of decentralized decision-making is not necessarily inconsistent with a large need for coordination (Alonso et al., 2008).[12] Our model departs from theirs mainly by (1) focusing on the case where information is "hard" (Grossman, 1981; Milgrom, 1981) or at least is costly to misrepresent (Kartik, 2009; Kartik et al., 2007),

[11] See "Microsoft overhauls, the Apple way", *New York Times*, 11 July 2013.

[12] While both Alonso et al. (2008) and Rantakari (2008) assume that the private information of the managers is exogenous, their main results are subsequently shown to be robust to endogenous information acquisition (Rantakari, 2013). Their models have also been extended to more than two divisions (Yang and Zhang, 2017).

and (2) relaxing the (implicit) assumption that the local markets where the organization operates exhibit no uncertainty in their profitability conditions. More important than the modeling differences, we add to Alonso et al. (2008) and Rantakari (2008, 2013), and more generally to the literature of organizational design and coordinated adaptation, by showing that the importance of coordination can even make decentralized decision-making optimal. In particular, this result holds despite the fact that in our model the allocation of decision rights does not affect the informativeness of communication at all, and that the conflicts of interests between the local managers are maximal (as they only care about their own divisions).

Within the literature on organizational design and coordinated adaptation, our paper is further related to Dessein et al. (2010), Friebel and Raith (2010), and Alonso et al. (2015). In Dessein et al. (2010), the organization can better exploit the benefits of cost-saving standardization by integrating its manufacturing activities. Standardization, however, also comes with a loss in revenues because it impedes the organization's ability to tailor its marketing activities to local conditions. Dessein et al. (2010) find that a more decentralized authority structure can better incentivize the managerial members of the organization to exert division-specific effort, but it is still dominated by a more centralized one if the expected value of synergies (akin to the importance of coordination in our model) is sufficiently large. Thus, unlike in our paper, the advantage of decentralized decision-making in incentivizing effort provision is thwarted rather than strengthened by the importance of coordinating activities across organizational units.

In line with Alonso et al. (2008) and Rantakari (2008), both Friebel and Raith (2010) and Alonso et al. (2015) consider settings where the top management of the organization is constrained (and often also harmed) by its informational disadvantage compared to the division managers. In Friebel and Raith (2010), delegating resource-allocating rights to the division managers can be optimal since they control the information about the marginal return of their projects. But delegation can also be sub-optimal because sometimes it is more profitable to concentrate all resources on a single project. In Alonso et al. (2015), the headquarter may be better off by letting the division managers choose their production plans independently given that they know more about the demand conditions of each market, but the opposite may also occur since the costs of production are interdependent. Nevertheless, if the division managers were *non-strategic* in communication, then both the models of Friebel and Raith (2010) and Alonso et al. (2015) would conclude that it is always optimal to have the decisions centrally made. In contrast, in our model, even without the help of message-contingent transfers, the division managers are always incentivized to be truthful when communicating their private information to the decision-making parties.

Finally, we contribute to the literature on delegation as an instrument to motivate information acquisition. The seminal work of Aghion and Tirole (1997) introduces an important trade-off between employee initiative and the loss of control. In their framework, an agent has to acquire decision-relevant information and has better incentives to do so when being able to formally control the decision. With multiple agents and partial coordination motives, the ability of an agent to influence the decisions is restricted by the optimal behavior of the other agents. In fact, in our multi-agent setting, absent the uncertainty in the principal's (interim) decision rule, the agents' incentives for information acquisition are always weaker under delegation (Theorem

1(i)). Nevertheless, we show that this pessimistic view of delegation need not hold once some uncertainty over the principal's decision rule is introduced (Theorems 1(ii) and 2). Thus, the driving force of the motivational advantage of delegation in our model is different from that in Aghion and Tirole (1997).

Recent contributions show that the incentive effect of delegation can be ambiguous if the communication between the principal and the agent is strategic.[13] For example, in Argenziano et al. (2016), the principal can benefit from retaining the decision-making authority while delegating the task of information acquisition to the agent. This is because the principal may either threaten the agent with a babbling off-path if information gathering is overt, or obstinately expect the information to be highly precise if it is acquired covertly. The finding that centralizing the authority to the principal can better motivate the agent to acquire information compared to delegation is shared by Che and Kartik (2009). A key driving force of their result is that the principal and the agent hold different priors about the state of nature ("opinions"), so under centralization whenever the latter fails to provide any evidence the former would make an adverse inference and take an unfavorable action.[14] All papers above focus on settings with a single agent, whereas ours feature multiple ones.[15] This modeling difference is not superfluous. As we show, the incentive effect of delegation (decentralization) crucially depends on the interaction between the need for coordinating the agents' actions and how their relative performance is valued by the principal.

4.3 The Model

An organization consists of two operating divisions, $i, j \in \{1, 2\}$, $i \neq j$. Division i's performance (e.g., profits/sales generated, number of patents obtained) is determined by its local conditions, described by $\theta_i \in \mathbb{R}$, and two actions $\mathbf{y} = (y_1, y_2) \in \mathbb{R}^2$:

$$\pi_i(\mathbf{y}, \theta_i) = K - (y_i - \theta_i)^2 - \delta(y_1 - y_2)^2,$$

where $K > 0$ is some constant, and $\delta > 0$ measures the importance of coordinating actions within the organization. Each *local* state θ_i is independently and identically distributed according to a commonly known distribution Γ with support $\Theta \subseteq \mathbb{R}$. We normalize the mean of the distribution to zero ($\mathbb{E}[\theta_i] = 0$) and assume that it has a finite variance $\sigma_\theta^2 = \mathbb{E}[\theta_i^2] > 0$.

Each division i is run by an agent (e.g., a division manager, he), which we will refer to as agent i. Before any action is taken, each agent i can privately invest effort $e_i \in E = [0, 1]$ in

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[13]Abstracting from strategic communication, the incentive view of delegation is also discussed by Rantakari (2012). He shows that formal delgation is unlikely to be optimal when the quality of implementable projects is determined by both the principal's and the agent's effort choices. The reason is that an unconstrained agent would only be interested in improving the private return of his project. In contrast, under centralization, for his project to be implemented the agent would also need to make it sufficiently attractive to the principal.

[14]A similar persuasive motive of information acquisition under centralization is also present in Newman and Novoselov (2009). In their setting, the principal and the agent share a common prior about the state of nature, but they differ in the costs of committing different types of statistical errors.

[15]Kartik et al. (2017) show that if the principal cannot commit to decision rules ex ante, then having multiple agents compete with each other does not necessarily encourage information acquisition. In their setting, the efforts of the agents are (endogenously) strategic substitutes, whereas in ours, the equilibrium effort choices are strategically independent (see Propositions 4.2 and 4.4).

acquiring information about the local state of his division. Specifically, by choosing an effort level $e_i \in E$, agent i incurs a cost of $c(e_i)$ and receives a perfectly revealing signal $s_i = \theta_i$ with probability e_i. With probability $1 - e_i$, the agent receives a null signal $s_i = \emptyset$. Thus, the agent's effort enhances the probability that the true state will be revealed by the signal (Green and Stokey, 1981).[16] The realization of the signal is referred to as the agent's type. We assume a twice-differentiable, strictly increasing and strictly convex cost function $c : E \to \mathbb{R}_+$. Each agent cares about the performance of his own division. In particular, the ex post payoff of agent i is given by

$$u_i(\mathbf{y}, \theta_i, e_i) = q\pi_i(\mathbf{y}, \theta_i) - c(e_i),$$

where $q > 0$ captures the marginal benefit for the agent to increase his division's performance (e.g., price of sales, monetary bonus, promotion opportunities). For analytical convenience, we assume throughout the paper that the marginal cost of information is sufficiently small at $e = 0$ (e.g., $c'(0) < q\sigma_\theta^2/2$) and is sufficiently large at $e = 1$ (e.g., $c'(1) > q\sigma_\theta^2$) to ensure an interior solution $e_i \in (0, 1)$.

The agents are led by a common and uninformed principal (e.g., a headquarter manager, she), whose payoff depends on the performance of both divisions and a stochastic vector $\boldsymbol{\eta} = (\eta_1, \eta_2)$:

$$\pi_P(\mathbf{y}, \boldsymbol{\theta}, \boldsymbol{\eta}) = \eta_1\pi_1(\mathbf{y}, \theta_1) + \eta_2\pi_2(\mathbf{y}, \theta_2). \tag{4.1}$$

Thus, η_i measures the marginal benefit for the principal from increasing division i's performance. As we have discussed in the introduction, there are many economic scenarios where the principal may care about the performance of different divisions to different extents (i.e., $\eta_1 \neq \eta_2$). In the context of multinational corporations, π_i can be profits measured in country i's currency, and η_i is the currency exchange rate between country i and the country where the headquarter is located. Another interpretation is that π_i is a measure (e.g., market share) which summarizes the firm's performance in market i *relative* to its competitors, while η_i reflects demand uncertainty such as changes in market size or in preference intensity ("fashion"). Alternatively, if π_i and q are the number and the price of sales in product market i, then we may have $\eta_i = q - \gamma_i$, where $\gamma_i \geq 0$ is the per unit cost for the headquarter to supply division i with necessary resources. Finally, we may instead assume that $\eta_i = q + \gamma_i$, and then interpret γ_i as a parameter that captures (in reduced-form) the strategic importance of succeeding in product market i (e.g., gaining competitive advantage through brand-building or consumer habit-forming).[17]

We assume that the random variables η_1 and η_2 are drawn according to some symmetric and commonly known joint probability distribution $F(\eta_1, \eta_2)$ on the support $\left[\underline{\eta}, \bar{\eta}\right]^2$, where $\bar{\eta} > \underline{\eta} > 0$. The values of η_1 and η_2 are realized and publicly observed *after* the agents have acquired

[16]The assumption that an agent's can only acquire an "all-or-nothing" signal simplifies the analysis, but it is not crucial. Our main results can be extended to more general settings where the precision of the signal is increasing in the agent's effort, in the sense that the expectation of the conditional variance $\sigma^2_{\theta_i|s_i} = \mathbb{E}\left[(\theta_i - \mathbb{E}[\theta_i|s_i])^2\right]$ is decreasing in e_i.

[17]The specification of the utility function (4.1) is also open to the "behavioral" interpretation that the principal has some intrinsic biases and thus favors the two agents unequally. This interpretation relates our model to the growing literature on behaviorally biased managers/supervisors. See, e.g., Prendergast and Topel (1996), Giebe and Gürtler (2012), and Letina et al. (2018).

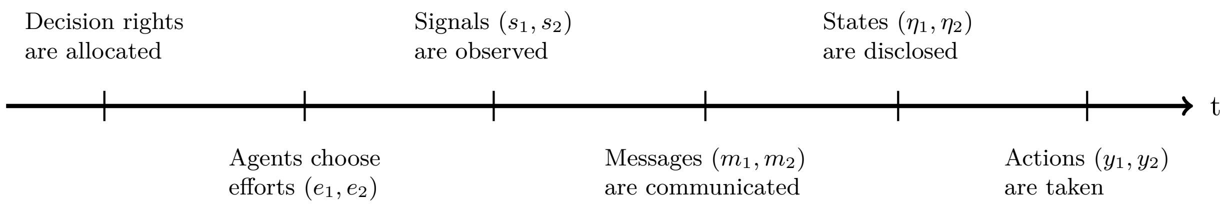

Figure 4.1: Timing of Events

information about their local states (θ_1, θ_2) and *before* the decisions (y_1, y_2) are taken.[18] The uncertainty due to (η_1, η_2) is different from the uncertainty coming from the local states (θ_1, θ_2). First, unlike the local states, η_1 and η_2 are not required to be independently distributed, reflecting the observation that various economic environments are correlated, possibly in a rather complex way (e.g., when η_1 and η_2 are currency exchange rates). Second, from the principal's perspective, how the decision rules of different divisions should be optimally interlinked is determined by the *relative* value of η_1 and η_2. If, for example, $\eta_1 > \eta_2$, the principal would prefer agent 1 to adapt more aggressively towards his local state and agent 2 to focus more on coordination. In other words, η_1 and η_2 determine which actions are globally optimal for the organization rather than locally optimal for individual divisions. We will therefore refer to them as the *global* states of our game. All model parameters are common knowledge.

We complete the model description by specifying how exactly information is communicated and decisions are taken under centralization and decentralization, respectively. Under centralization, the principal takes the decisions (y_1, y_2) after communicating with both agents.[19] Under decentralization, each agent takes the decision of his own division after communicating with each other. Figure 4.1 summarizes the timing of events in our model.

Independent of the allocation of decision rights, we assume that in the communication stage the agents can credibly reveal their findings about the local states if they want to do so. In particular, conditional on receiving a signal $s_i \in \mathcal{S} = \Theta \cup \{\emptyset\}$, agent i can send a message $m_i \in \mathcal{M}(s_i)$ to either agent j (under decentralization) or the principal (under centralization), where we denote $\mathcal{M} = \cup_{s_i \in \mathcal{S}} \mathcal{M}(s_i)$ and assume that the signal-dependent message spaces satisfy the condition below.[20]

Assumption 4.1. $\emptyset \in \mathcal{M}(\emptyset)$*, and* $\forall s_i \neq \emptyset$, $\exists\, m^{s_i} \in \mathcal{M}(s_i) \setminus \cup_{s_i' \neq s_i} \mathcal{M}(s_i')$

The essential requirement of assumption 4.1 is that an *informed* agent can always certify his type (Seidmann and Winter, 1997). In particular, whenever the message m^{s_i} is communicated, the receiving party will know for sure that agent i has learned the value of the local state θ_i, which is equal to s_i. However, since the message $m = \emptyset$ is not necessarily only available to type

[18]Information about (η_1, η_2) may also arrive exogenously before the agents have exerted efforts. Here, an implicit assumption is that such information can be summarized in the common prior F.

[19]As will become clear in Section 4.4.2, our main results hold regardless of whether the uncertainty of (η_1, η_2) resolves before or after the agents communicate with the principal under centralization.

[20]Games with signal-independent message spaces ($\mathcal{M}_i(s_i) = \mathcal{S}\ \forall s_i \in \mathcal{S}$) and costly lying are studied in Section 4.6.2.

$\emptyset$, 4.1 allows the possibility that an agent may not be able to prove that he is uninformed. The assumption thus accomodates a large class of communication games. For example, it is satisfied by the evidence game introduced by Dye (1985), where $\mathcal{M}(s_i) = \{s_i, \emptyset\}$ $\forall s_i \in \mathcal{S}$, i.e., the agents can always hide but cannot fake their findings about the local conditions. It is also satisfied by the persuasion game studied by Grossman (1981), Milgrom (1981), and Milgrom and Roberts (1986b), where $\mathcal{M}(s_i) = \{S \subseteq \mathcal{S} : s_i \in S\}$ $\forall s_i \in \mathcal{S}$, i.e., the agents cannot lie but they may send "vague" messages about their findings. Finally, while the assumption rules out pure cheap talk communication, it nevertheless permits the following game of cheap talk with certification: $\emptyset \in \mathcal{M}(\emptyset)$, and $\mathcal{M}(s_i) = \mathcal{S} \cup \{c^{s_i}\}$ if $s_i \neq \emptyset$, where $c^{s_i} \neq c^{s_i'}$ $\forall s_i \neq s_i'$. The interpretation is that agent i can either send a non-verifiable message to claim that his type is $\tilde{s}_i \in \mathcal{S}$, or provide a certification to truthfully reveal the signal he has received. However, such a certification is not necessarily available when the agent has failed to obtain any informative signal.

We are interested in how the overall organizational performance is shaped by the interaction between authority allocation and the model's primitives, in particular δ and F (i.e, the coordination motive and the uncertainty/volatility of local market profitability). To answer this question, we first derive and analyze the respective perfect Bayesian equilibria (PBE; Fudenberg and Tirole, 1991, p. 333) of the games under centralization and decentralization (see Section 4.4). We show that under either of the two organizational structures, full revelation of agents' private signals can always be sustained as part of an equilibrium. Moreover, this is essentially the unique equilibrium outcome of the communication game. We then characterize (i) the agents' effort provision and (ii) the principal's expected payoff in the corresponding PBE, which are uniquely pinned down given the full-revelation communication, and use them to measure the performance of the organization. The main results on the optimal allocation of decision rights are presented in Section 4.5.

4.4 Equilibrium Analysis

4.4.1 Decentralized authority

We first analyze the game under decentralization. As mentioned, decentralization means that each agent has full control over the decision of his own division. Since the global states (η_1, η_2) only affect the principal's payoff, they are irrelevant for the agents' incentives under decentralization.

Formally, the strategy of each agent $i \in \{1, 2\}$ is a triple (e_i^d, m_i^d, y_i^d) where $e_i^d \in E$ is his effort to acquire decision-relevant information, m_i^d is a mapping that specifies for every given effort-signal pair (e_i, s_i) which message $m_i^d(e_i, s_i) \in \mathcal{M}(s_i)$ agent i will send to agent j, and y_i^d is a decision rule specifying the agent's action $y_i^d(e_i, s_i, m_i, m_j)$ conditional on the effort-signal pair (e_i, s_i) and the messages (m_i, m_j). In equilibrium, each agent i's choices of effort, messages and actions must be sequentially rational with respect to his beliefs (about θ_i, e_j and s_j), which are formed using Bayes' rule whenever applicable. In addition, since the message sets are signal-dependent, we further require that for every $m_j \in \mathcal{M}$ agent i's posterior belief about agent j's signal s_j, which we denote by $\mu_i^j(\cdot|m_j) \in \Delta(\mathcal{S})$, must be *consistent* (Milgrom and Roberts, 1986b). Mathematically, this requires that $\mu_i^j(\mathcal{S}^{m_j}|m_j) = 1$ $\forall m_j \in \mathcal{M}$, where

$\mathcal{S}^{m_j} = \{s_j \in \mathcal{S} : m_j \in \mathcal{M}(s_j)\}$ is the set of signals which could possibly make the message m_j available to agent j.

Our first proposition shows that under decentralization there is essentially a unique equilibrium outcome of the communication stage: despite the conflicts of interests, both agents are incentivized to reveal all their private information.

Proposition 4.1. *Consider the decentralized authority structure.*

(i) Suppose that $\forall m \in \mathcal{M}$, $\mathcal{S}^m$ is closed. Then, there exists a fully revealing PBE in which $m_i^d(e_i, s_i) = m^{s_i}$ and $m_i^d(e_i, \emptyset) = \emptyset$, $\forall s_i \in \Theta$, $\forall e_i \in E, \forall i = 1, 2$.

(ii) If a communication strategy m_i^* is part of a PBE, then $\mu_j^i(\{s_i\}|m_i^*(e_i^*, s_i)) = 1$ for almost all $s_i \in \mathcal{S} \setminus \{0, \emptyset\}$ with respect to Γ.

The existence of a fully revealing equilibrium is not obvious. In particular, it is known that complete voluntary disclosure need not arise in equilibrium when information is costly to acquire (e.g., Shavell, 1994) and/or when the possibility that the sender is uninformed cannot be ruled out (e.g., Shin, 1994). To gain the intuition for Proposition 4.1(i), consider an agent i who observes $s_i > 0$ (and thus knows θ_i) and contemplates a deviation from the fully revealing strategy m_i^d. As we require in the proof, agent j always *assumes the worst* in the spirit of Milgrom and Roberts (1986b): for every message $m_i \in \mathcal{M}$ observed, j would think that i's type is for sure $\underline{s}^{m_i} \in \arg\min_{s_i \in \mathcal{S}^{m_i}} |\mathbb{E}[\theta_i|s_i]|$, i.e., the one that minimizes the distance between j's posterior and prior expectations about θ_i among all types who have access to the message m_i. This implies that by deviating to any message $m_i \neq m^{s_i}$, i could only manipulate j to think that on average the local state θ_i is lower than s_i (i.e., $\mathbb{E}[\theta_i|m_i] \leq s_i$).

Now imagine, for the sake of the argument, that agent i knows that j has either received a signal $s_j = \emptyset$ or $s_j = \theta_j \leq \theta_i$. Given that the sequentially rational action for agent j is a weighted average of his posterior expectations of θ_j and y_i, the above manipulation is not profitable for agent i because it will mislead j to take an action even further away from what would have been ideal for i. In contrast, if agent j is known to have received a signal $s_j = \theta_j > \theta_i$, deceiving j to underestimate the value of θ_i could be tempting for agent i, since it may move j's action closer to i's local state θ_i than what j would have chosen otherwise. Of course, as the communication game is simultaneous, when deciding which message to send agent i does not know which of the above two cases j's signal falls into. However, since $\mathbb{E}[\theta_j] = 0$ and $\theta_i = s_i > 0$, agent i does know that either or both of the followings must hold: (i) $\Pr(\theta_j \leq \theta_i) \geq \Pr(\theta_j > \theta_i)$, i.e., a priori $\theta_j \leq \theta_i$ is a more likely scenario compared to $\theta_j > \theta_i$; (ii) $|\mathbb{E}[\theta_j|\theta_j \leq \theta_i]| \geq \mathbb{E}[\theta_j|\theta_j > \theta_i]$, i.e., the distribution Γ assigns a substantial weight to values that are far smaller than θ_i. Hence, on average the losses from mis-coordination and mis-adaptation are minimized when agent i reveals his type by sending the message m^{s_i} to j.[21]

The second part of Proposition 4.1 establishes that full revelation of private information is essentially the unique prediction of the communication game under decentralization. In any

[21] One may envision invoking the general results of Hagenbach et al. (2014) to prove the existence of a fully revealing equilibrium in the current model. But their results are not directly applicable to our problem because they do not consider endogenous information acquisition.

equilibrium, after the bilateral communication the agents can always be (almost) sure about each other's types, except possibly when the distribution Γ admits an atom at $\theta_i = 0$ and an agent may use the same message for types 0 and $\emptyset$. However, this exception is not payoff-relevant because knowing whether $s_i = 0$ or $s_i = \emptyset$ will not affect the subsequent decisions of the agents. If the distribution Γ is discrete, then the result can be proved by adapting the well-known unraveling argument (Grossman, 1981; Milgrom, 1981). More specifically, in our setting, if several types of agent i are using the same message $m \in \mathcal{M}$, then at least one of them, say s_i, would find that his finding is being understated ($|E[\theta_i|m]| < |E[\theta_i|m^{s_i}]| = s_i$). Thus, by deviating to the type-revealing message m^{s_i} agent i could convince j to take decisions that are more favorable to i in expectation. In the proof, we show how this intuitive argument can be generalized to arbitrary distributions, including the ones that are partly discrete and partly continuous.

Given that the private signals are truthfully revealed in equilibrium, the decision rules (y_1^d, y_2^d) are uniquely pinned down on the equilibrium path. Thus, when calculating the expected payoffs of the agents, the decision rules can be written as functions of the private signals $\mathbf{s} = (s_1, s_2)$ only. Taking these action functions and agent j's effort e_j as given, agent i then solves the following maximization problem at the information acquisition stage:

$$\max_{e_i \in [0,1]} U_i^d(e_i, e_j) = \mathbb{E}_{\boldsymbol{\theta}} \left[\mathbb{E}_{\mathbf{s}} \left[u_i \left(y_1^d(\mathbf{s}), y_2^d(\mathbf{s}), \theta_i, e_i \right] |e_i, e_j \right] \right]. \tag{4.2}$$

It turns out that (4.2) admits a unique solution $e_i^d \in (0, 1)$, which is independent of the effort choice of agent j. Hence, the equilibrium outcome at the information acquisition stage is also unique under decentralization. The findings about the stages of decision-making and information acquisition are summarized in the next proposition.

Proposition 4.2. *In any fully revealing PBE under decentralization, the on-path equilibrium decisions are given by*

$$y_i^d(s_i, s_j) = \frac{1+\delta}{1+2\delta} \cdot \mathbb{E}[\theta_i|s_i] + \frac{\delta}{1+2\delta} \cdot \mathbb{E}[\theta_j|s_j], \ \forall i, j = 1, 2.$$

In addition, both agents exert the same effort

$$e_1^d = e_2^d = e^d \equiv (c')^{-1} \left(\left(1 - \frac{\delta^2 + \delta}{(1+2\delta)^2} \right) q\sigma_\theta^2 \right).$$

Thus, the equilibrium effort level e^d is increasing in q and σ_θ^2. Intuitively, this is because an increase in q or σ_θ^2 leads to a larger expected loss of being uninformed. Further, as formally shown in Appendix D, e^d is decreasing in δ. This is also intuitive: a higher need for coordination makes adaptation less important from the agents' perspective and thus decreases the value of information.

4.4.2 Centralized authority

In this section, we analyze the game under centralization. Recall that in this case the principal has full control over the decisions of both divisions. Thus, in contrast to the case of decentralization, when making their effort choices and communicating their signals, the agents take into account how the global states (η_1, η_2) may affect the principal's decisions.

Under centralization, each agent i's strategy is a pair (e_i^c, m_i^c), where $e_i^c \in E$ is his effort to acquire information about his local state θ_i and m_i^c is a mapping that specifies for every given effort-signal pair (e_i, s_i) which message $m_i^c(e_i, s_i)$ he reports to the principal. The principal's strategy is a pair of mappings (y_1^c, y_2^c), where $y_i^c(m_i, m_j, \eta_1, \eta_2)$ is the action that the principal takes for division i when receiving messages (m_i, m_j) from the agents and observing the global states (η_1, η_2). In equilibrium, each agent i chooses the effort level and signal-dependent messages that maximize his expected payoff, and the principal chooses actions that are sequentially rational with respect to his beliefs (about $\boldsymbol{\theta}$, $\boldsymbol{s}$ and $\boldsymbol{e}$), which are formed using Bayes' rule whenever applicable. Similar to the case of decentralization, we require that for every $m_j \in \mathcal{M}$ and $j \in \{1, 2\}$ the principal's posterior belief about agent j's type, which we denote by $\mu_p^j(\cdot|m_j) \in \Delta(\mathcal{S})$, must be consistent. That is, $\mu_p^j(\mathcal{S}^{m_j}|m_j) = 1 \; \forall m_j \in \mathcal{M}$.

The next result parallels Proposition 4.1 in the previous section. It shows that the principal need not be concerned about the agents strategically manipulating their reports under centralization, as they are incentivized to fully reveal their private information in equilibrium.

Proposition 4.3. *Consider the centralized authority structure.*

(i) Suppose that $\forall m \in \mathcal{M}$, $\mathcal{S}^m$ is closed. Then, there exists a fully revealing PBE in which $m_i^c(e_i, s_i) = m^{s_i}$ and $m_i^c(e_i, \emptyset) = \emptyset$, $\forall s_i \in \Theta$, $\forall e_i \in E, \forall i = 1, 2$.

(ii) If a communication strategy m_i^* is part of a PBE, then $\mu_p^i(\{s_i\}|m_i^*(e_i^*, s_i)) = 1$ for almost all $s_i \in \mathcal{S} \setminus \{0, \emptyset\}$ with respect to Γ.

The intuition of Proposition 4.3 is similar to that of Proposition 4.1. Together, our full-revelation results suggest that the allocation of decision rights does not affect the quality of communication in the organization. This finding can even be extended to settings where the message sets are type-independent. In Section 6 we show that fully revealing equilibria also arise in a communication game with $\mathcal{M}(s_i) = \mathcal{S} \; \forall s_i \in \mathcal{S}$ and costly exaggeration. Our results are in sharp contrast to Alonso et al. (2008) and Rantakari (2008), who show that if information about local states is dispersed and held by agents who communicate via cheap talk, the relative performance of different authority structures depends crucially on their endogenous quality of communication. Moreover, our results show that centralized decision-making does not suffer from the usual problem of information asymmetry, as the principal can elicit all the information from the agents even without the help of contingent transfers. Thus, different from related works such as Dessein (2002) and Deimen and Szalay (2018), strategic communication does not give rise to a trade-off between loss of control under delegation/decentralization and loss of information under centralization in our setting.

Given that the private signals are truthfully revealed in equilibrium, the principal's decision rules (y_1^c, y_2^c) are uniquely pinned down on the equilibrium path. Thus, when calculating the expected payoffs of the agents, the decision rules can be written as functions of the private signals $\mathbf{s} = (s_1, s_2)$ only. Especially, the action chosen by the principal for each division will be a weighted sum of the conditional expectations $\mathbb{E}[\theta_i|s_i]$ and $\mathbb{E}[\theta_j|s_j]$, while the weights will depend on the realization of the global states. Taking the principal's on-path decision rules and agent j's effort e_j as given, agent i then solves the following maximization problem at the information acquisition stage:

$$\max_{e_i \in [0,1]} U_i^c(e_i, e_j) = \mathbb{E}_{\boldsymbol{\theta}} \left[\mathbb{E}_{\mathbf{s}} \left[\mathbb{E}_{\boldsymbol{\eta}} \left[u_i(y_1^c(\mathbf{s}, \boldsymbol{\eta}), y_2^c(\mathbf{s}, \boldsymbol{\eta}), \theta_i, e_i)\right] | e_i, e_j\right]\right]. \tag{4.3}$$

Similar to the parallel problem under decentralization, (4.3) admits a unique solution $e_i^c \in (0,1)$, which is independent of e_j. Using the symmetry of the distribution of (η_1, η_2), we then obtain the following result:

Proposition 4.4. *In any fully revealing PBE under centralization, the on-path equilibrium decisions are given by*

$$y_i^c(s_i, s_j, \eta_i, \eta_j) = \frac{\frac{\eta_i}{\eta_i+\eta_j} \cdot \left(\frac{\eta_j}{\eta_i+\eta_j} + \delta\right) \mathbb{E}\left[\theta_i|s_i\right] + \frac{\delta\eta_j}{\eta_i+\eta_j}\mathbb{E}[\theta_j|s_j]}{\frac{\eta_i}{\eta_i+\eta_j} \cdot \frac{\eta_j}{\eta_i+\eta_j} + \delta}, \ \forall i, j = 1, 2.$$

In addition, both agents exert the same effort

$$e_1^c = e_2^c = e_F^c \equiv (c')^{-1}\left(\left(1 - \mathbb{E}_\lambda\left[\frac{\delta^2(\lambda^2 + (1-\lambda)^2) + 2\delta\lambda^2(1-\lambda)^2}{2(\lambda(1-\lambda)+\delta)^2}\right]\right) q\sigma_\theta^2\right),$$

where $\lambda \equiv \eta_1/(\eta_1 + \eta_2)$.[22]

Similar to the case of decentralization, the equilibrium effort level e_F^c is unambiguously increasing in q and σ_θ^2. In addition, as we show in Appendix D, e_F^c is decreasing in δ. Intuitively, the value of information decreases in the importance of coordination because it makes adaptation less important from the perspective of all players. It is less clear, however, how the equilibrium effort level depends on the distribution of the global states (η_1, η_2). We investigate this question in the next section as we compare the effort provision under both organizational forms.

4.5 Comparing Organizational Structures

Having analyzed separately the fully revealing equilibria under centralization and decentralization, we now ask which allocation of decision rights is optimal for the organization. In our model, an immediate candidate for the criterion of optimality is the principal's expected payoff. Since communication is fully revealing and the principal directly controls the divisional decisions under centralization, a sufficient (necessary) condition for her to benefit more from a centralized

[22]Note that the distribution of λ can be derived from the joint distribution of (η_1, η_2):

$$\Pr(\lambda \le x) = \int_{[\underline{\eta}, \bar{\eta}]^2} \mathbb{1}_{\{\eta_1/(\eta_1+\eta_2) \le x\}} dF(\eta_1, \eta_2) \forall x \in \mathbb{R}.$$

(decentralized) authority structure is the extent of the agents' effort provision.[23] Hence, comparing agents' efforts under centralization and decentralization provides a useful stepping stone for answering the question of which allocation of decision rights is optimal for the principal. Moreover, the comparison of effort provision can be of interest per se, especially if one is concerned that our model may not capture *all* the benefits of learning for the organization. With these motivations in mind, in what follows we will start by analyzing the relative performance of the organization in terms of effort provision (Section 4.5.1). The analysis of the principal's expected payoff will then be presented in Section 4.5.2.

4.5.1 Effort provision

Propositions 4.2 and 4.4 directly imply that the equilibrium effort level is higher under decentralization than that under centralization ($e^d > e_F^c$) if and only if

$$D(\delta) = \frac{\delta^2 + \delta}{(1+2\delta)^2} < C_F(\delta) = \mathbb{E}\left[\frac{\delta^2(\lambda^2 + (1-\lambda)^2) + 2\delta\lambda^2(1-\lambda)^2}{2(\lambda(1-\lambda)+\delta)^2}\right], \tag{4.4}$$

where we recall that $\lambda = \eta_1/(\eta_1+\eta_2)$, and we drop the subscript λ from the expectation operator for brevity.

To understand the above condition, note that when choosing his effort, an optimizing agent balances the marginal benefit and the marginal cost of information. Condition (4.4) is then equivalent to the statement that the marginal benefit of information is higher under decentralization than that under centralization. More specifically, if the agent had the right to choose actions for *both* divisions, he can always make sure that they are perfectly coordinated ($y_j = y_i$). In this case, the expected benefit of exerting an additional unit of effort will be $q\sigma_\theta^2$, which is exactly the payoff difference between taking an informed and ideal decision ($y_i = \theta_i$) and an uninformed one ($y_i = \mathbb{E}[\theta_i]$). When decision rights are decentralized, each agent only has control over his own division, and coordination is no longer perfect due to conflicting interests between the agents. Hence, from the agents' perspective, the value of information is impaired by the need of coordination, and so the marginal benefit of being better informed decreases to $\text{MB}^d = (1 - D(\delta))q\sigma_\theta^2$. Under centralization, the principal may better coordinate the actions of the divisions. However, because the *relative* profitability of the local markets (η_i/η_j) is uncertain, a forward-looking agent would be concerned that the principal may prioritize the adaptation problem of the other division and thus pay little attention to his acquired information. These effects jointly determine the marginal benefit of information under centralization, which is $\text{MB}^c = (1 - C_F(\delta))q\sigma_\theta^2$. It is then straightforward to check that $\text{MB}^d > \text{MB}^c$ if and only if (4.4) holds.

Exploiting the limiting properties of the functions $D(\delta)$ and $C_F(\delta)$ in (4.4), our first theorem below shows that, except for the knife-edge case where the global states are perfectly and positively correlated, a decentralized organization outperforms its centralized counterpart in terms of incentivizing effort provision (or information gathering) whenever coordination is sufficiently important.

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[23] If the set of feasible effort choices is binary and is given by $E = \{0, \bar{e}\}$, where $\bar{e} \in (0, 1]$, then $e^d > e_F^c$ is not only necessary but also sufficient for concluding that the principal is better off under decentralization.

Theorem 4.1. *Let $corr(\eta_1, \eta_2)$ be the correlation of the global states.*

(i) If $corr(\eta_1, \eta_2) = 1$, then $e^d < e^c_F$ $\forall \delta > 0$.

(ii) If $corr(\eta_1, \eta_2) < 1$, then $\exists\, \bar{\delta} \in [0, +\infty)$, such that $e^d > e^c_F$ $\forall \delta > \bar{\delta}$. In addition, the difference $e^d - e^c_F$ is increasing in δ $\forall \delta > \bar{\delta}$.

To see the intuition, remember that under decentralization the agents are free to adjust their actions according to the acquired information. However, as in many settings with partial coordination motives and lack of commitment, the decentralized equilibrium outcome is not Pareto efficient for the agents. In contrast, whenever the global states are perfectly and positively correlated, under centralization the principal acts as a neutral party maximizing both agents' payoffs. In this case, a centralized authority structure effectively allows the agents to commit to the efficient action plans for *any* given information acquired. As a result, regardless of the importance of coordination, from an individual agent's perspective the marginal benefit of information is highest when decisions are centrally made by the principal.

However, the picture changes as we move away from the knife-edge case of a perfectly predictable profitability ratio $\eta_1/\eta_2 = 1$. Ex post, the performances of the two divisions may not be equally profitable/important for the principal, so she only aims to maximize a *weighted* sum of the agents' surplus. Thus, even though the principal values both divisions equally *on average* (i.e., $\mathbb{E}[\eta_1]/\mathbb{E}[\eta_2] = 1$), centralization is less valuable for the agents as a commitment device because of the uncertainty of the global states. Importantly, this negative uncertainty effect is further amplified by the need for coordination: as the latter increases, a biased principal gives substantially less consideration to her unfavored agent's report. This happens because she primarily wants the more profitable division to adapt more aggressively to its local state while minimizing the mis-coordination costs.[24] Eventually, the positive commitment effect of centralized decision-making is dominated by the negative effect due to the volatile profitability of the local markets, leading to an increasing gap in effort provision between decentralization and centralization.

Part (ii) of Theorem 4.1 establishes that a decentralized organization induces more efforts from the agents *if* the importance of coordination exceeds some cutoff value $\bar{\delta} \geq 0$. One may wonder whether the converse of this statement also holds, i.e., whether it is the case that a centralized organization is better in terms of effort provision whenever coordination is sufficiently *unimportant*. Note that this question is only meaningful if $\bar{\delta} \neq 0$. In the next subsection, we show that the lower bound $\bar{\delta} = 0$ can indeed be achieved by some distributions, implying that in those cases decentralization outperforms centralization in terms of effort provision whenever coordination is of *any* importance. Nevertheless, as we will also show by example in the next subsection, when the cut-off is strictly positive it is not necessarily the case that $e^d < e^c_F$ $\forall \delta \in (0, \bar{\delta})$. In particular, while for intermediate values of δ centralization may indeed outperform

[24]To formalize this intuition, let $w = \frac{\eta_i}{\eta_i+\eta_i} \cdot \left(\frac{\eta_j}{\eta_i+\eta_j} + \delta\right) \Big/ \left(\frac{\eta_i}{\eta_i+\eta_j} \cdot \frac{\eta_j}{\eta_i+\eta_j} + \delta\right)$ be the strategic weight that the principal would assign to agent i's private information when making decision y_i under centralization (see Proposition 4.4). It can be shown that $\frac{\partial w}{\partial \eta_i} > 0$ and $\frac{\partial^2 w}{\partial \eta_i \partial \delta} > 0$ $\forall \delta > 0$ and $\forall \eta_1, \eta_2 \in \left[\underline{\eta}, \bar{\eta}\right]$, i.e., the principal's decision weights will respond more aggressively to the profitability conditions of the local markets as the need for coordination increases.

decentralization in terms of effort provision, it may fail to do so when the need for coordination is relatively small. The next result shows that this is likely to happen when the profitability conditions of the local markets are very volatile.

Theorem 4.2. *If* $\mathbb{E}\left[\frac{1}{\lambda^2}\right] > \mathbb{E}\left[\frac{2}{\lambda(1-\lambda)}\right] - 3$, *then* $\exists\, \underline{\delta} \in (0, +\infty]$, *such that* $e^d > e^c_F\ \forall \delta \in (0, \underline{\delta})$.

To understand Theorem 4.2, note that its condition is violated if $corr(\eta_1, \eta_2) = 1$, as this implies that $\Pr(\lambda = 0.5) = 1$. By continuity, it must also be violated if η_1 and η_2 are sufficiently positively correlated, meaning that the principal is unlikely to strongly bias her decisions in favor of the more profitable division ex post. As the profitability conditions become less and less positively correlated, the strategic weights η_1 and η_2 that the principal assigns to the two divisions are more likely to be extreme (i.e, λ is more likely to take values that are close to 0 and 1). This makes the condition of Theorem 4.2 more likely to be satisfied.[25] Thus, Theorem 4.2 captures the intuition that if the principal is likely to be highly biased ex post, then centralizing the decision rights can strongly discourage the agents from acquiring valuable information even when the motive of coordination is small. Therefore, the scope for decentralization to outperform centralization in effort provision is larger when the profitability conditions of the local markets are more volatile.

4.5.1.1 Binary distributions: characterizations

In this section, we use a class of binary distributions $\{F_\omega\}_{\omega \in [0,1)}$ to illustrate our main findings regarding the effect of decision right allocation on effort provision: for every $\omega \in [0, 1)$ the distribution F_ω is characterized by

$$\Pr(\eta_1 = 1 + \omega, \eta_2 = 1 - \omega) = \Pr(\eta_1 = 1 - \omega, \eta_2 = 1 + \omega) = \frac{1}{2}. \tag{4.5}$$

Thus, ω can be interpreted as a measure of both the volatility of the local markets' profitability conditions and the ex post bias of the principal: the larger ω, the more volatile are the local markets (since $\mathbb{E}[(\eta_i - \mathbb{E}[\eta_i])^2] = \omega^2$ and $Cov(\eta_1, \eta_2) = -\omega^2$) and the more biased is the principal ex post (as $|(\eta_i - \eta_j)/(\eta_i + \eta_j)| = \omega$).

For the above class of binary distributions, we fully characterize when a decentralized organization outperforms its centralized counterpart in providing incentives to the agents for exerting costly yet valuable effort. Fixing the volatility of the profitability conditions, or the degree of the principal's ex post bias, the next result shows how this regime is shaped by the importance of promoting synergies in the organization.

Proposition 4.5. *Consider any binary distribution* F_ω *with* $\omega \in [0, 1)$.

(i) If $\omega \le \sqrt{2} - 1$, *then* $e^d > e^c_F$ *if and only if* $\delta \in (0, \max\{0, \underline{\delta}(\omega)\}) \cup (\bar{\delta}(\omega), +\infty)$, *where*

$$\underline{\delta}(\omega) \equiv -\frac{\omega^4 + 4\omega^2 - 1}{8\omega^2} - \frac{(1+\omega^2)\sqrt{\omega^4 - 6\omega^2 + 1}}{8\omega^2},$$

[25] While both $\mathbb{E}[\frac{1}{\lambda^2}]$ and $\mathbb{E}\left[\frac{2}{\lambda(1-\lambda)}\right]$ may increase if the distribution of λ puts more weight toward to endpoints of the interval $[0, 1]$, the first term increases much faster because of its quadratic form.

and

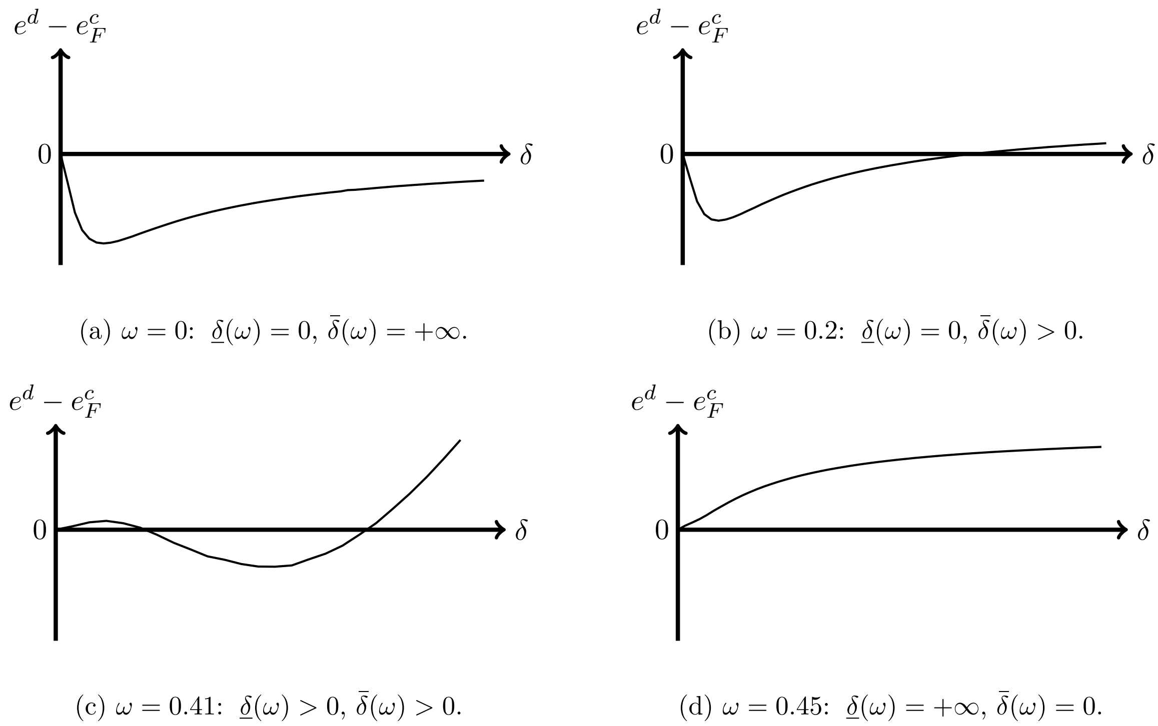
$$\bar{\delta}(\omega) \equiv -\frac{\omega^4 + 4\omega^2 - 1}{8\omega^2} + \frac{(1+\omega^2)\sqrt{\omega^4 - 6\omega^2 + 1}}{8\omega^2},$$

with $\underline{\delta}(\omega) = 0$ *if and only if* $\omega \leq \sqrt{\frac{2\sqrt{3}}{3} - 1} \approx 0.393$, *and* $\lim_{\omega \to 0} \bar{\delta}(\omega) = +\infty$.

(ii) If $\omega > \sqrt{2} - 1$, *then* $e^d > e_F^c$ $\forall \delta > 0$.

Figure 4.2 provides an illustration of Proposition 4.5 as well as the key messages of Theorems 4.1 and 4.2. For the benchmark case of no uncertainty in the profitability conditions ($\omega = 0$), Figure 4.1(a) shows that the agents always work harder under centralization ($e^d - e_F^c < 0$). This illustrates Theorem 4.1(i) and the asymptotic result $\lim_{\omega \to 0} \bar{\delta}(\omega) = +\infty$ from Proposition 4.5(i). As we start introducing uncertainty to the profitability conditions (η_1, η_2), both Theorem 4.1(ii) and Proposition 4.5(i) suggest that a decentralized authority structure is superior in guaranteeing effort provision (and thus also in information production) whenever the need for coordination is sufficiently strong. Figure 4.2(b) demonstrates that a strong coordination motive is also necessary for the equilibrium effort level to be higher under decentralization if the uncertainty of profitability conditions is sufficiently small. If the degree of uncertainty takes an intermediate value, then additionally we have $e^d > e_F^c$ when coordination is sufficiently unimportant relative to adaptation ($\delta < \underline{\delta}(\omega)$). As depicted in Figure 4.2(c), in this case centralizing the decision rights improves the effort provision if and only if the need for coordination is also intermediate. This echoes the finding of Theorem 4.2.[26] Finally, when the degree of uncertainty becomes sufficiently large ($\omega > \sqrt{2} - 1$), the agents anticipate that the principal will be heavily biased when making decisions. This substantially impairs the marginal benefit of information under centralization from the agents' perspectives. Proposition 4.5(ii) and Figure 4.2(d) show that in such scenarios decentralization is optimal for guaranteeing effort provision regardless of the importance of coordination.

[26] With the binary distributions (4.5), it can be verified that the inequality condition in Theorem 4.2 is equivalent to $\omega > ((2\sqrt{3})/3 - 1)^{1/2}$, which is also necessary and sufficient for the cutoff $\underline{\delta}(\omega)$ in Proposition 4.5 to be strictly positive.



(a) $\omega = 0$: $\underline{\delta}(\omega) = 0$, $\bar{\delta}(\omega) = +\infty$.

(b) $\omega = 0.2$: $\underline{\delta}(\omega) = 0$, $\bar{\delta}(\omega) > 0$.

(c) $\omega = 0.41$: $\underline{\delta}(\omega) > 0$, $\bar{\delta}(\omega) > 0$.

(d) $\omega = 0.45$: $\underline{\delta}(\omega) = +\infty$, $\bar{\delta}(\omega) = 0$.

Figure 4.2: The dependence of effort difference on δ, with $c(e) = e^2$.

To further sharpen our understanding of the effort provision under both organizational forms we next fix the degree of the coordination requirement and ask, how does the effort provision depend on the volatility of the profitability conditions? As one may already expect, the equilibrium effort level is higher under decentralization if and only if the profitability conditions are sufficiently volatile (i.e., ω is sufficiently large). Perhaps less intuitively, the range of the volatility parameter ω for which decentralization provides more powerful incentives (i.e. the set $\{\omega \in (0,1) : e^d > e^c_F\}$) does not change monotonically with respect to the coordination motives. Starting with a situation where coordination is of little (large) importance, an increase in the need for coordination makes it more likely that the agents exert more effort under centralization (decentralization). These observations are summarized in the next proposition.

Proposition 4.6. *Given a binary distribution F_ω with $\omega \in (0,1)$ and $\delta > 0$, $e^d > e^c_F$ if and only if $\omega > \hat{\omega}(\delta)$, where*

$$\hat{\omega}(\delta) = \sqrt{\frac{(4\delta+2)\sqrt{4\delta^2+4\delta+3} - 2\delta - 3}{4\delta+3} - 2\delta}.$$

The cutoff $\hat{\omega}(\delta)$ is strictly increasing on $\left(0, \frac{\sqrt{2}-1}{2}\right)$, and it is strictly decreasing on $\left(\frac{\sqrt{2}-1}{2}, +\infty\right)$, with $\hat{\omega}\left(\frac{\sqrt{2}-1}{2}\right) = \sqrt{2} - 1$ and $\lim_{\delta \to +\infty} \hat{\omega}(\delta) = 0$.

The insight of Proposition 4.6 is further highlighted in Figure 4.3, where the hatched area indicates the regime of parameters for which the equilibrium effort level is higher under decentralization. Notably, this graphic representation does not require any specification of the effort cost function. This shows the generality and robustness of the qualitative results that we have

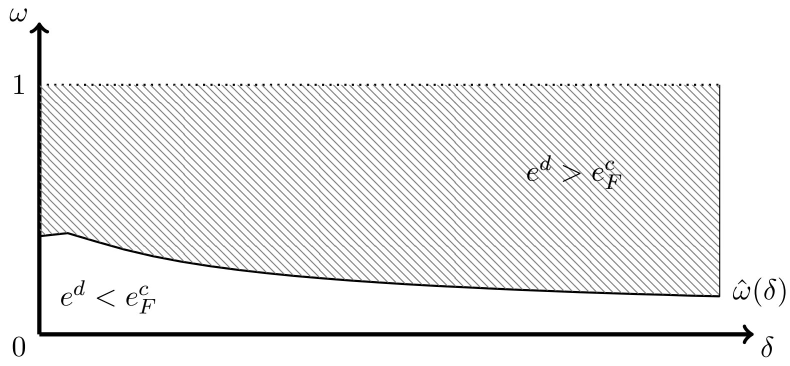


Figure 4.3: The cutoff $\hat{\omega}(\delta)$ and the regimes for $e^d > e^c_F$ and $e^d < e^F_c$.

obtained so far.

4.5.2 The principal's payoff

In this section, we turn to the question of when the principal can benefit from centralization (decentralization). The immediate implication of the full-revelation results (Propositions 4.1 and 4.2) is that centralization is optimal for the principal whenever it can better motivate the agents than decentralization ($e^d < e^c_F$), since this allows her to adjust the relevant organizational activities to better support the (ex post) more profitable division without sacrificing the (ex ante) informativeness of the decisions. As suggested by the characterization results Propositions 4.5 and 4.6, the principal is more likely to confront such a straightforward comparison between organizational forms when the need for coordination is small or intermediate and the local markets are not too volatile.

However, in the previous section we have also shown that the agents' incentives for information gathering are lower under centralization whenever the need for coordination is sufficiently large and/or the local markets are sufficiently volatile in their profitability conditions. If the disadvantage of centralization in motivating information gathering is substantial enough, having the flexibility to adapt decisions to the actual profitability conditions may not be so valuable for the principal after all.[27] The next result provides a sufficient condition under which the gap in effort provision between centralization and decentralization is large enough for the principal to prefer the latter. Specifically, we show that decentralization will outperform centralization in terms of the principal's expected payoff provided the effort cost function is not too convex and coordination is sufficiently important.

Theorem 4.3. *Suppose that $corr(\eta_1, \eta_2) < 1$. There exists $\zeta > 0$, such that if $c''(e) \cdot e < \zeta$ $\forall e \in [0, 1]$, then we have $\Pi^c_P < \Pi^d_P$ for sufficiently large δ.*

[27]This argument can be best understood by considering the extreme case where both agents exert very little effort under centralization: given the poor quality of information, the principal often have to take the uninformed decisions ($y_i = y_j = \mathbb{E}[\theta_i]$). Thus, the option of tailoring decisions to (η_1, η_2) is not quite useful.

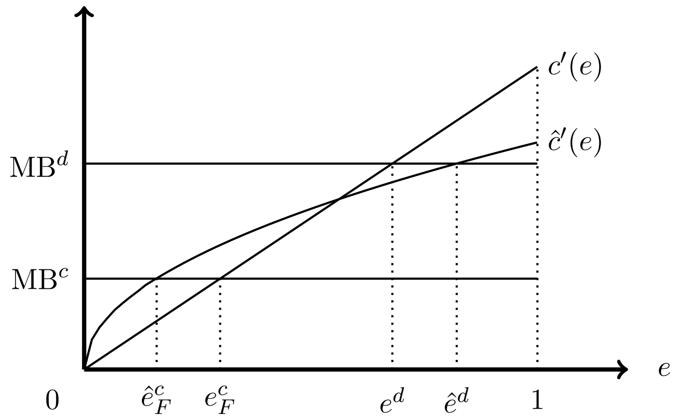


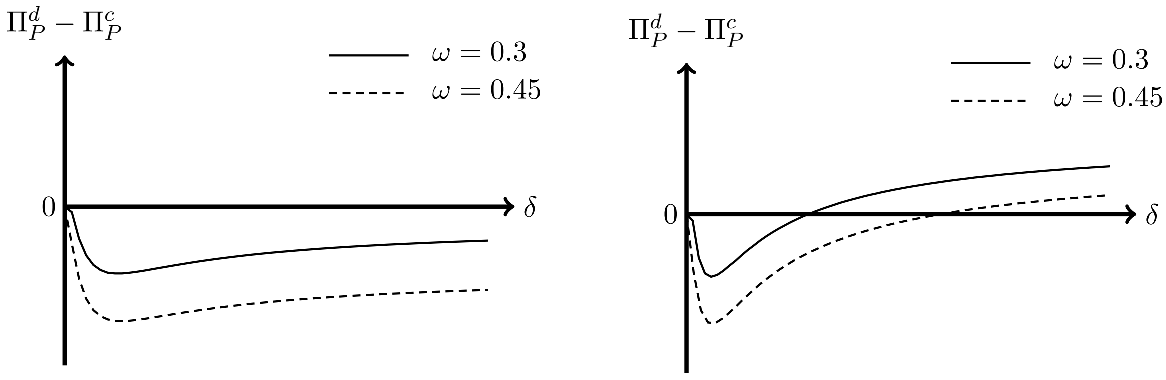
Figure 4.4: Differences in equilibrium effort levels with $c(e) = e^2$ and $\hat{c}(e) = e^{1.5}$.

To understand the above theorem, consider again an individual agent who is deciding on how much effort to invest in the task of information acquisition. As Theorem 4.1 shows, if the local markets exhibit *any* uncertainty in their relative profitability ($corr(\eta_1, \eta_2) < 1$) and coordination is sufficiently important, the marginal benefit of effort is higher when decision rights are allocated to the agents. The gap in the marginal benefits of effort between centralization and decentralization then translates into a gap in effort provision. Intuitively, this gap in effort provision will be larger if the derivative c' does not increase very fast, because the equilibrium effort levels are chosen to balance the corresponding marginal benefits and marginal costs. Figure 4.4 provides a graphical illustration of this intuition: Consider two cost functions $c(e) = e^2$ and $\hat{c}(e) = e^{1.5}$. The marginal cost is arguably increasing faster (on average) in the former case than in the latter (since $\mathbb{E}[c''(e)] > \mathbb{E}[\hat{c}''(e)]$). As we can see from the figure, for given marginal benefits of effort under centralization (MB^c) and decentralization (MB^d) with $\mathrm{MB}^d - \mathrm{MB}^c > 0$, the gap in effort provision is larger when the cost function is $\hat{c}$ than when it is c (i.e., $\hat{e}^d - \hat{e}_F^c > e^d - e_F^c$). In fact, in this case, the argument $\hat{e}^d - \hat{e}_F^c > e^d - e_F^c$ also follows from the observation that the marginal cost function $\hat{c}'(e)$ is a concave transformation of $c'(e)$. More generally, if the cost function takes the form $c(e) = ke^\alpha$, where $k > 0$ and $\alpha > 1$, then the "sufficiently small ζ" condition in Theorem 4.3 can be replaced by the requirement that the power parameter α is sufficiently close to one - in other words, the marginal cost function is sufficiently concave.[28] However, the proof of Theorem 4.3 shows that what is crucial is not the concavity of the marginal cost function, but rather the bound of speed at which it grows.

We close this section with a result that parallels Theorem 4.2: if the profitability conditions of the local markets are sufficiently volatile and the cost function is not too convex, then, even when the need of coordination is relatively small, the resulting gap in effort provision can be substantial enough to make decentralization optimal for the principal.

Theorem 4.4. *Suppose that* $\mathbb{E}\left[\frac{1}{\lambda^2}\right] > \mathbb{E}\left[\frac{2}{\lambda(1-\lambda)}\right] - 3$. *For sufficiently small* $\delta > 0$, *there exists* $\zeta(\delta) > 0$, *such that if* $c''(e) \cdot e < \zeta(\delta)$ $\forall e \in [0, 1]$, *then* $\Pi_P^c < \Pi_P^d$.

[28] In fact, one can show that with the cost function $c(e) = ke^\alpha$, there exists a cutoff $\alpha^* > 1$, such that the conclusion of Theorem 4.3 holds if *and* only if $\alpha \leq \alpha^*$ (See Figure 4.5 for further illustration). More generally and similar to Theorem 4.3, if information cost is sufficiently convex, the motivational advantage of decentralization need not make it optimal for the principal even when coordination is extremely important.



(a) $c(e) = e^2$, $q\sigma_\theta^2 = 1.25$.

(b) $c(e) = e^{1.25}$, $q\sigma_\theta^2 = 1.25$.

Figure 4.5: The principal's payoff and cost functions with different degrees of convexity.

Unlike the uniform cutoff ζ in Theorem 4.3, the cutoff $\zeta(\delta)$ in Theorem 4.4 is δ-specific. From a technical point of view, this is because regardless of the distribution of the global states, the expected payoffs of the principal under both authority structures (i.e., Π_P^c and Π_P^d) converge to each other as δ goes to zero. A deeper insight we can gain from this exercise is that the optimal authority structure is more ambiguous when coordination is not so important *and* the local markets are highly volatile in their profitability conditions. In such cases, while decentralization can lead to more motivated agents (see Theorem 4.2 and Proposition 4.5), given the large uncertainty in relative market profitability the principal would also find the power of making contingent decisions especially valuable.

4.5.2.1 Binary distributions: examples

To sharpen our understanding on the role of the convexity of the cost function in determining the relative expected payoff of the principal under centralization and decentralization, we consider again the class of binary distributions $\{F_\omega\}_{\omega\in[0,1)}$ introduced in section 4.5.1.1. Note that assuming general cost functions makes it difficult to obtain characterization results that parallel Propositions 4.5 and 4.6. Thus, we look at particular cost functions and specify the degree of market volatility (ω) to illustrate how the principal's optimal organizational form depends on the coordination requirement.

Consider two cost functions, $c(e) = e^2$ and $c(e) = e^{1.25}$, and two situations of volatility, $\omega = 0.3$ and $\omega = 0.45$. The choices of ω are meant to be representative. According to Proposition 4.5, for $\omega = 0.3$ the agents exert higher effort under decentralization if and only if coordination is sufficiently important. In contrast, for $\omega = 0.45$ decentralization outperforms centralization for any degree of the coordination requirement.

In Figure 4.5(a), we let the cost function be $c(e) = e^2$. For both cases $\omega = 0.3$ and $\omega = 0.45$, the principal's expected payoff is always higher under centralization independent of the coordination parameter δ. Thus, with the quadratic cost function, the negative uncertainty effect of centralization on effort provision is not too severe a concern from the principal's perspective. Thus, centralization dominates decentralization by its advantage of allowing the principal to tailor the organizational activities to the actual profitability conditions across markets. Moreover, as Figure 4.5(a) shows, the value of such flexibility in decision-making is particularly high for the principal when the volatility of the profitability conditions is large. In the figure, the dashed curve lies strictly below the non-dashed one, meaning that regardless of the importance of coordination the payoff difference $\Pi_P^d - \Pi_d^c$ is more negative for $\omega = 0.45$ compared to $\omega = 0.3$.

The pattern that centralization is relatively more attractive to the principal when the volatility/bias measure ω is larger is shared by Figure 4.5(b), where we use a less convex cost function $c(e) = e^{1.25}$. However, unlike in the previous case, here the negative effect of centralization on effort provision is amplified *sufficiently* by the need for coordination. In both cases $\omega = 0.3$ and $\omega = 0.45$, as the coordination parameter δ increases, the difference in effort provision (and thus also in the quality of information) eventually becomes so large that the principal have to take the uninformed decisions much more often under centralization. Hence, confirming the finding of Theorem 4.3, when the effort cost is not too convex and the need for coordination is sufficiently large the principal is worse off by having the decisions centrally made.

4.6 Extensions

4.6.1 Introducing transfers

So far, we have assumed that the agents care only about their own performance. This can be interpreted as that an agent only get paid based on the performance of his own division. However, as Athey and Roberts (2001) and Rantakari (2013) point out, due to informational externalities the organization designer may want to align the incentives of the managerial members by tying their compensation to each other's performance. Indeed, while in extreme cases an interdependent pay structure may discourage information acquisition (e.g., if agent i's reward is *primarily* determined by j's performance), an appropriate level of interdependence can lead to a more efficient use of information (from the principal's perspective) when decision rights are decentralized to the divisions. Under centralization, however, there is no room for such improvement given that a central manager can elicit all information from the local ones for free. The implication of this analysis is that decentralized decision-making is even more likely to be optimal when performance-based transfers are available, echoing Milgrom and Roberts (1992)'s view that the alignment of incentives is complementary to the delegation of authority.

Since the central trade-off of our model comes from strategic information acquisition rather than strategic communication, one may also envision improving the organization's performance by directly rewarding information collection. Formally, suppose that the principal can commit to pay a fixed bonus $b \geq 0$ to an agent provided that he credibly discloses that his information experiment is successful ($s_i \neq \emptyset$). In general, allowing for such information-based transfers will make centralization more likely to be optimal. This is because, other things equal, an additional unit effort of an agent will be more valuable for the principal when she can decide how to use the resulting information. Thus, in contrast to performance-based transfers, information-based transfers are complimentary to centralization. However, it is worth to note that ex ante it may be optimal for the principal *not* to provide any direct rewards for information collection (i.e., $b^* = 0$). For instance, under centralization, implementing an effort level $\tilde{e} > e_F^c$ would require the principal to set $b = c'(\tilde{e}) - \mathrm{MB}^c > 0$, where MB^c is an agent's marginal benefits of effort under centralization (see Section 4.4.1). Compared to the case where $b = 0$, this yields a higher expected payoff for the principal if and only if

$$[\Pi_P^c(\tilde{e}, \tilde{e}) - 2 \cdot (c'(\tilde{e}) - \mathrm{MB}^c) \cdot \tilde{e}] - \Pi_P^c(e_F^c, e_F^c) > 0, \tag{4.6}$$

where $\Pi_P^c(e, e)$ is the principal's expected payoff under centralization when both agents choose the effort level e (see Appendix D.10). It is straightforward to check that

$$\Pi_P^c(\tilde{e}, \tilde{e}) - \Pi_P^c(e_F^c, e_F^c) = \Xi_F \cdot (\tilde{e} - e_F^c),$$

where

$$\Xi_F = 2 \cdot \left(\mathbb{E}[\eta_i] - \mathbb{E}\left[\frac{\delta^2 \cdot \frac{\eta_1\eta_2}{\eta_1+\eta_2} + \delta \cdot \frac{\eta_1^2\eta_2^2}{(\eta_1+\eta_2)^3}}{\left(\frac{\eta_1\eta_2}{(\eta_1+\eta_2)^2} + \delta\right)^2} \right] \right) \cdot \sigma_\theta^2.$$

Thus, other things equal, (4.6) is more likely to be violated if the term Ξ_F is small or if $c'(\tilde{e})$ is large. In particular, if Ξ_F is sufficiently small, then (4.6) will not hold for any $\tilde{e} > e_F^c$.[29] In those cases, it would be optimal for the principal to choose $b = 0$ under centralization.

4.6.2 Costly exaggeration

So far, the communication stages under both centralization and decentralization have been modeled as a game with verifiable information: an agent can always send a certified message and reveal the finding of his information acquisition experiment to the receiving party. The crucial implication of this assumption is that in our model, the fundamental difference between centralization and decentralization is not the endogenous quality of communication - in both cases the messages communicated by agents will be truthful and fully informative - but rather the quality of information, which is endogenously determined by the effort of the agents.

The verifiability assumption is intended to capture situations where the decision-relevant information held by organizational agents is in the form of hard evidence, or at least it can be supported by objective measures. For instance, a division manager may conduct marketing research with statistical analysis to convey information about the consumer demand of his responsible market. However, there are certainly settings where one may view the assumption of perfect verifiability restrictive. For example, when a specialized manager provides marketing research showing that the consumer demand is high, others in the organization may not be able to tell for sure whether the conclusion is driven by a deliberate (and possibly biased) choice of statistical methods of analysis. If the manager wants to exaggerate the consumer demand by manipulating his data, the imperfect verifiability of information can be problematic because it seems conceivable that an exaggerated report may not be caught by his colleagues, especially when it is not too far away from the truth. In what follows, we will show that the insights from our full revelation results are robust provided that such exaggeration is not entirely costless.

[29] To see this, note that the first derivative of the LHS of (4.6) with respect to $\tilde{e}$ is negative for all $\tilde{e} > e_F^c$ if Ξ_F is sufficiently small.

Specifically, suppose that $\Theta = \mathbb{R}$, and when communicating (either with the principal or with each other) the agents are allowed to send any message $m_i \in \mathcal{M} = \mathbb{R} \cup \{\emptyset\}$, irrespective of the true findings of their experiments. However, given the true signal is $s_i \in \mathcal{S} = \mathbb{R} \cup \{\emptyset\}$, sending a message $m_i \in \mathcal{M} = \mathbb{R} \cup \{\emptyset\}$ will incur a non-negative cost $z(m_i, s_i)$ to agent i. This communication game converges to one with verifiable disclosure when the function z satisfies $z(m, s) = 0$ if $m \in \{s, \emptyset\}$, and $z(m, s) = +\infty$ otherwise. We now consider general cases which only require the following less restrictive assumption on the communication cost function z:

Assumption 4.2. *Function* $z : \mathcal{M} \times \mathcal{S} \to \mathbb{R}_+ \cup \{+\infty\}$ *satisfies*

(i) $z(s, s) = 0 \quad \forall s \in \mathcal{S}$, *and*

(ii) $z(m, s) = \kappa(m - s)^2$ *if either* $m > s > 0$ *or* $m < s < 0$, *where* $\kappa > 0$.

In words, condition (i) states that telling the truth is always costless. However, as condition (ii) states, it is costly for the agents to exaggerate their findings. In particular, the further away an agent's message is from the truth, the more costly it is to send such a message. It may be natural to further assume that $z(m, s) = 0$ if $0 \leq m < s$, $s < m \leq 0$ or $m = \emptyset$ (i.e., understating or concealing one's finding is also costless), and $z(m, s) = +\infty$ if $m \cdot s < 0$ (i.e., lying about the *sign* of the local state is never feasible). One may also want to extend condition (ii) to the case where $s \in \{0, \emptyset\}$. None of these additional assumptions will be necessary for our analysis in this section.

Although assumption 4.2 rules out pure cheap talk communication, it still provides arbitrarily rich possibilities to lie (under consideration of lying costs). The next proposition states that despite the non-verifiability of agents' private information, provided that assumption 4.2 is satisfied the endogenous quality of communication under both centralization and decentralization will be identical and maximal: just as in the main model, in either case a fully revealing equilibrium exists.

Proposition 4.7. *Suppose that the communication cost satisfies assumption 4.2.*

(i) Under decentralization, there exists a fully revealing PBE in which

$$\check{m}_i^d(e_i, s_i) = \begin{cases} t^d s_i & \textit{if } s_i \in \mathbb{R} \\ \emptyset & \textit{if } s_i = \emptyset \end{cases}, \ \forall e_i \in [0, 1], \textit{ and } \forall i = 1, 2,$$

where $t^d = \frac{1}{2} + \sqrt{\frac{q\delta^2}{\kappa(1+2\delta)^2} + \frac{1}{4}}$. *In equilibrium, both agents exert the same effort*

$$\check{e}^d \equiv (c')^{-1} \left(\left(1 - \frac{\delta^2 + \delta}{(1 + 2\delta)^2} \right) q\sigma_\theta^2 - \kappa(t^d - 1)^2 \sigma_\theta^2 \right).$$

(ii) Under centralization, there exists a fully revealing PBE in which

$$\check{m}_i^c(e_i, s_i) = \begin{cases} t^c s_i & \textit{if } s_i \in \mathbb{R} \\ \emptyset & \textit{if } s_i = \emptyset \end{cases}, \ \forall e_i \in [0, 1], \textit{and } \forall i = 1, 2,$$

where $t^c = \frac{1}{2} + \sqrt{\mathbb{E}\left[\frac{q\lambda(1-\lambda)[2\delta^2+\delta(\lambda^2+(1-\lambda)^2)]}{2\kappa(\lambda(1-\lambda)+\delta)^2}\right] + \frac{1}{4}}$, *and* $\lambda = \eta_1/(\eta_1+\eta_2)$. *In equilibrium, both agents exert the same effort*

$$\check{e}^c_F \equiv (c')^{-1}\left(\left(1 - \mathbb{E}_\lambda\left[\frac{\delta^2(\lambda^2+(1-\lambda)^2) + 2\delta\lambda^2(1-\lambda)^2}{2\left(\lambda(1-\lambda)+\delta\right)^2}\right]\right) q\sigma^2_\theta - \kappa\left(t^c-1\right)^2\sigma^2_\theta\right).$$

Proposition 4.7 shows that in equilibrium, the agents "lie" in such a way that their private signals can be perfectly inferred from the messages communicated. To relate it to our previous results in Section 4.4 (Propositions 4.1 - 4.4), note that as $\kappa \to +\infty$, both coefficients t^d and t^c converge to 1. In addition, the total expected communication costs $\kappa(t^d-1)^2$ and $\kappa(t^c-1)^2$ converge to zero. This implies that as exaggeration becomes infinitely costly, under both centralization and decentralization the agents simply disclose their acquired signals $(m^d_i(e_i,s_i), m^c_i(e_i,s_i) \to s_i \; \forall s_i \in \mathbb{R}\cup\{\emptyset\})$ and exert the same amount of effort as in the main model $(\check{e}^d \to e^d, \check{e}^c_F \to e^c_F)$.

Perhaps a more interesting observation is that the equilibria under both centralization and decentralization feature language inflation $(t^c, t^d > 1)$. This is reminiscent of the findings of Kartik et al. (2007) and Kartik (2009) on strategic communication with credulous receivers or with exogenous lying costs. Compared to the existing papers, a novelty of our language inflation results is that they hold in settings that feature either bilateral communication (in the case of decentralization) or competing senders with differentiated private information (in the case of centralization).[30]

4.7 Conclusion

When operating in multiple markets which exhibit uncertainty in their relative profitability, how should an organization optimally allocate decision-making authority to its managerial members? In this paper we addressed this question in a model where decision-relevant information is collected and transmitted by strategic and self-interested division managers, and the objective of the organization is to solve the problem of coordinated adaptation.

Our paper makes two main contributions. The first is that if information is verifiable or if lying is not costless, then the quality of communication is not affected by where the decision-making authority is lodged in the organization. Moreover, since the principal of the organization can elicit all private information from its local delegates, the fact that the principal is not well informed per se does not make centralized decision-making inferior. However, as a second contribution, we show that the quality of endogenously acquired information depends crucially on the allocation of decision rights. In particular, if the local markets exhibit any uncertainty in their relative profitability, a large coordination motive can strongly discourage information gathering under centralization, which in turn makes decentralized decision-making optimal. Yet it is also worth noting that when the need for coordination is small or intermediate, centralized

[30] Emons and Fluet (2012) were the first to show that the feature of language information can also arise in a setting with multiple senders. However, the senders in their model (plaintiff and defendant) have perfectly correlated types (they share the private knowledge about the amount of damages). This is not the case in our model because the private types θ_i and θ_j are independently distributed.

decision-making is often optimal because it allows the organization to better cope with inter-market uncertainty, while not necessarily making the division managers less motivated. Overall, our results call for a more careful examination of the Delegation Principle, which is well-known in the management literature (see, e.g., Milgrom and Roberts, 1992) and emphasizes that "the power to make decisions should reside in the hands of those with relevant information" (Krishna and Morgan, 2008, p. 905).

We suggest two venues for future research. First, given that the communication of decision-relevant information in organizations is often not entirely cheap talk (e.g., marketing reports must contain survey evidence or data analysis in order to be taken serious, lying to colleagues may result in retaliation or even being fired), it is worth reconsidering how essential the informational constraints are in various organizational design problems. A conjecture based on the analysis of our paper is that in settings with verifiable information, the incentive constraints for communication can be much less important than the physical or technological ones (Aoki, 1986; Dessein and Santos, 2006). Second, when the uncertainty in the profitability of different product markets is substantial, the principal of the organization may prefer a more moderate way to mitigate her commitment problem than unconditionally delegating the decisions to the division managers. It is an open question whether the principal can benefit from conditional delegation, e.g., committing to only execute her authority when it is reported that the local states take extreme values.

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5 On the Equivalence of Bayesian and Dominant Strategy Implementation for Environments with Non-Linear Utilities[1]

Joint with Alexey Kushnir

5.1 Introduction

Fundamental advances in mechanism design have found vast practical applications including auctions for radio spectrum licenses, carbon emission permits, and online advertising (Siegfried, 2010). One of the most important practical challenges facing mechanism designers is to ensure that they propose *robust* mechanisms, i.e., mechanisms that are not sensitive to the fine details of the environment such as the beliefs of the agents. As argued by Bergemann and Morris (2005), robustness in private value settings is equivalent to dominant strategy incentive compatibility.[2] Indeed, dominant strategy implementation has the significant advantage that it only requires that agents have common knowledge about the specification of the payoff environment, and it is resistant to deviations from rationality that are often observed in practice.

However, dominant-strategy implementation is arguably a rather strong implementation concept (compared to, e.g., Bayesian implementation). Thus, an important question is whether the mechanism designer can benefit from designing possibly more complex mechanisms with Bayes-Nash equilibria. An important setting where one could ask this question is when the designer knows that the agents share a common prior and only differ in their payoff types. As highlighted in recent works by Manelli and Vincent (2010) and Gershkov et al. (2013), the answer to the above question is negative for many common-prior settings often studied in applied work. In particular, they show that if agents have linear utilities and independent, one-dimensional private types, then for any Bayesian incentive compatible (BIC) mechanism there exists an equivalent dominant strategy incentive compatible (DIC) mechanism that yields the same interim expected utilities to all agents and generates the same expected social surplus.[3]

The main contribution of this paper is to extend the BIC-DIC equivalence result to quasi-linear environments where agent's *non-linear utility* from physical allocations satisfies two as-

[1] A version of this paper is published as Kushnir, A. and S. Liu (2018): "On the Equivalence of Bayesian and Dominant Strategy Implementation for Environments with Non-Linear Utilities," *Economic Theory*, forthcoming.

[2] Bergemann and Morris (2005)'s results apply only to quasi-linear environments with unrestricted transfers and when the mechanism designer seeks to implement a single-valued social choice function that only depends on agents' payoff types. We are thankful to a referee for bringing to our attention a more precise statement.

[3] Goeree and Kushnir (2017) provide an alternative proof of this equivalence result using a novel geometric approach to mechanism design. Kushnir (2015) extends the result to environments with correlated types. Kushnir and Liu (2017) explain how the BIC-DIC equivalence problem reduces to a purely mathematical question of when a linear transformation of intersection of two closed convex sets coincides with the intersection of their images.

sumptions. The first one demands that each agent's utility comply with the *increasing differences over distributions* property, which is a natural extension of the standard increasing differences (or supermodularity) property to the space of lotteries. This novel property delineates the settings where all BIC mechanisms can be conveniently described by a monotonicity condition and an envelope formula. We also fully characterize the set of functions satisfying the increasing differences over distributions property.

The second assumption demands that the mapping of all agents' utilities, as a mapping from the set of feasible allocations to the space of utilities, has *a convex image* for each profile of types.[4] Though this condition might be restrictive in general, it is trivially satisfied for linear utilities defined on a convex set (as in Gershkov et al., 2013) and for any symmetric settings.

Assuming the *increasing differences over distributions* property and the mapping of all agents' utilities being *convex-valued*, we establish the BIC-DIC equivalence for non-linear environments. For settings where our main equivalence theorem does not apply, we provide further conditions on agents' utilities when for any given BIC mechanism one could find a DIC mechanism that yields the same interim expected utilities to all agents and generates *at least as large* expected social surplus. The latter requirement captures the economic intuition that one does not need to insert additional money to achieve a more robust solution concept.

Finally, we demonstrate the usefulness of our results by revisiting several important applications, for which the previous works have little bite (e.g. Gershkov et al., 2013; Manelli and Vincent, 2010). We first consider the principal-agent problem in a procurement context and illustrate that many influential papers satisfy our main assumptions (e.g. Laffont and Martimort, 1997; Mookherjee and Tsumagari, 2004). In the same context, we study settings with allocative externalities, when agents care not only about their own contracts, but also about contracts received by other agents (e.g. Jehiel et al., 1996; Segal, 1999). If agents face non-decreasing convex (concave) contracting costs and positive (negative) concave externalities, then for any BIC mechanism one could find a DIC mechanism yielding the same interim expected utilities to all agents and generating *at least as large* social surplus. We also establish that the above result holds for environmental mechanism design problems (Baliga and Maskin, 2003; Martimort and Sand-Zantman, 2013, 2016) when agents have linear (concave) benefits and concave (linear) costs of pollution reduction. We finally consider a problem of public good provision, where in addition to incentive compatibility and individual rationality constraints the budget-balance constraint is of huge importance (e.g. Hellwig, 2003; Ledyard and Palfrey, 1999; Mailath and Postlewaite, 1990; Norman, 2004). When agents have concave utilities and the cost of public good provision is convex, we show that for any BIC mechanism that is *ex ante budget balanced* there exists an equivalent DIC mechanism that satisfies the same requirement.

The paper is organized as follows. Section 5.2 presents the model. Section 5.3 introduces the increasing differences over distributions property. We prove our main equivalence results in Section 5.4. Section 5.5 presents applications and Section 5.6 concludes. Appendix E contains omitted proofs.

[4]Similar convexity assumptions on the utility possibility set are also made in many seminal papers in the literature of bargaining theory (e.g. Crawford, 1982; Kalai et al., 1975; Nash, 1950).

5.2 The Model

We consider environments with a finite set of agents $\mathcal{I} = \{1, 2, .., I\}$ and a compact set of available alternatives $A \subset \mathbb{R}^k$ for some natural k. Agent i's utility when alternative $a \in A$ is chosen equals $v_i(a, x_i) + t_i$, where x_i is agent i's type that is independently distributed according to some probability distribution λ_i with one-dimensional connected support $X_i = [\underline{x}_i, \overline{x}_i] \subset \mathbb{R}$, function $v_i : A \times X_i \to \mathbb{R}$ is continuous in a, is absolutely continuous in x_i, and has a bounded derivative with respect to x_i (i.e., $\exists K_1, ..., K_I \geq 0$, such that $|v_{ix}(a, x_i)| \leq K_i, \forall a \in A, x_i \in X_i, i \in I$), and $t_i \in \mathbb{R}$ is a monetary transfer. We denote $\mathbf{x} = (x_1, ..., x_I)$, $\mathbf{X} = \prod_{i \in \mathcal{I}} X_i$, and $\boldsymbol{\lambda} = \prod_{i \in \mathcal{I}} \lambda_i$.[5]

We consider only direct mechanisms (q, t), where $q : \mathbf{X} \to A$ defines an allocation rule and $t = \{t_i\}_{i \in \mathcal{I}}$, with $t_i : \mathbf{X} \to \mathbb{R}$ defines monetary transfers to agents. A mechanism (q, t) is Bayesian incentive compatible or BIC (dominant strategy incentive compatible or DIC) if truthful reporting by all agents constitutes a Bayes-Nash equilibrium (a dominant strategy equilibrium). We also say that an allocation rule q is BIC (DIC) if there exists a payment rule t such that mechanism (q, t) is Bayesian incentive compatible (dominant strategy incentive compatible).

When all agents report their types truthfully and agent i's type is x_i, we denote his utility by $u_i(\mathbf{x}) = v_i(q(\mathbf{x}), x_i) + t_i(\mathbf{x})$ and his interim expected utility by $U_i(x_i) = E_{\mathbf{x}_{-i}}(v_i(q(\mathbf{x}), x_i) + t_i(\mathbf{x}))$. The expected social surplus is defined as $E_{\mathbf{x}}\left(\sum_{i \in \mathcal{I}} v_i(q(\mathbf{x}), x_i)\right)$ or, equivalently, as the sum of agents' ex ante expected utilities minus the sum of agents' ex ante expected transfers. As in Gershkov et al. (2013), we employ the following notion of equivalence.

Definition 5.1. *Two mechanisms (q, t) and $(\tilde{q}, \tilde{t})$ are equivalent if and only if they yield the same interim expected utilities to all agents and generate the same expected social surplus.*

5.3 The Increasing Difference over Distributions

In this section, we introduce and characterize the *increasing differences over distributions* property. We use this novel property to characterize Bayesian incentive compatible mechanisms in terms of a monotonicity condition and an envelope formula, which is similar to how the standard increasing differences property is used to characterize dominant strategy incentive compatible mechanisms.

To motivate our novel property, let us first consider the standard property of increasing differences or supermodularity (see Topkis, 1998).

Definition 5.2. *Function v_i satisfies the increasing differences property if for any pair of alternatives $a, a' \in A$ the difference $v_i(a, x) - v_i(a', x)$ is either increasing, decreasing, or constant in x.*[6]

Assuming that v_i satisfies the increasing differences property for each $i \in \mathcal{I}$, Mookherjee and Reichelstein (1992) showed that dominant strategy incentive compatibility can be characterized

[5] Our main results Theorems 5.1 and 5.2 can also be extended to discrete types similar to Gershkov et al. (2013).

[6] Throughout the paper, "increasing" ("decreasing") refers to a strictly increasing (decreasing).

by a monotone-marginal condition and an envelope formula.[7]

Proposition 5.1. *(Mookherjee and Reichelstein 1992) Suppose v_i satisfies the increasing differences property for each $i \in \mathcal{I}$. A mechanism (q,t) is DIC if and only if for each $i \in \mathcal{I}$ and $\mathbf{x} \in \mathbf{X}$: (i) $v_{ix}(q(s,\mathbf{x}_{-i}),x_i)$ is non-decreasing in s and (ii) agent i's utility can be expressed as*

$$u_i(x_i,\mathbf{x}_{-i}) \;=\; u_i(\underline{x}_i,\mathbf{x}_{-i}) + \int_{\underline{x}_i}^{x_i} v_{ix}(q(s,\mathbf{x}_{-i}),s)ds.^8 \tag{5.1}$$

Proposition 5.1 is a powerful result as it provides a tractable analysis of incentive compatibility constraints in many important applications (e.g. Laffont and Martimort, 1997; Segal, 2003). In Appendix E, we further show that the increasing differences property is a necessary condition for the characterization of Proposition 5.1. In particular, if some agent's function v_i does not satisfy the increasing differences property then one can always construct a DIC mechanism that does not have non-decreasing marginals (see Proposition E.1).

To obtain a similar characterization for BIC mechanisms, we first need an appropriate extension of the increasing differences property to Bayesian settings. Note that from the perspective of each agent, who knows only the distribution of the types of other agents, every allocation rule induces a probability distribution over possible outcomes. This logically leads to the following definition.

Definition 5.3. *Function v_i satisfies the increasing differences over distributions property if for any pair $G,F \in \Delta(A)$, the difference $\int v_i(a,x)dG(a) - \int v_i(a,x)dF(a)$ is either increasing, decreasing, or constant in x.*

The following proposition shows that, given the increasing differences over distributions property, BIC mechanisms can be indeed characterized by a monotone-expected-marginal condition and an envelope formula.

Proposition 5.2. *Suppose v_i satisfies the increasing differences over distributions property for each $i \in \mathcal{I}$. A mechanism (q,t) is BIC if and only if for each $i \in \mathcal{I}$ and $x_i \in X_i$: (i) $E_{\mathbf{x}_{-i}} v_{ix}(q(s,\mathbf{x}_{-i}),x_i)$ is non-decreasing in s and (ii) agent i's interim expected utility can be expressed as*

$$U_i(x_i) \;=\; U_i(\underline{x}_i) + \int_{\underline{x}_i}^{x_i} E_{\mathbf{x}_{-i}} v_{ix}(q(s,\mathbf{x}_{-i}),s)ds,\;\; \forall x_i \in X_i \text{ and } i \in \mathcal{I}.^9 \tag{5.2}$$

Parallel to the result of Proposition 5.1, the increasing differences over distributions is a necessary condition for the characterization of Proposition 5.2 (see Proposition E.2 in Appendix E).

[7]The result below follows from Propositions 1, 2, and 3 of Mookherjee and Reichelstein (1992), because when v_i is differentiable with respect to its second argument the increasing differences property is equivalent to the weak single-crossing property used in their paper.

[8]See also Milgrom and Segal (2002). The sufficiency part holds even without imposing the increasing differences.

[9]As in Proposition 5.1, the sufficiency part holds even without imposing increasing differences over distributions.

Propositions 5.1 and 5.2 are connected to the literature on monotonicity and incentive compatibility. For general quasi-linear environments, Rochet (1987) showed that incentive compatibility constraint can be characterized by a condition called cycle-monotonicity. For convex type-spaces, Saks and Yu (2005) advanced Rochet (1987)'s result by establishing that it is sufficient to consider only two-cycle monotonicity.[10] The two-cycle monotonicity condition reduces to the standard monotonicity of the allocation rule when agents have dot product valuations (e.g., $A \subset \mathbb{R}$ and $v_i(a, x_i) = a \cdot x_i$). When agents have non-linear differentiable valuations (and one-dimensional types), the two-cycle monotonicity is equivalent to the monotone-marginal condition (see Proposition 5.1). Thus, Propositions 5.1 and 5.2, together with Propositions E.1 and E.2, determine the largest set of differentiable quasi-linear utility functions that permit the characterization of incentive compatibility with the two-cycle monotonicty condition for one-dimensional types.

The increasing differences over distributions property gives us a readily workable characterization of Bayesian incentive compatibility. This property is, however, novel, and we want to understand how it restricts agents' utilities before proceeding with further analysis. First of all, if the feasible set A is the set of all possible lotteries over some set of alternatives, increasing differences over distributions and increasing differences properties are equivalent, because in this case any probability distribution over A simply defines a compound lottery over the underlying set of alternatives. In general, however, the increasing differences over distributions property only implies the increasing differences property. To see this, simply note that one can always consider a pair of deterministic distributions in the definition of the increasing differences over distributions. Finally, we provide a full characterization of utility functions that satisfy the increasing differences over distributions property.

Proposition 5.3. *Function $v_i : A \times X_i \to \mathbb{R}$ satisfies increasing differences over distributions if and only if there exist functions $f_i, g_i : A \to \mathbb{R}$ and $M_i, m_i : X_i \to \mathbb{R}$, where f_i and g_i are continuous and M_i is increasing, such that for all $a \in A$ and $x_i \in X_i$,*

$$v_i(a, x_i) = f_i(a)M_i(x_i) + m_i(x_i) + g_i(a). \tag{5.3}$$

In a concurrent paper, Kartik et al. (2018) study a less demanding property of the single-crossing expectational differences, which extends the standard single-crossing differences property to the space of lotteries. They show that their novel property admits the characterization $v_i(a, x_i) = f_i(a)M_i(x_i) + g_i(a)\hat{M}_i(x) + m_i(x_i)$, where $f_i, g_i : A \to \mathbb{R}$ and $M_i, \hat{M}_i, m_i : X_i \to \mathbb{R}$ with M_i and $\hat{M}_i$ being each single crossing and ratio ordered. The ratio-ordered requirement reduces to M_i being increasing function when $\hat{M}_i \equiv 1$. Celik (2015) also employs a weaker version of increasing differences over distributions condition to analyze the implementation with gradual-revelation. These weaker properties, however, do not allow a convenient characterization of Bayesian incentive compatibility as Proposition E.2 in Appendix highlights.

[10]See also Ashlagi et al. (2010) and Bikhchandani et al. (2006) for the analysis of incentive compatibility in convex domains. Mishra et al. (2014) and Kushnir and Galichon (2017) analyze two-cycle monotonicity condition in important non-convex domains.

Liu and Pei (2018) also consider a related but more demanding property of the increasing absolute differences over distributions. They show that this property together with monotone-supermodularity are sufficient to guarantee the monotonicity of sender's equilibrium strategy with respect to her type in signaling games.

The increasing differences over distributions property is also related to the aggregation of the single-crossing property analyzed by Quah and Strulovici (2012). They consider function $v(a, x, t)$ that satisfies the single-crossing differences property in (a, x) for each t. They ask under what conditions the aggregate function $\int v(a, x, t)dF(t)$ will also satisfy the single-crossing differences property for all distributions F. While this question is not trivial, the answer to the parallel question for the increasing differences property is rather straightforward if one fixes the direction of monotonicity: If for given a and a' the difference $v(a', x, t) - v(a, x, t)$ is increasing in x for each t, the aggregate difference has to be increasing.[11] However, requiring the increasing differences property to hold in the space of lotteries is different from requiring it to be preserved under aggregation, as Proposition 5.3 shows.

Given the result of Proposition 5.3, we assume in the rest of the paper that agent i's value function v_i takes the form of (5.3). With this specification, for each $i \in \mathcal{I}$, the monotonicity conditions in the characterizations of DIC and BIC mechanisms are now equivalent to $f_i(q(s, \mathbf{x}_{-i}))$ being non-decreasing in s for $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$ and $E_{\mathbf{x}_{-i}} f_i(q(s, \mathbf{x}_{-i}))$ being non-decreasing in s, respectively.[12]

5.4 The BIC-DIC Equivalence

We use the following logic to prove the equivalence between Bayesian and dominant strategy implementation. The characterizations of DIC and BIC mechanisms (Propositions 5.1 and 5.2) imply that the interim expected utilities of agents are determined by the allocation rule (up to a constant). Therefore, to match agents' interim expected utilities, we need to match $E_{\mathbf{x}_{-i}} f_i(q(x_i, \mathbf{x}_{-i}))$ for each $x_i \in X_i$ and $i \in \mathcal{I}$. To respect the incentive compatibility, we need to satisfy the monotone-marginal condition, i.e. $f_i(q(\cdot, \mathbf{x}_{-i}))$ is non-decreasing for each $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$ and $i \in \mathcal{I}$. Finally, we need to make sure that the equivalent mechanisms generate the same expected social surplus.

To state our main result, we introduce first the notion of convex-valued mappings. A mapping $\mathbf{f} : A \to \mathbb{R}^I$ with $\mathbf{f}(a) = (f_1(a), ..., f_I(a))$ is *convex-valued* if its image is convex, i.e., for any $a, b \in A$ and $\alpha \in [0, 1]$ there exists $c \in A$ such that $\mathbf{f}(c) = \alpha\mathbf{f}(a) + (1 - \alpha)\mathbf{f}(b)$. We also note a useful property of mappings $\mathbf{g} = (g_1, ..., g_I)$ and $\mathbf{f} = (f_1, ..., f_I)$ in (5.3): If $\mathbf{g}$ is a linear transformation of $\mathbf{f}$, i.e., $\mathbf{g} \equiv M\mathbf{f}$ for some $I \times I$ matrix M, then $\mathbf{f}$ is convex-valued if and only if the mapping of all agents utilities $(v_1(\cdot, x_1) + t_1, ..., v_I(\cdot, x_I) + t_I)$ is convex-valued for each $(x_1, ..., x_I) \in \mathbf{X}$.[13]

[11] We thank Navin Kartik, SangMok Lee, and Daniel Rappoport for pointing out this connection to us.

[12] In specification (5.3) we could redefine types $\tilde{x}_i \sim M_i(x_i)$ and drop function $m_i(x_i)$ because it does not interact with allocation. We cannot, however, modify g_i and f_i as it becomes clear from applications of Section 5.5.

[13] The necessity part actually holds only under an additional mild condition. If we denote the matrix transforming $\mathbf{f}$ to $\mathbf{g}$ as A and the diagonal matrix with elements $M_i(x_i)$ as $M(\mathbf{x})$ with $\mathbf{x} = (x_1, ..., x_I)$, the additional condition states that the sum of matrices $M(\mathbf{x}) + A$ has a full rank.

Theorem 5.1. *Assume mapping* $\mathbf{f}$ *is convex-valued, and* $\mathbf{g}$ *is a linear transformation of* $\mathbf{f}$*. Then for any BIC mechanism* $(\tilde{q}, \tilde{t})$ *there exists an equivalent DIC mechanism* (q, t)*.*

The main part of the argument proving the theorem establishes that for a given BIC allocation rule $\tilde{q}$ there exists a feasible allocation q that satisfies

$$E_{\mathbf{x}_{-i}} f_i(q(x_i, \mathbf{x}_{-i})) \;=\; E_{\mathbf{x}_{-i}} f_i(\tilde{q}(x_i, \mathbf{x}_{-i})), \forall x_i \in X_i, \forall i \in \mathcal{I}, \tag{5.4}$$

and that has non-decreasing marginals $f_i(q(\cdot, \mathbf{x}_{-i}))$ for all $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$ and $i \in \mathcal{I}$. We establish this statement for discrete and uniformly distributed types in Lemma 5.1 below. In particular, we develop an algorithm that finds a feasible allocation that satisfies (5.4) and that has non-decreasing marginals.[14] We then extend the proof to continuous types and arbitrary distributions (see Lemmas E.1 and E.2). Finally, we construct transfers that lead to the same interim expected utilities and generate the same expected social surplus using the envelope formula (see Proposition 5.1).

Lemma 5.1. *Suppose, for all* $i \in \mathcal{I}$*,* X_i *is a finite discrete set and* λ_i *is the uniform distribution on* X_i*. For any BIC allocation* $\tilde{q}$ *there exists a feasible allocation* q *satisfying (5.4) and* $f_i(q(\cdot, \mathbf{x}_{-i}))$ *being non-decreasing for all* $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$ *and* $i \in \mathcal{I}$*.*

Proof of Lemma 5.1. Consider an arbitrary BIC allocation $\tilde{q}$, and let us assume $f_j(\tilde{q}(\cdot, \mathbf{x}_{-j}))$ is not non-decreasing for some j and $\mathbf{x}_{-j}$; otherwise the statement is trivial. Then, there exists some $x'_j > x_j$ such that $f_j(\tilde{q}(x'_j, \mathbf{x}_{-j})) < f_j(\tilde{q}(x_j, \mathbf{x}_{-j}))$. Since agent j's expected marginal $E_{\mathbf{x}_{-j}} f_j(\tilde{q}(\cdot, \mathbf{x}_{-j}))$ is non-decreasing there also exists $\mathbf{X}'_{-j} \subset \mathbf{X}_{-j}$ such that $f_j(\tilde{q}(x'_j, \mathbf{x}'_{-j})) > f_j(\tilde{q}(x_j, \mathbf{x}'_{-j}))$ for all $\mathbf{x}'_{-j} \in \mathbf{X}'_{-j}$. Now consider a new allocation $\hat{q} \neq \tilde{q}$ such that

$$\begin{aligned}
\mathbf{f}(\hat{q}(x_j, \mathbf{x}_{-j})) &= \frac{1}{2}\mathbf{f}(\tilde{q}(x_j, \mathbf{x}_{-j})) + \frac{1}{2}\mathbf{f}(\tilde{q}(x'_j, \mathbf{x}_{-j})), \\
\mathbf{f}(\hat{q}(x'_j, \mathbf{x}_{-j})) &= \frac{1}{2}\mathbf{f}(\tilde{q}(x'_j, \mathbf{x}_{-j})) + \frac{1}{2}\mathbf{f}(\tilde{q}(x_j, \mathbf{x}_{-j})), \\
\mathbf{f}(\hat{q}(x_j, \mathbf{x}'_{-j})) &= (1-\delta)\mathbf{f}(\tilde{q}(x_j, \mathbf{x}'_{-j})) + \delta\mathbf{f}(\tilde{q}(x'_j, \mathbf{x}'_{-j})), \\
\mathbf{f}(\hat{q}(x'_j, \mathbf{x}'_{-j})) &= (1-\delta)\mathbf{f}(\tilde{q}(x'_j, \mathbf{x}'_{-j})) + \delta\mathbf{f}(\tilde{q}(x_j, \mathbf{x}'_{-j})),
\end{aligned}$$

for all $\mathbf{x}'_{-j} \in \mathbf{X}'_{-j}$ and $\hat{q}(\mathbf{x}) = \tilde{q}(\mathbf{x})$ for all other $\mathbf{x} \in \mathbf{X}$, where

$$\delta = \frac{1}{2} \cdot \frac{f_j(\tilde{q}(x_j, \mathbf{x}_{-j})) - f_j(\tilde{q}(x'_j, \mathbf{x}_{-j}))}{\sum_{\mathbf{x}'_{-j} \in X'_{-j}} (f_j(\tilde{q}(x'_j, \mathbf{x}'_{-j})) - f_j(\tilde{q}(x_j, \mathbf{x}'_{-j})))}. \tag{5.5}$$

Since $E_{\mathbf{x}_{-j}} f_j(\tilde{q}(\cdot, \mathbf{x}_{-j}))$ is non-decreasing we have $0 \le \delta \le \frac{1}{2}$. In addition, a feasible allocation $\hat{q}$ with $\hat{q}(\mathbf{x}) \in A$, for each $\mathbf{x} \in \mathbf{X}$, is guaranteed to exist, because mapping $\mathbf{f}$ is convex-valued. Equation (5.5) guarantees that the equal expected marginal condition (5.4) is satisfied for agent j having types x_j and x'_j. For agent j having other types, condition (5.4) follows trivially. For agent i, $i \neq j$, condition (5.4) follows from $\mathbf{f}(\hat{q}(x_j, \mathbf{x}_{-j})) + \mathbf{f}(\hat{q}(x'_j, \mathbf{x}_{-j})) = \mathbf{f}(\tilde{q}(x_j, \mathbf{x}_{-j})) + \mathbf{f}(\tilde{q}(x'_j, \mathbf{x}_{-j}))$

[14] Gershkov et al. (2013) use a minimization problem to find a feasible allocation that satisfies (5.4) and that has non-decreasing marginals. Their approach could also be adapted to our settings. We use an algorithmic proof because of its convenience in the proofs of our Theorem 5.2 and the applications presented in Section 5.5.

and $\mathbf{f}(\hat{q}(x_j, \mathbf{x}'_{-j})) + \mathbf{f}(\hat{q}(x'_j, \mathbf{x}'_{-j})) = \mathbf{f}(\tilde{q}(x_j, \mathbf{x}'_{-j})) + \mathbf{f}(\tilde{q}(x'_j, \mathbf{x}'_{-j}))$.

Let us now define $\hat{s} = E_{\mathbf{x}}(||\mathbf{f}(\hat{q}(\mathbf{x})||^2)$ and $\tilde{s} = E_{\mathbf{x}}(||\mathbf{f}(\tilde{q}(\mathbf{x})||^2)$, where $||\cdot||$ denotes the Euclidean norm $||\mathbf{f}(q(\mathbf{x}))||^2 = \sum_{i\in\mathcal{I}} f_i(q(\mathbf{x}))^2$. Taking into account that λ_i is uniformly distributed, we have

$$\begin{aligned}\hat{s} - \tilde{s} \;=\; & -\frac{1}{2|X|}||\mathbf{f}(\tilde{q}(x_j, \mathbf{x}_{-j})) - \mathbf{f}(\tilde{q}(x'_j, \mathbf{x}_{-j}))||^2 \\ & - \frac{2\delta(1-\delta)}{|X|}\,||\mathbf{f}(\tilde{q}(x'_j, \mathbf{x}'_{-j})) - \mathbf{f}(\tilde{q}(x_j, \mathbf{x}'_{-j}))||^2 \;<\; 0.\end{aligned}$$

If $f_j(\hat{q}(\cdot, \mathbf{x}_{-j}))$ is not non-decreasing for some j and $\mathbf{x}_{-j}$, we repeat the above procedure. Iterating the procedure, we finally obtain a sequence of allocations $q^n \in A$ and a sequence of values $s^n \geq 0$ for $n = 1, 2, ...$ If for some n we find that $f_j(q^n(\cdot, \mathbf{x}_{-j}))$ is non-decreasing for all j and $\mathbf{x}_{-j}$, we set $q^{n+1} \equiv q^n$ and $s^{n+1} \equiv s^n$. By construction, s^n is a weakly decreasing sequence that is bounded below by 0. Hence, s^n has a limit that we denote as s. Since set A is compact, there also exists a convergent subsequence of q^n with a limit q such that $q(\mathbf{x}) \in A$ for all $\mathbf{x} \in X$. By construction, $s = E_{\mathbf{x}}(||\mathbf{f}(q(\mathbf{x})||^2)$.

We argue for the limit allocation q that $f_j(q(\cdot, \mathbf{x}_{-j}))$ has to be non-decreasing for each j and $\mathbf{x}_{-j}$. Suppose, in contradiction, that for some $j \in \mathcal{I}$ and $\mathbf{x}_{-j} \in \mathbf{X}_{-j}$ $f_j(q(\cdot, \mathbf{x}_{-j}))$ is not non-decreasing. Using the above construction, we can obtain an allocation q' with $s' = E_{\mathbf{x}}(||\mathbf{f}(q'(\mathbf{x})||^2) < s$. This contradicts to the fact that s is a limit of decreasing sequence s^n constructed above. □

Proof of Theorem 5.1. Lemmas E.1 and E.2 (postponed to Appendix E) extend Lemma 5.1 to show that, given any set $X_i \subset \mathbb{R}$ and any distribution λ_i, for any BIC allocation $\tilde{q}$ there exists a feasible allocation q satisfying (5.4) with non-decreasing marginals $f_i(q(\cdot, \mathbf{x}_{-i}))$ for all $i \in \mathcal{I}$ and $\mathbf{x}_{-i} \in X_{-i}$. To complete the construction of an equivalent DIC mechanism we consider transfers t defined by

$$\begin{aligned}t_i(x_i, \mathbf{x}_{-i}) \;=\; & t_i(\underline{x}_i, \mathbf{x}_{-i}) + v_i(q(\underline{x}_i, \mathbf{x}_{-i}), \underline{x}_i) \\ & - v_i(q(x_i, \mathbf{x}_{-i}), x_i) + \int_{\underline{x}_i}^{x_i} v_{ix}(q(s, \mathbf{x}_{-i}), s)ds, \end{aligned} \tag{5.6}$$

for all $\mathbf{x} \in X$, $i \in \mathcal{I}$, where $t_i(\underline{x}_i, \mathbf{x}_{-i}) = E_{\mathbf{x}_{-i}}(v_i(\tilde{q}(\underline{x}_i, \mathbf{x}_{-i}), \underline{x}_i) + \tilde{t}_i(\underline{x}_i, \mathbf{x}_{-i})) - v_i(q(\underline{x}_i, \mathbf{x}_{-i}), \underline{x}_i)$. Proposition 5.1 guarantees that mechanism (q, t) is DIC. In addition, mechanism (q, t) leads to the same interim expected utilities as in BIC mechanism $(\tilde{q}, \tilde{t})$. In particular,

$$\begin{aligned}U_i(x_i) \;&=\; E_{\mathbf{x}_{-i}}(\tilde{t}_i(\underline{x}_i, \mathbf{x}_{-i}) + v_i(\tilde{q}(\underline{x}_i, \mathbf{x}_{-i}), \underline{x}_i)) + \int_{\underline{x}_i}^{x_i} E_{\mathbf{x}_{-i}} v_{ix}(q(s, \mathbf{x}_{-i}), s)ds \\ &=\; E_{\mathbf{x}_{-i}}(\tilde{t}_i(\underline{x}_i, \mathbf{x}_{-i}) + v_i(\tilde{q}(\underline{x}_i, \mathbf{x}_{-i}), \underline{x}_i)) + \int_{\underline{x}_i}^{x_i} E_{\mathbf{x}_{-i}} v_{ix}(\tilde{q}(s, \mathbf{x}_{-i}), s)ds \\ &=\; \tilde{U}_i(x_i),\end{aligned} \tag{5.7}$$

where the first equality follows from (5.6), the second one from (5.4), and the third one from the characterization of BIC mechanisms (Proposition 5.2). When mapping $\mathbf{g}$ is a linear transformation of $\mathbf{f}$, the equal expected marginal conditions in (5.4) also imply $E_{\mathbf{x}}[\sum_{i\in\mathcal{I}} g_i(q(\mathbf{x}))] = E_{\mathbf{x}}[\sum_{i\in\mathcal{I}} g_i(\tilde{q}(\mathbf{x}))]$. Hence, both mechanisms also generate the same social surplus. $\square$

Theorem 5.1 extends the BIC-DIC equivalence result to non-linear environments where each agent's utility satisfies the increasing differences over distributions property and the mapping of all agents' utilities is convex-valued. The convex-valued assumption is generally indispensable for the equivalence result as Example E.4 in Appendix shows. In addition, the new proof requires only that the set of feasible allocations A is compact instead of being a simplex as in Gershkov et al. (2013).

The requirement that $\mathbf{g}$ is a linear transformation of $\mathbf{f}$ is satisfied, for example, if $g_i \equiv 0\,\forall i \in \mathcal{I}$ as in some applications of Section 5.5. For general $\mathbf{g}$, the constructed DIC mechanism, however, does not necessarily match the expected social surplus of the BIC mechanism.[15] We now analyze the conditions when for any BIC mechanism one could find a DIC mechanism that produces the same interim expected utilities and generates *at least as large* expected social surplus. In addition to being more flexible than the equivalence, this way of comparing the implementation concepts better captures the economic intuition that one does not need to insert additional money to achieve a more robust solution concept.

For this purpose, we consider environments where the set of feasible allocations A is a convex and compact subset of $\mathbb{R}^I$ with $\mathbf{q} = (q_1, ..., q_I)$, where $q_i \in \mathbb{R}$ for each $i \in \mathcal{I}$. We also assume that functions f_i depend on different components of allocations, i.e., $f_i(\mathbf{q}) = \check{f}_i(q_i)$, for all $i \in \mathcal{I}$, $\mathbf{q} \in A$.

Theorem 5.2. *Assume mapping $\mathbf{f}$ is convex-valued. For any BIC mechanism there exists a DIC mechanism that delivers the same interim expected utilities for all agents. In addition, the DIC mechanism generates at least as large expected social surplus, if*

(i) for each $i \in \mathcal{I}$ $\check{f}_i(q_i)$ is non-decreasing and concave (or non-increasing and convex) and $g_i(\mathbf{q})$ is continuous, non-increasing, and concave in each component, or

(ii) for each $i \in \mathcal{I}$ $\check{f}_i(q_i)$ is non-increasing and concave (or non-decreasing and convex) and $g_i(\mathbf{q})$ is continuous, non-decreasing, and concave in each component.

The theorem also extends to settings where the set of feasible allocations A is compact, mapping $\mathbf{f}$ is convex-valued, and the utility of each agent satisfies the following condensation property: Functions f_i and g_i can be written as $f_i(\mathbf{q}) = \check{f}_i(h_i(\mathbf{q}))$ and $\sum_i g_i(\mathbf{q}) = G(h_1(\mathbf{q}), ..., h_I(\mathbf{q}))$ for all $\mathbf{q} \in A$, where $h_i : A \to \mathbb{R}$, $\check{f}_i$ is non-decreasing and concave (or non-increasing and convex), and the aggregate function $G : \mathbb{R}^I \to \mathbb{R}$ is continuous, non-increasing, and concave in each component.[16] The proof of this extension repeats the steps of the proof of Theorem 5.2 presented in Appendix E, and we omit it to avoid repetition. We exploit this observation when we consider the environmental mechanism design applications in Section 5.5.

The requirement of Theorem 5.2 that the DIC mechanism produces only at least as large expected social surplus compared to the original BIC mechanism is less demanding than the one

[15]For general $\mathbf{g}$ the constructed DIC mechanism still delivers the same interim expected utilities.

[16]Similar to condition (ii) in Theorem 5.2, the result also extends to settings when $\check{f}_i$ is non-increasing and concave (or non-decreasing and convex) and G is continuous, non-decreasing, and concave in each component.

of mechanisms equivalence (see Definition 5.1). Hence, it also has a broader range of meaningful economic applications, which we illustrate in Section 5.5.

5.5 Applications

In this section, we demonstrate that Theorems 5.1 and 5.2 apply to many important environments where previous works have little bite (e.g. Manelli and Vincent 2010; Gershkov et al. 2013). In addition, they produce several novel implications that are of independent interest.

5.5.1 Principal-Agent Problem with Allocative Externalities

Consider a standard contracting setting where a principal needs to procure I goods from I agents. Assume the principal chooses a production plan $\mathbf{q} = (q_1, ..., q_I) \in A \equiv \Pi_{i=1}^{I}[\underline{q}_i, \overline{q}_i] \subseteq \mathbb{R}^I$ and a transfer scheme $(t_1, ..., t_I) \in \mathbb{R}^I$. The payoff of agent i is then given by $-c_i(q_i)x_i + t_i$, where $c_i : [\underline{q}_i, \overline{q}_i] \to \mathbb{R}$ is some continuous non-decreasing function with an interpretation of $c_i(q_i)x_i$ being agent i's cost of supplying q_i units of good i. Many influential papers analyzing the optimal procurement contracts fall into this setting (e.g. Duenyas et al., 2013; Laffont and Martimort, 1997; Mookherjee and Tsumagari, 2004; Severinov, 2008). In this setting, we have $f_i(\mathbf{q}) = -c_i(q_i)$ and $g_i(\mathbf{q}) = 0$ for each $i \in \mathcal{I}$. Since functions c_i are continuous, the Intermediate Value Theorem implies that mapping $\mathbf{f}(\cdot) = (-c_1(\cdot), ..., -c_I(\cdot))$ is convex-valued. Thus, Theorem 5.1 leads to the following corollary.

Corollary 5.1. *Consider the standard procurement setting. If c_i is continuous for $i \in \mathcal{I}$, then for any BIC mechanism there exists an equivalent DIC mechanism.*

In many contracting situations, agents may not only care about their own contracts with the principal, but also have preferences about contracts received by other agents. For instance, a country may prefer its ally rather than its enemy to receive a weapon contract (see Jehiel et al., 1996). Similar concerns arise in the presence of downstream competition among firms (Segal, 1999). Within the current framework, type-independent allocative externalities can be captured by incorporating an additional term into agent's utility function, i.e., $-c_i(q_i)x_i + g_i(\mathbf{q}) + t_i$. Assuming that the cost and externality functions satisfy the conditions of Theorem 5.2, we establish the following result.

Corollary 5.2. *Consider a procurement setting with allocation externalities. If c_i is continuous for each $i \in \mathcal{I}$, then for any BIC mechanism there exists a DIC mechanism that delivers the same interim expected utilities for all agents. If c_i is also non-decreasing and convex (concave) and g_i is continuous, non-decreasing (non-increasing), and concave in each component for each $i \in \mathcal{I}$, then the DIC mechanism generates at least as large expected social surplus as the BIC mechanism.*

Corollary 5.2 identifies environments with allocative externalities where a mechanism designer can rely on dominant strategy implementation and gains nothing from designing more complex BIC mechanisms. This is in sharp contrast to results pertaining to environments with both allocative and information externalities, where more robust solution concepts appear to be much more restrictive (see Jehiel and Moldovanu (2006) for an excellent survey).

5.5.2 Environmental Mechanism Design

Let us first consider the environmental mechanism design model of (Martimort and Sand-Zantman, 2013, 2016), who analyze feasible agreements in reducing the aggregate pollution of I countries. Each country i can exert effort $q_i \in [\underline{q}, \overline{q}] \subseteq \mathbb{R}_+$ that has both local benefits of size αq_i (with $\alpha \in [0,1)$) and global benefits of size $(1-\alpha)q_i$, which accrue worldwide. The countries differ in their costs of effort $q_i^2 x_i/2$, with x_i being country i's efficiency parameter. Efficiency parameters are drawn independently from the same cumulative distribution λ with support $[\underline{x}, \overline{x}] \subseteq \mathbb{R}$. Overall, country i's payoff is given by $-q_i^2 x_i/2 + \alpha q_i + (1-\alpha)Q + t_i$, where $Q = \sum_{i=1}^{I} q_i$ is aggregate global benefits and t_i is a monetary transfer to country i. Taking into account that function $-q_i^2/2$ is non-increasing and concave and the externality function $\alpha q_i + (1-\alpha)Q$ is non-decreasing and linear (and, hence, concave), the following result directly follows from Theorem 5.2.

Corollary 5.3. *Consider the setting of (Martimort and Sand-Zantman, 2013, 2016). Then, for any BIC mechanism there exists a DIC mechanism producing the same interim expected utilities to all agents and generating at least as large expected social surplus.*

Baliga and Maskin (2003) also study feasible agreements to efficiently reduce the aggregate pollution level, but consider a slightly different model. Although they assume that agents' costs are type-independent, agents have private information about their value of the pollution reduction. More specifically, agent i's utility equals $x_i Q^{1/2} - q_i + t_i$, where $x_i Q^{1/2}$ is the gross benefits to agent i from aggregate reduction Q. Though Theorem 5.2 does not formally apply to this environment, each agent i's benefits and costs from pollution reduction satisfy the condensation property defined in the remark after Theorem 5.2. In particular, agent i's benefits equal $\check{f}_i(h_i(\mathbf{q})) = \sqrt{h_i(\mathbf{q})}$ and the aggregate costs equal $\sum_{i\in\mathcal{I}} q_i = \sum_{i\in\mathcal{I}} q_i = -G(h_1(\mathbf{q}), ..., h_I(\mathbf{q}))$, where the condensation function $h_i(\mathbf{q}) = \sum_{i\in\mathcal{I}} q_i$, $i \in \mathcal{I}$, is the same for all agents. The mapping $\mathbf{f} = (f_1(\cdot), ..., f_I(\cdot))$ is symmetric and, hence, the Intermediate Value Theorem implies that it is convex-valued. In addition, $\check{f}_i$ is non-decreasing and concave, and function G is non-increasing and linear (and, hence, concave). Hence, the following result is implied by the extension discussed in the remark after Theorem 5.2.

Corollary 5.4. *Consider the environmental mechanism design setting of Baliga and Maskin (2003). Then, for any BIC mechanism there exists a DIC mechanism producing the same interim expected utilities to all agents and generating at least as large expected social surplus.*

Corollaries 5.3 and 5.4 imply that the mechanism designer would lose nothing by restricting himself/herself to DIC mechanisms for environmental design problems, if he/she wanted to maintain the same level of agents' interim expected utility without the influx of additional money into the system. This result, however, may no longer hold when additional constraints - such as ex post budget balance - are imposed, as thoroughly discussed in Baliga and Maskin

(2003). Though Bayesian implementation is more permissive when ex post budget balance is imposed, the mechanism designer can still rely only on DIC mechanisms if the budget balance constraint needs to be satisfied in expectations. We show this result in the next application.

5.5.3 Public Good Provision

Consider a standard setting of public good provision with $I \geq 2$ agents. If $q \in A = [\underline{q}, \overline{q}]$ units of public good are provided, agent i's utility is given by $f(q)x_i + t_i$, where $f(q)x_i$ is agent i's valuation of the public good and $t_i \in \mathbb{R}$ is the units of private good that he receives. Many influential papers on public good provision fall into this setting (e.g. Hellwig, 2003; Ledyard and Palfrey, 1999; Mailath and Postlewaite, 1990; Norman, 2004). If $f : A \to \mathbb{R}$ is continuous in q, it again follows from the Intermediate Value Theorem that the mapping $\mathbf{f}(\cdot) = (f(\cdot), ..., f(\cdot))$ is convex-valued and, hence, Theorem 5.1 can be applied here.

Corollary 5.5. *Consider the public good provision setting. If f is continuous, then for any BIC mechanism there exists an equivalent DIC mechanism.*

While the equivalent DIC mechanism, constructed in Theorem 5.1, inherits interim individual rationality from the BIC mechanism,[17] there is no guarantee that other constraints imposed on the BIC mechanism will remain satisfied as well. For example, when designing a mechanism for public good provision, it is typical to require that the private goods raised from the agents are enough to cover the cost of the public good. Formally, a direct mechanism (q, t) is *ex ante budget balanced* if

$$\int_{\mathbf{x} \in X} \left[K(q(\mathbf{x})) + \sum_{i=1}^{I} t_i(\mathbf{x}) \right] d\lambda(\mathbf{x}) \leq 0, \tag{5.8}$$

where $K : A \to \mathbb{R}$ is the cost function of producing the public good. The following corollary of Theorem 5.2 provides a sufficient condition under which the equivalent DIC mechanism constructed in Theorem 5.1 also inherits ex ante budget balance from the original BIC mechanism.[18]

Corollary 5.6. *Suppose f is continuous, non-decreasing, and concave and K is continuous, non-decreasing, and convex. For any BIC mechanism that is ex ante budget balanced, the equivalent DIC mechanism, constructed in Theorems 5.1, is also ex ante budget balanced.*

Intuitively, the monotonicity and concavity of utility functions imply that the provision of public good is more balanced across states in the equivalent DIC mechanism than that in the BIC mechanism. Consequently, the expected cost of providing the public good is lower. Since the expected transfers remain unchanged in the equivalent DIC mechanism constructed in Theorem 5.1, the property of ex ante budget balance is preserved.

Our result thus suggests that for a quite general class of public good provision problems it is without loss of generality to insist on dominant-strategy incentive compatibility, even when

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[17]The constructed DIC mechanism satisfies even a stronger notion of ex post individual rationality.

[18]The result of Corollary 5.6 extends without any change to non-symmetric settings with mapping $\mathbf{f} = (f_1(\cdot), ..., f_I(\cdot))$ being convex-valued and functions f_i, $i \in \mathcal{I}$, being continuous, non-decreasing, and concave.

the additional ex ante budget balance constraint is imposed.[19] For example, the second-best allocation rule in Hellwig (2003) can be equivalently implemented in dominant strategies without violating the ex ante budget balance condition if functions f and K are concave and convex respectively.

5.6 Conclusion

This paper extends the equivalence between Bayesian and dominant strategy implementation to environments where each agent's utility satisfies *the increasing differences over distributions property* and the mapping of all agents' utilities is *convex-valued.* These assumptions are satisfied by many important models that are studied in the literature on principal-agent problems with allocative externalities, environmental mechanism design, and public good provision. Since the results of the previous papers (Manelli and Vincent 2010; Gershkov et al. 2013) do not apply to these environments, the current paper significantly enlarges the set of settings where the mechanism designer can rely on a more robust solution concept of dominant strategy implementation.

In this paper, we also provide sufficient conditions when for a given BIC mechanism there exists a DIC mechanism that yields the same interim expected utilities to all agents and generates *at least as large* social surplus (see also Kushnir, 2015). Using this result, we provide several novel implications for the above-mentioned environments. In addition, being less demanding than the notion of equivalence due to Gershkov et al. (2013), this way of comparing two implementation concepts broadens the set of environments when the mechanism designer could insist on a more robust notion of implementation without sacrificing his/her objectives. Hence, we believe this notion will be useful for future studies.

Our proof of the BIC-DIC equivalence result relies heavily on the characterization of incentive compatibility using the novel *increasing differences over distributions property.* We show that increasing differences over distributions is necessary and sufficient for Bayesian incentive compatibility to be conveniently characterized in terms of a monotone-expected-marginal condition and an envelope formula.[20] The equivalence result could potentially hold in environments where the above properties are not satisfied. The proof should then employ quite different techniques.

One possible approach has been discussed in our recent work. In Kushnir and Liu (2017), we explain how the BIC-DIC equivalence reduces to a purely mathematical question when a linear transformation of intersection of two closed convex sets coincides with the intersection of their images. Another possible approach has been proposed by Goeree and Kushnir (2017) who develop a novel geometric approach to mechanism design using basic tools from convex analysis. Applying these techniques to the question of the BIC-DIC equivalence in non-linear environments without increasing differences over distributions condition and environments with multidimensional types is an exciting prospect for future research.

[19] For some applications, it is natural to require mechanisms to be *ex post budget balanced*, i.e., inequality (5.8) holds for each $\mathbf{x} \in \mathbf{X}$. Börgers and Norman (2009) show that for every ex ante budget balanced DIC mechanism (q, t) there exist transfers t' such that (q, t') is (i) BIC for all agents and DIC for all but one agent and (ii) ex post budget balanced. Agents also have the same interim expected payments in both mechanisms (see also Börgers, 2015).

[20] We also establish that the standard increasing differences property is necessary for dominant strategy incentive compatibility to be conveniently characterized in terms of a monotone-marginal condition and an envelope formula.

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Part III

Appendices

A Appendix: Chapter 1

A.1 Proof of Proposition 1.1

In this appendix, we establish Proposition 1.1 by proving a series of equivalence statements.

Lemma A.1. *u_1 has IADD if and only if for every $a_1 \in A_1$ and every $\alpha_2, \alpha_2' \in \Delta(A_2)$, we have*

$$\exists\, \tilde{\theta} \in \Theta, u_1(\tilde{\theta}, a_1, \alpha_2) > u_1(\tilde{\theta}, a_1, \alpha_2') \implies u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_1, \alpha_2') \text{ is increasing in } \theta. \quad \text{(A.1)}$$

Proof. The only-if part of the lemma is straightforward. Let us focus on the if part. To show that (A.1) implies IADD, it suffices to show that if $u_1(\tilde{\theta}, a_1, \alpha_2) > u_1(\tilde{\theta}, a_1, \alpha_2')$ for some $\tilde{\theta}$, then $u_1(\theta, a_1, \alpha_2) \geq u_1(\theta, a_1, \alpha_2')$ for all $\theta \in \Theta$.

Suppose towards a contradiction that there exist $a_1 \in A_1, \alpha_2, \alpha_2' \in \Delta(A_2)$, and $\tilde{\theta}, \hat{\theta} \in \Theta$, such that $u_1(\tilde{\theta}, a_1, \alpha_2) > u_1(\tilde{\theta}, a_1, \alpha_2')$ and $u_1(\hat{\theta}, a_1, \alpha_2) < u_1(\hat{\theta}, a_1, \alpha_2')$. Then, condition (A.1) implies that we have both $u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_1, \alpha_2')$ and $u_1(\theta, a_1, \alpha_2') - u_1(\theta, a_1, \alpha_2)$ being increasing in θ. Hence, $u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_1, \alpha_2')$ must be constant for every θ, which leads to a contradiction. □

Next, notice that an immediate implication of u_1 satisfying IADD is that for every $a_1 \in A_1$ and $\alpha_2, \alpha_2' \in \Delta(A_2)$, the expected payoff difference $u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_1, \alpha_2')$ is monotone in θ. The following lemma fully characterizes this necessary condition of IADD.

Lemma A.2. *$u_1(\theta, a_1, \alpha_2) - u_1(\theta, a_1, \alpha_2')$ is monotone in θ for every $(a_1, \alpha_2, \alpha_2') \in A_1 \times \Delta(A_2) \times \Delta(A_2)$ if and only if the sender's payoff has the following representation:*

$$u_1(\theta, a_1, a_2) = f(a_1, a_2)v(\theta, a_1) + c(\theta, a_1) + g(a_1, a_2), \quad \text{(A.2)}$$

where $v : \Theta \times A_1 \to \mathbb{R}$ is an increasing function of θ.

The proof of Lemma A.2 is omitted as it immediately follows from the characterization results in Kartik et al. (2018) and Kushnir and Liu (2018). Therefore, it is without loss of generality to assume u_1 taking the functional form in (A.2), which we will do for the rest of the proof.

We now proceed to characterize condition (A.1). To do this, let us first introduce some useful notation. Let $A_2 \equiv \{a_2^1, .., a_2^n\}$ with $n \geq 2$. For every $a_1 \in A_1$, let

$$\underline{v}^{a_1} \equiv \min_{\theta \in \Theta} v(\theta, a_1) \in \mathbb{R},$$

and

$$f^{a_1} \equiv (f(a_1, a_2^1), ..., f(a_1, a_2^n)), \ g^{a_1} \equiv (g(a_1, a_2^1), ..., g(a_1, a_2^n)) \in \mathbb{R}^n.$$

Finally, let $\Gamma \equiv \{\gamma \in \mathbb{R}^n | \mathbf{1} \cdot \gamma = 0\}$, where $\mathbf{1} \equiv (1, 1, ..., 1) \in \mathbb{R}^n$ and '$\cdot$' denotes the inner product of two vectors. We establish the following result.

Lemma A.3. *Suppose that u_1 has representation (A.2). Then, u_1 satisfies* (A.1) *if and only if*

$$\forall\, (a_1, \gamma) \in A_1 \times \Gamma, \ \ (\underline{v}^{a_1} f^{a_1} + g^{a_1}) \cdot \gamma > 0 \implies f^{a_1} \cdot \gamma \geq 0. \tag{A.3}$$

Proof. (*If statement*) First, note that given the representation (A.2), condition (A.1) is equivalent to the requirement that for every $(a_1, \gamma) \in A_1 \times \Gamma$ and every $v \geq \underline{v}^{a_1}$, $(v f^{a_1} + g^{a_1}) \cdot \gamma > 0$ $\Rightarrow f^{a_1} \cdot \gamma \geq 0$. Suppose towards a contradiction that this does not hold for some $(a_1, \gamma) \in A_1 \times \Gamma$ and some $v \geq \underline{v}^{a_1}$. That is, we have $(v f^{a_1} + g^{a_1}) \cdot \gamma > 0$ but $f^{a_1} \cdot \gamma < 0$. Then, (A.3) implies that $(\underline{v}^{a_1} f^{a_1} + g^{a_1}) \cdot \gamma \leq 0$. Hence, we have

$$0 < (v f^{a_1} + g^{a_1}) \cdot \gamma = \underbrace{(\underline{v}^{a_1} f^{a_1} + g^{a_1}) \cdot \gamma}_{\leq 0} + \underbrace{(v - \underline{v}^{a_1}) f^{a_1} \cdot \gamma}_{\leq 0} \leq 0,$$

which leads to a contradiction.

(Only-if statement) Suppose that (A.3) is violated for some $(a_1, \gamma) \in A_1 \times \Gamma$, i.e. $(\underline{v}^{a_1} f^{a_1} + g^{a_1}) \cdot \gamma > 0$ but $f^{a_1} \cdot \gamma < 0$. Let $\xi > 0$ be small enough such that:

$$\max\{|\xi\gamma_1|, ..., |\xi\gamma_n|\} < 1/n.$$

Consider two probability distributions $\alpha_2, \alpha_2' \in \Delta(A_2)$, where $\alpha_2 \equiv \sum_{i=1}^n \frac{1}{n} \delta_{a_2^i}$, $\alpha_2' \equiv \sum_{i=1}^n (\frac{1}{n} - \xi\gamma_i) \delta_{a_2^i}$, and $\delta_{a_2^i}$ denotes the Dirac measure on $a_2^i \in A_2$. Let $\underline{\theta}$ be the smallest element in Θ, which exists since Θ is a complete lattice. By construction, when playing a_1, type $\underline{\theta}$ sender strictly prefers α_2 to α_2'. However, since $f^{a_1} \cdot \gamma < 0$, $u_1(\cdot, a_1, \alpha_2) - u_1(\cdot, a_1, \alpha_2')$ is strictly decreasing in θ. Hence, condition (A.1) is violated. □

Next, consider the linear operator $\tau : \mathbb{R}^n \to \mathbb{R}^{n-1}$ with

$$\tau(w) \equiv (w_1 - w_n, ..., w_{n-1} - w_n), \quad \forall w \in \mathbb{R}^n.$$

By construction, $\tau(w) = 0$ if and only if w is a constant vector. In addition, for every $\gamma \in \Gamma$ and $w \in \mathbb{R}^n$, we have $w \cdot \gamma = \sum_{i=1}^{n-1} (w_i - w_n)\gamma_i = \tau(w) \cdot \gamma$. Our next lemma provides a further characterization of condition (A.1) via the linear mapping τ.

Lemma A.4. *Suppose that u_1 has the representation (A.2). Then, u_1 satisfies condition* (A.3) *if and only if for every $a_1 \in A_1$, there exist $\lambda, \mu \in [0, +\infty)$ with $(\lambda, \mu) \neq (0, 0)$ such that*

$$\lambda \tau(f^{a_1}) = \mu \tau(\underline{v}^{a_1} f^{a_1} + g^{a_1}) \tag{A.4}$$

Proof. For every $w \in \mathbb{R}^n$, let us partition Γ into $\Gamma^+(w), \Gamma^-(w), \Gamma^0(w)$, such that $w \cdot \gamma > 0$

(resp., $w \cdot \gamma < 0$) for every $\gamma \in \Gamma^+(w)$ (resp., $\gamma \in \Gamma^-(w)$), and $\Gamma^0(w) = \Gamma \setminus (\Gamma^+(w) \cup \Gamma^-(w))$. Now we can equivalently state condition (A.3) as

$$\Gamma^+(\underline{v}^{a_1} f^{a_1} + g^{a_1}) \subset \Gamma^0(f^{a_1}) \cup \Gamma^+(f^{a_1}), \quad \forall a_1 \in A_1. \tag{A.5}$$

(*If statement*) Pick any $a_1 \in A_1$ and suppose there exist λ and μ such that (A.4) holds. If either λ or μ is 0, then since $(\lambda, \mu) \neq (0, 0)$, we have either $\tau(f^{a_1}) = 0$ or $\tau(\underline{v}^{a_1} f^{a_1} + g^{a_1}) = 0$. In both cases, (A.5) is satisfied. If $\lambda\mu \neq 0$, then by (A.4) we have for every $\gamma \in \Gamma^+(\underline{v}^{a_1} f^{a_1} + g^{a_1})$,

$$f^{a_1} \cdot \gamma = \tau(f^{a_1}) \cdot \gamma = \frac{\mu}{\lambda} \tau(\underline{v}^{a_1} f^{a_1} + g^{a_1}) \cdot \gamma = \frac{\mu}{\lambda} (\underline{v}^{a_1} f^{a_1} + g^{a_1}) \cdot \gamma > 0.$$

Hence, $\gamma \in \Gamma^+(f^{a_1})$.

(*Only-if statement*) Pick any $a_1 \in A_1$ and consider the two $n-1$ dimensional vectors $\tau(f^{a_1})$ and $\tau(\underline{v}^{a_1} f^{a_1} + g^{a_1})$. Suppose that the required λ and μ do not exist. Then, there exists no $\kappa \geq 0$ such that $\kappa\tau(f^{a_1}) = \tau(\underline{v}^{a_1} f^{a_1} + g^{a_1})$. By Farkas' Lemma, there exists $\tilde{\gamma} \equiv (\tilde{\gamma}_1, ..., \tilde{\gamma}_{n-1}) \in \mathbb{R}^{n-1}$ such that

$$\tau(f^{a_1}) \cdot \tilde{\gamma} < 0 \text{ but } \tau(\underline{v}^{a_1} f^{a_1} + g^{a_1}) \cdot \tilde{\gamma} > 0.$$[1]

Let $\gamma \equiv (\tilde{\gamma}_1, ..., \tilde{\gamma}_{n-1}, \tilde{\gamma}_n)$, where $\tilde{\gamma}_n \equiv -\sum_{i=1}^{n-1} \tilde{\gamma}_i$. The construction of $\tilde{\gamma}$ implies that $\gamma \in \Gamma^+(\underline{v}^{a_1} f^{a_1} + g^{a_1})$ but $\gamma \in \Gamma^-(f^{a_1})$. This violates (A.5) and thus also violates (A.3). □

To conclude the proof of Proposition 1.1, we derive (1.14) from (A.4). According to the definition of τ, Lemma A.4 implies that for every $(a_1, a_2) \in A_1 \times A_2$,

$$\lambda \left(f(a_1, a_2) - f(a_1, a_2^n)\right) = \mu\Big[\left(\underline{v}^{a_1} f(a_1, a_2) + g(a_1, a_2)\right) - \left(\underline{v}^{a_1} f(a_1, a_2^n) + g(a_1, a_2^n)\right) \Big],$$

or, equivalently,

$$\mu g(a_1, a_2) = (\lambda - \mu\underline{v}^{a_1}) f(a_1, a_2) + h(a_1),$$

where

$$h(a_1) = \mu(\underline{v}^{a_1} f(a_1, a_2^n) + g(a_1, a_2^n) - \lambda f(a_1, a_2^n).$$

On the one hand, if $\mu \neq 0$, let

$$\hat{v}(\theta, a_1) \equiv v(\theta, a_1) + (\lambda - \mu\underline{v}^{a_1})/\mu \text{ and } \hat{c}(\theta, a_1) \equiv c(\theta, a_1) + h(a_1),$$

which obtains representation (1.14). Note that by construction, $\min_{\theta \in \Theta} \hat{v}(\theta, a_1) = \lambda/\mu \geq 0$.

On the other hand, if $\mu = 0$, then we must have $\lambda \neq 0$ and $f(a_1, a_2) = h(a_1)/\lambda$. In this case,

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[1] Farkas' Lemma implies the existence of $\hat{\gamma} \in \mathbb{R}^{n-1}$ such that $\tau(f^{a_1}) \cdot \hat{\gamma} \leq 0$ and $\tau(\underline{v}^{a_1} f^{a_1} + g^{a_1}) \cdot \hat{\gamma} > 0$. But given that $\tau(f^{a_1}) \neq 0$, if $\tau(f^{a_1}) \cdot \hat{\gamma} = 0$, there must exist $\tilde{\gamma} \in \mathbb{R}^{n-1}$ close to $\hat{\gamma}$ such that $\tau(f^{a_1}) \cdot \tilde{\gamma} < 0$ and $\tau(\underline{v}^{a_1} f^{a_1} + g^{a_1}) \cdot \tilde{\gamma} > 0$.

let

$$\hat{f}(a_1, a_2) \equiv g(a_1, a_2),\ \hat{v}(\theta, a_1) \equiv 1 \text{ and } \hat{c}(\theta, a_1) \equiv c(\theta, a_1) + h(a_1)v(\theta, a_1)/\lambda,$$

which obtains representation (1.14). $\square$

A.2 Characterizing the Quasiconcavity-Preserving Property

In this appendix, we provide a characterization of the quasiconcavity-preserving property based on the primitives of the model. We first introduce a strict version of the signed-ratio monotonicity condition in Quah and Strulovici (2012).

Definition A.1. *(Strict Signed-Ratio Monotonicity). A pair of functions $\gamma_\theta^{a_1}, \gamma_{\theta'}^{a_1} : I \to \mathbb{R}$ obeys strict signed-ratio monotonicity (or SSRM) if*

(1) for every i such that $\gamma_\theta^{a_1}(i) < 0$ and $\gamma_{\theta'}^{a_1}(i) > 0$, we have

$$-\frac{\gamma_\theta^{a_1}(i)}{\gamma_{\theta'}^{a_1}(i)} > -\frac{\gamma_\theta^{a_1}(j)}{\gamma_{\theta'}^{a_1}(j)} \quad \textit{for every } j > i, \textit{ and}$$

(2) for every i such that $\gamma_\theta^{a_1}(i) > 0$ and $\gamma_{\theta'}^{a_1}(i) < 0$, we have

$$-\frac{\gamma_{\theta'}^{a_1}(i)}{\gamma_\theta^{a_1}(i)} > -\frac{\gamma_{\theta'}^{a_1}(j)}{\gamma_\theta^{a_1}(j)} \quad \textit{for every } j > i.$$

The next result characterizes the quasiconcavity-preserving property in our setting, which is a straightforward extension of Theorem 1 in Quah and Strulovici (2012):

Proposition A.1. *The receiver's payoff is quasiconcavity-preserving if and only if (i) $\gamma_\theta^{a_1}$ satisfies SSCP for every $(\theta, a_1) \in \Theta \times A_1$, and (ii) $\gamma_\theta^{a_1}$ and $\gamma_{\theta'}^{a_1}$ obey SSRM for every $a_1 \in A_1$ and every $\theta, \theta' \in \Theta$.*

Proof. (*Only-if statement*) Suppose that the receiver's payoff is quasiconcavity-preserving, i.e., $\Gamma_{\tilde{\pi}}^{\theta}$ has the strict single-crossing property for every $(a_1, \tilde{\pi}) \in A_1 \times \Delta(\Theta)$. Then, (i) immediately follows by taking the degenerate distributions over $\Delta(\Theta)$. For (ii), pick any pair of functions $\gamma_\theta^{a_1}$ and $\gamma_{\theta'}^{a_1}$. Suppose that $\gamma_\theta^{a_1}(i) < 0$ and $\gamma_{\theta'}^{a_1}(i) > 0$. Let

$$\beta = \frac{-\gamma_\theta^{a_1}(i)/\gamma_{\theta'}^{a_1}(i)}{1 - \gamma_\theta^{a_1}(i)/\gamma_{\theta'}^{a_1}(i)},$$

so that $\beta \in (0, 1)$ and $\beta\gamma_\theta^{a_1}(i) + (1-\beta)\gamma_{\theta'}^{a_1}(i) = 0$. Since $\beta\gamma_\theta^{a_1} + (1-\beta)\gamma_{\theta'}^{a_1}$ has the strict single-crossing property, we have $\beta\gamma_\theta^{a_1}(j) + (1-\beta)\gamma_{\theta'}^{a_1}(j) > 0$ for all $j > i$. Given that $\gamma_{\theta'}^{a_1}$ must satisfy SSCP and thus $\gamma_{\theta'}^{a_1}(j) > 0$, we can further obtain

$$\frac{1-\beta}{\beta} = -\frac{\gamma_\theta^{a_1}(i)}{\gamma_{\theta'}^{a_1}(i)} > -\frac{\gamma_\theta^{a_1}(j)}{\gamma_{\theta'}^{a_1}(j)}.$$

Hence, $\gamma_\theta^{a_1}$ and $\gamma_{\theta'}^{a_1}$ must obey SSRM for every $a_1 \in A_1$ and every $\theta, \theta' \in \Theta$.

(*If-statement*) Let $\Theta \equiv \{\theta_1, ..., \theta_K\}$. We need to show that $\forall \mu \equiv (\mu_1, ..., \mu_K) \in [0,1]^K$ such that $\sum_{k=1}^K \mu_i = 1$, the function $\Gamma_\mu^{a_1} : I \to \mathbb{R}$ with $\Gamma_\mu^{a_1}(i) \equiv \sum_{k=1}^K \mu_k \gamma_{\theta_k}^{a_1}(i)$ satisfies the strict single-crossing property. Since SSCP is preserved under positive scalar multiplication, and if $\gamma_\theta^{a_1}$ and $\gamma_{\theta'}^{a_1}$ obey SSRM then so do $\beta\gamma_\theta^{a_1}$ and $\gamma_{\theta'}^{a_1}$ for all $\beta \geq 0$, it suffices for us to show that $\Gamma^{a_1} \equiv \sum_{k=1}^K \gamma_{\theta_k}^{a_1}$ satisfies SSCP.

Suppose that $\Gamma^{a_1}(i) \geq 0$. We want to show that $\Gamma^{a_1}(j) > 0$ for every $j > i$. If $\gamma_{\theta_k}^{a_1}(i) \geq 0$ for all $k = 1, ..., K$, then we are done because each $\gamma_{\theta_k}^{a_1}$ satisfies SSCP. Now suppose that $\gamma_{\theta_k}^{a_1}(i) < 0$ for some $\theta_k \in \Theta$. In this case, let us partition Θ into three subsets, Θ^+, Θ^0 and Θ^-, such that $\theta_{k'} \in \Theta^+$ if $\gamma_{\theta_{k'}}^{a_1}(i) > 0$, $\theta_{k'} \in \Theta^0$ if $\gamma_{\theta_{k'}}^{a_1}(i) = 0$, and $\theta_{k'} \in \Theta^-$ if $\gamma_{\theta_{k'}}^{a_1}(i) < 0$. Hence, we have

$$\sum_{\theta_k \in \Theta^+ \cup \Theta^-} \gamma_{\theta_k}^{a_1} = \sum_{\ell=1}^{L} \gamma_\ell^{a_1},$$

where each function $\gamma_\ell : I \to \mathbb{R}$ is a positive linear combination of at most two functions $\gamma_{\theta_k}^{a_1}, \gamma_{\theta_{k'}}^{a_1}$ such that $\theta_k, \theta_{k'} \in \Theta^+ \cup \Theta^-$, and $\gamma_\ell(i) \geq 0$ for all $\ell = 1, ..., L$.

To complete the proof, it now suffices to show that for every $\ell = 1, ..., L$, if $\gamma_\ell^{a_1} = \alpha\gamma_{\theta_k}^{a_1} + \beta\gamma_{\theta_{k'}}^{a_1}$ for some $\alpha, \beta > 0$ and $\gamma_{\theta_k}^{a_1}, \gamma_{\theta_{k'}}^{a_1}$ such that $\gamma_{\theta_k}^{a_1}(i) < 0$ and $\gamma_{\theta_{k'}}^{a_1}(i) > 0$, we would then obtain $\gamma_\ell^{a_1}(j) > 0$ for every $j > i$. This is true because by SSRM, we have

$$\frac{\beta}{\alpha} \geq -\frac{\gamma_{\theta_k}^{a_1}(i)}{\gamma_{\theta_{k'}}^{a_1}(i)} > -\frac{\gamma_{\theta_k}^{a_1}(j)}{\gamma_{\theta_{k'}}^{a_1}(j)} \quad \text{for every } j > i,$$

and hence $\gamma_\ell^{a_1}(j) = \alpha\gamma_{\theta_k}^{a_1}(j) + \beta\gamma_{\theta_{k'}}^{a_1}(j) > 0$. □

A.3 Strongly Monotone Equilibria

Theorems 1.1 - 1.3 in the main text provide sufficient conditions under which the sender uses a monotone strategy in every Nash equilibrium. As discussed in section 1.2, our notion of monotonicity can be strengthened in environments when A_1 is multi-dimensional. This leads to the definitions of strongly monotone strategy and strongly monotone equilibrium:

Definition A.2. *σ_1 is a strongly monotone strategy if* $\min_{a_1}\{supp(\sigma_1^\theta)\} \succeq \max_{a_1}\{supp(\sigma_1^{\theta'})\}$ *for every $\theta \succ \theta'$. An equilibrium (σ_1, σ_2) is strongly monotone if σ_1 is strongly monotone.*

In words, a strategy is strongly monotone if a high type sender always plays a higher action than a low type. Plainly, strong monotonicity is strictly more demanding than monotonicity when A_1 is multi-dimensional. In what follows, we use an example to illustrate why it is fundamentally more challenging to establish a result stating that all equilibria are strongly monotone.

Let $\Theta = A_2 = \{0, 1, 2, 3\}$, and $A_1 = \{(0,0), (1,0), (0,1), (1,1)\}$. All sets are endowed with the product order. Note that A_1 is two-dimensional, and the sender's actions are only partially ordered. For all $\theta \in \Theta$, $a_1 = (a_{11}, a_{12}) \in A_1$ and $a_2 \in A_2$, the sender's payoff is given by

$$u_1(\theta, a_1, a_2) = \sqrt{a_2} - c(\theta, a_1),$$

where

$$c(\theta, a_1) = \begin{cases} 3a_{11} + 3a_{22} & \text{if } \theta = 0, \\ 0.9a_{11} + 2a_{22} & \text{if } \theta = 1, \\ 0.8a_{11} + a_{22} & \text{if } \theta = 2, \\ 0.2a_{11} + 0.2a_{22} & \text{if } \theta = 3. \end{cases}$$

The receiver's payoff is given by $u_2(\theta, a_1, a_2) = 1$ if $a_2 = \theta$, and $u_2(\theta, a_1, a_2) = 0$ otherwise. We leave the receiver's prior belief π unspecified as it plays no role.

In this example, the sender's payoff is *separable* and monotone-supermodular, which is sufficient to guarantee that she will use a monotone strategy in every equilibrium (Theorem 1.3). However, even the sender's payoff takes such a simple form, one cannot assert that all equilibria are strongly monotone. In particular, consider the following strategy profile: Type $\theta = 0$ sender chooses $a_1 = (0, 0)$, type 1 chooses $a_1 = (1, 0)$, type 2 chooses $a_1 = (0, 1)$, and type 3 chooses $a_1 = (1, 1)$. The receiver plays $a_2 = 0$ if he observes $a_1 = (0, 0)$, 1 if he observes $a_1 = (1, 0)$, 2 if he observes $a_1 = (0, 1)$, and 3 if $a_1 = (1, 1)$ is observed. One can check that this strategy profile constitutes a sequential equilibrium, in which every action of the sender is played with positive probability, and the players' incentives are strict. The sender's strategy is monotone but not strongly monotone, because the action $(0, 1)$ taken by type 2 is not higher than the action $(1, 0)$ taken by type 1.

In sum, the above example suggests that strong assumptions on the players' payoffs would need to be made if we want to further strengthen the robust monotonicity prediction about the sender's equilibrium strategies. In particular, the difficulty due to the incompleteness of the order on A_1 cannot be easily bypassed even in the simplest settings where the sender's payoff is separable.

A.4 Generalized Results with Infinite A_2

In this appendix, we generalize our monotonicity results to cases where A_1 is infinite. For simplicity, we shall assume that $A_1 \subset \mathbb{R}^n$ with $n \geq 1$ and it is a complete lattice with to the product order on the Euclidean space. With infinite A_1, a technical difficulty is that some of the actions in the support of σ_1^θ can be suboptimal. Therefore, the notion of monotonicity in Definition 1.2 does not apply.

For every $a_1 \in A_1$ and $\alpha_1 \in \Delta(A_1)$, let $\Pr(\alpha_1 \succ a_1)$ be the probability that the realization of α_1 is strictly higher than a_1. To accommodate the above-mentioned technical difficulty, we introduce the following weaker version of monotonicity:

Definition A.3. *σ_1 is an almost surely monotone strategy if for every $\theta \succ \theta'$ and $a_1 \in supp(\sigma_1^\theta)$, we have $\Pr(\sigma_1^{\theta'} \succ a_1) = 0$. An equilibrium (σ_1, σ_2) is almost surely monotone if σ_1 is almost* *surely monotone.*

We establish the following result, which generalizes Theorem 1.1.

Theorem A.1. *If $|A_2| = 2$ and the sender's payoff is monotone-supermodular, then every Nash equilibrium is almost surely monotone.*

PROOF. The proof of Theorem 1.1 implies the following lemma.

Lemma A.5. *Given the receiver's strategy σ_2, for every $\theta \succ \theta'$ and $a_1 \succ a_1'$, if a_1' is a best response to σ_2 for type θ, then a_1 is not a best response to σ_2 for type θ'.*

For every $x \in \mathbb{R}^n$ and $r > 0$, let $B(x, r)$ be the open ball around x with radius r. For every $\theta \succ \theta'$ and $a_1 \in \text{supp}(\sigma_1^\theta)$, we have $\sigma_1^\theta(B(a_1, r)) > 0$ for every $r > 0$. That is to say, there exists $a_1' \in B(a_1, r)$ such that a_1' is optimal for type θ. Let a_1^r be the smallest element that is above every element in $B(a_1, r)$. Lemma A.5 implies that $\Pr(\sigma_1^{\theta'} \succ a_1^r) = 0$ for every $r > 0$. For every strictly positive decreasing sequence $\{r_i\}_{i=1}^\infty$ with $\lim_{i\to\infty} r_i = 0$, we have:

$$\lim_{i\to\infty} \{a_1' | a_1' \succ a_1^{r_i}\} = \{a_1' | a_1' \succ a_1\} \text{ and } \{a_1' | a_1' \succ a_1^{r_i}\} \supset \{a_1' | a_1' \succ a_1^{r_j}\} \text{ for every } i > j.$$

The monotone convergence theorem implies that:

$$\Pr(\sigma_1^{\theta'} \succ a_1) = \Pr(\sigma_1^{\theta'} \succ \lim_{i\to\infty} a_1^{r_i}) = \lim_{i\to\infty} \Pr(\sigma_1^{\theta'} \succ a_1^{r_i}) = 0.$$

The statement of the theorem then immediately follows. $\square$

The corresponding generalizations of Theorems 1.2 and 1.3 are similar and, therefore, omitted.

B Appendix: Chapter 2

B.1 Proof of Lemma 2.1

We first show that $g_{\mathbf{a}}(0) > 0$ $\forall \mathbf{a} \in \mathcal{A}$, and hence $p^*_{\mathbf{a}}$ is well-defined. If $\varepsilon_{\mathbf{a}}$ is degenerate, this follows directly from (A2.1.3). Now suppose that $\varepsilon_{\mathbf{a}}$ is non-degenerate. By (A2.1.5), $\Gamma_{\mathbf{a}}$ then has a density $\gamma_{\mathbf{a}}$ which is log-concave on its support. By definition, it follows that $supp(\gamma_{\mathbf{a}})$ must be a convex set, i.e., an interval on $\mathbb{R}$. It then follows from (A2.1.4) that $0 \in supp(\gamma_{\mathbf{a}})$, for if $0 \notin supp(\gamma_{\mathbf{a}})$ then $supp(\gamma_{\mathbf{a}})$ must reside entirely either in $(-\infty, 0)$ or in $(0, \infty)$, contradicting the symmetry of $\Gamma_{\mathbf{a}}$ at zero. Symmetry of $\Gamma_{\mathbf{a}}$ further assures that $supp(\gamma_{\mathbf{a}})$ is an interval symmetric around zero. By (A2.1.3), g_0 is continuous and strictly positive at the point $x = 0$. Hence, there must exist $\delta > 0$ such that $g_0(x) > 0$ $\forall x \in [-\delta, \delta]$. In particular, we can choose this $\delta > 0$ so small to assure that $[-\delta, \delta] \subset supp(\gamma_{\mathbf{a}})$. Accordingly, we have

$$g_{\mathbf{a}}(0) = \int_{-\infty}^{+\infty} g_0(-\varepsilon)\gamma_a(\varepsilon)d\varepsilon \geq \int_{-\delta}^{\delta} g_0(-\varepsilon)\gamma_a(\varepsilon)d\varepsilon > 0.$$

Now consider the profit maximization problems of the firms. As G_0 is non-degenerate, choosing $p_i = 0$ would never be optimal for any firm. For every $p_2 > 0$, let

$$\underline{p}(p_2) \equiv \sup\left\{p_1 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 1\right\}, \text{ and } \bar{p}(p_2) \equiv \inf\left\{p_1 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 0\right\}.$$

Similarly, for every $p_1 > 0$, define

$$\underline{p}(p_1) \equiv \sup\left\{p_2 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 0\right\}, \text{ and } \bar{p}(p_2) \equiv \inf\left\{p_2 \in \mathbb{R} \,|\, G_{\mathbf{a}}(p_2 - p_1) = 1\right\}.$$

Taking firm j's price p_j as given, it is clear that, from firm i's perspective, any $p_i < \underline{p}(p_j)$ is strictly dominated by $\underline{p}(p_j)$, and any $p_i > \bar{p}(p_j)$ is strictly dominated by $\bar{p}(p_j)$. Hence, when solving the firms' optimization problems, we can assume without loss of generality that, by taking $p_j > 0$ as given, each firm i will only choose a price p_i from the interval $\left[\max\{\underline{p}(p_j), 0\}, \bar{p}(p_j)\right]$. This leads to the following first-order conditions of the firms:

$$\frac{\partial \Pi_1(p_1, p_2)}{\partial p_1} = G_{\mathbf{a}}(p_2 - p_1) - p_1 g_{\mathbf{a}}(p_2 - p_1) = 0,$$
$$\frac{\partial \Pi_2(p_1, p_2)}{\partial p_2} = 1 - G_{\mathbf{a}}(p_2 - p_1) - p_2 g_{\mathbf{a}}(p_2 - p_1) = 0.$$

By (A2.1.1) and (A2.1.4), it is easy to verify that $G_{\mathbf{a}}$ is symmetric for all $\mathbf{a} \in \mathcal{A}$. Hence, by plugging $p_1 = p_2$ into the first-order conditions, we obtain the unique candidate for a symmetric equilibrium in the pricing subgame:

$$p_1 = p_2 = p_a^* \equiv \frac{G_{\mathbf{a}}(0)}{g_{\mathbf{a}}(0)} = \frac{1}{2g_a(0)},$$

where the last equality follows again the symmetry of $G_{\mathbf{a}}$.

Finally, we argue that the log-concavity assumptions (A2.1.2) and (A2.1.5) guarantee that the price profile $(p_1, p_2) = (p_a^*, p_a^*)$ indeed constitutes an equilibrium. To see this, first note that $\forall \mathbf{a} \in \mathcal{A}$, the distribution function $G_{\mathbf{a}}$ is log-concave on $supp(g_{\mathbf{a}})$. This is trivial if $G_{\mathbf{a}} = G_0$, i.e., if $\varepsilon_{\mathbf{a}}$ is degenerate at zero. If $\varepsilon_{\mathbf{a}}$ is non-degenerate, the claim holds since both G_0 and $\gamma_{\mathbf{a}}$ are log-concave ((A2.1.2) and (A2.1.5)), and the convolution of log-concave functions is also log-concave.[1] Since the function $f(p) = p$ is strictly log-concave on $[0, +\infty)$, the profit function $\Pi_i(p_i, p_j)$ is strictly log-concave (and hence strictly quasi-concave) in p_i on $\left[\max\{\underline{p}(p_j), 0\}, \bar{p}(p_j)\right]$ for all $p_j > 0$. Since $\Pi_i(\underline{p}(p_j), p_j) = \Pi_i(\bar{p}(p_j), p_j) = 0$, strict quasi-concavity implies that $p_1 = p_a^*$ must be a global maximum of the function $\Pi_1(p_1, p_{\mathbf{a}}^*)$. Hence, when firm j plays $p_j = p_{\mathbf{a}}^*$, it is a best response for firm i to choose the same price $p_i = p_{\mathbf{a}}^*$. Thus $(p_{\mathbf{a}}^*, p_{\mathbf{a}}^*)$ constitutes a unique symmetry equilibrium for the pricing subgame where firms choose $\mathbf{a} \in \mathcal{A}$ in the first-stage. □

B.2 Proof of Theorem 2.1

Part (i): The claim is trivial if $\varepsilon_{\mathbf{a}}$ is degenerate for all $\mathbf{a} \in \mathcal{A}$, so suppose otherwise. Then, for any $\mathbf{a} \in \mathcal{A}$ such that $\varepsilon_{\mathbf{a}}$ is non-degenerate, we have

$$g_{\mathbf{a}}(0) = \int_{supp(\gamma_{\mathbf{a}})} g_0(-\varepsilon)\gamma_{\mathbf{a}}(\varepsilon)d\varepsilon > \int_{supp(\gamma_{\mathbf{a}})} g_0(0)\gamma_{\mathbf{a}}(\varepsilon)d\varepsilon = g_0(0),$$

where the inequality follows from $supp(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$ and that the true preferences are weakly polarized. Hence, by Lemma 2.1 we can conclude that $p_{\mathbf{a}}^* < p_{\mathbf{0}}^* = \frac{1}{2g_0(0)}$ for all such $\mathbf{a} \in \mathcal{A}$. It immediately follows that any choice of $\mathbf{a} \in \mathcal{A}$ which leaves $\varepsilon_{\mathbf{a}}$ degenerate must be an equilibrium of the marketing stage, followed by $p_1^* = p_2^* = \frac{1}{2g_0(0)}$ in the pricing stage. In this SPE, marketing strategies are thus chosen such that no consumer confusion results. Part (ii) of the theorem can be proven analogously. □

B.3 Proof of Theorem 2.2

Part (i): By Theorem 2.1, for this part of the proof it is without loss to assume that $\mathbf{0} \notin \mathcal{A}$, since even if it is available the marketing profile $\mathbf{a} = \mathbf{0}$ will not be chosen in any SPE anyway. Consider first the case where (a) holds. Take any $\mathbf{a}, \mathbf{a}' \in \mathcal{A}$ such that $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$. By the

[1] For an overview of the properties of log-concave distributions, see Bagnoli and Bergstrom (2005).

MPS assumption, $\varepsilon_{\mathbf{a}'}$ has the same distribution as $\varepsilon_{\mathbf{a}} + \eta$, where $\eta \neq O$. Thus

$$\begin{aligned} g_{\mathbf{a}'}(0) = \textstyle\int_{-\delta}^{\delta} g_0(-e)d\Gamma_{\mathbf{a}'}(e) &= E\left[g_0(\varepsilon_{\mathbf{a}'})\right] \\ &= E\left[E\left[g_0\left(\varepsilon_{\mathbf{a}'}\right)|\varepsilon_{\mathbf{a}}\right]\right] \\ &= E\left[E\left[g_0\left(\varepsilon_{\mathbf{a}}+\eta\right)|\varepsilon_{\mathbf{a}}\right]\right] \\ &< E\left[g_0\left(E[\varepsilon_{\mathbf{a}}+\eta|\varepsilon_{\mathbf{a}}]\right)\right] \\ &= E\left[g_0\left(\varepsilon_{\mathbf{a}}\right)\right] = \textstyle\int_{-\delta}^{\delta} g_0(-e)d\Gamma_{\mathbf{a}}(e) = g_{\mathbf{a}}(0). \end{aligned} \tag{B.1}$$

The second line follows from the law of iterated expectations, the third because $\varepsilon_{\mathbf{a}'}$ and $\varepsilon_{\mathbf{a}} + \eta$ are equal in distribution, and the forth line follows from Jensen's inequality because g_0 is strictly concave on $[-\delta, \delta] \supset supp(\gamma_{\mathbf{a}})$. Hence, $g_{\mathbf{a}}(0)$ achieves its minimum on $\mathcal{A}$ if and only if $\mathbf{a} = (\bar{a}, \bar{a})$. By Lemma 2.1, this also maximizes the equilibrium price and the expected profits of the firms. For this reason, the marketing profile $(\bar{a}, \bar{a})$ is part of an SPE. Moreover, $(\bar{a}, \bar{a})$ is the only possible equilibrium outcome, because for any alternative $(a_1, a_2) \in \mathcal{A}$, the firm with $a_i < \bar{a}$ would always deviate to $\bar{a}$.

Now suppose that (b) holds. Take any $\mathbf{a}, \mathbf{a}'$ such that $\mathbf{a} \neq \mathbf{a}'$ and $\mathbf{a} \leq \mathbf{a}'$. Let $supp(\gamma_{\mathbf{a}}) = [-\omega_{\mathbf{a}}, \omega_{\mathbf{a}}]$ and $supp(\gamma_{\mathbf{a}'}) = [-\omega_{\mathbf{a}'}, \omega_{\mathbf{a}'}]$. Condition (A2.3.2) implies that there exists a unique $\hat{e} \in (0, \omega_{\mathbf{a}'})$, such that $\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) < 0$ $\forall e \in [0, \hat{e})$, and $\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e) > 0$ $\forall e \in (\hat{e}, \omega_{\mathbf{a}'}]$. Since g_0 is strictly decreasing on $[0, \omega_{\mathbf{a}'}] \subset [0, \delta]$, we further have

$$\begin{aligned} \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(e)\left(\gamma_{a'}(e) - \gamma_a(e)\right) de &< \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(\hat{e})\left(\gamma_{a'}(e) - \gamma_a(e)\right) de \\ &= \int_0^{\hat{e}} g_0(\hat{e})(\gamma_a(e) - \gamma_{a'}(e))de \\ &< \int_0^{\hat{e}} g_0(e)(\gamma_a(e) - \gamma_{a'}(e))de, \end{aligned} \tag{B.2}$$

where the equality makes use of the fact that, by symmetry and $\omega_{\mathbf{a}} \leq \omega_{\mathbf{a}'}$, we have

$$\frac{1}{2} = \int_0^{\omega_{\mathbf{a}'}} \gamma_{\mathbf{a}'}(e)de = \int_0^{\omega_{\mathbf{a}}} \gamma_{\mathbf{a}}(e)de = \int_0^{\omega_{\mathbf{a}'}} \gamma_{\mathbf{a}}(e)de.$$

Exploiting again the symmetry of g_0, $\gamma_{\mathbf{a}}$ and $\gamma_{\mathbf{a}'}$, we further have

$$\begin{aligned} &\int_{-\omega_{\mathbf{a}'}}^{\omega_{\mathbf{a}'}} g_0(-e)\gamma_{\mathbf{a}'}(e)de - \int_{-\omega_{\mathbf{a}}}^{\omega_{\mathbf{a}}} g_0(-e)\gamma_{\mathbf{a}}(e)de \\ &= \int_{-\omega_{\mathbf{a}'}}^{\omega_{\mathbf{a}'}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de \\ &= 2\int_0^{\omega_{\mathbf{a}'}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de \\ &= 2\left[\int_0^{\hat{e}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de + \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(-e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de\right] \\ &= 2\left[\int_0^{\hat{e}} g_0(e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de + \int_{\hat{e}}^{\omega_{\mathbf{a}'}} g_0(e)\left(\gamma_{\mathbf{a}'}(e) - \gamma_{\mathbf{a}}(e)\right) de\right] \\ &< 0, \end{aligned}$$

where the last inequality follows from (B.2). We have thus shown that $g_{\mathbf{a}'}(0) < g_{\mathbf{a}}(0)$ for any feasible $\mathbf{a} \neq \mathbf{a}'$ with $\mathbf{a} \leq \mathbf{a}'$. Hence, if preferences are δ-indecisive, $g_{\mathbf{a}}(0)$ must be uniquely minimized at $\mathbf{a}^* = (\bar{a}, \bar{a})$. By arguments analogous to the case with MPS ordering, we can conclude that there exists a unique SPE, and $\mathbf{a}^* = (\bar{a}, \bar{a})$ is the unique equilibrium outcome in the first stage.

Part (ii): As the proof with (b) is similar, we only provide arguments for the case that (a) holds. Since now g_0 is strictly convex, the inequality in (B.1) is reversed. By Lemma 2.1, any $\varepsilon_{\mathbf{a}'}$ which is MPS of $\varepsilon_{\mathbf{a}}$ therefore is payoff dominated by $\varepsilon_{\mathbf{a}}$. Further, any $\varepsilon_{\mathbf{a}} \neq O$ trivially is an MPS of O. Thus, regardless of whether $\mathbf{0} \in \mathcal{A}$ or not, $\mathbf{a}^* = (\underline{a}, \underline{a})$ is the only possible equilibrium outcome. Indeed, setting $a_i = \underline{a}$ is a dominant action for each firm i, because for any given $a_j \in A$, with any alternative $a_i > \underline{a}$ the resulting $\varepsilon_{(a_i, a_j)}$ is a MPS of $\varepsilon_{(\underline{a}, a_j)}$, which can only lead to a lower equilibrium price in the second stage. □

B.4 Proof of Theorem 2.3

For every $\omega \geq 0$, let

$$g_\omega(0) \equiv \int_{-\omega}^{\omega} g_0(\varepsilon) d\Gamma_\omega,$$

where Γ_ω is the uniform distribution on $[-\omega, \omega]$:

$$\Gamma_\omega(x) = \begin{cases} 1 & \text{if } x > \omega, \\ \frac{x+\omega}{2\omega} & \text{if } x \in [-\omega, \omega], \\ 0 & \text{otherwise.} \end{cases}$$

Note that under the assumption of the theorem, we have $\Gamma_{\mathbf{a}}(x) = \Gamma_{\omega_{\mathbf{a}}}(x)\ \forall x \in \mathbb{R}, \mathbf{a} \in \mathcal{A}$.

Consider first case (i). We start by showing that $g_\omega(0)$ is strictly decreasing in ω on $[0, +\infty)$. Since preferences are indecisive on $supp(g_0)$ and $g_0(x) = 0\ \forall x \notin supp(g_0)$, we have $g_0(\varepsilon) < g_0(0)$ $\forall \varepsilon \neq 0$. It then follows that, $\forall \omega > 0$,

$$g_\omega(0) = \int_{-\omega}^{\omega} g_0(\varepsilon)\Gamma_\omega < \int_{-\omega}^{\omega} g_0(0) d\Gamma_\omega = g_0(0).$$

Furthermore, since the partial derivative of $g_\omega(0)$ with respect to ω exists for all $\omega > 0$, we have

$$\begin{aligned} \frac{\partial g_\omega(0)}{\partial \omega} &= \frac{g_0(-\omega)}{2\omega} + \frac{g_0(\omega)}{2\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2}\, g_0(-\varepsilon) d\varepsilon \\ &= \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2}\, g_0(\varepsilon) d\varepsilon \\ &< \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{1}{2\omega^2}\, g_0(\omega) d\varepsilon \\ &= \frac{g_0(\omega)}{\omega} - \frac{g_0(\omega)}{\omega} \\ &= 0, \end{aligned}$$

where the inequality follows that preferences are indecisive on $supp(g_0)$, g_0 is zero-symmetry, and $g_0(x) = 0 \ \forall x \notin supp(g_0)$.

By Lemma 2.1, in every pricing subgame there exists a unique symmetric equilibrium in which both firms choose the price $p^* = \frac{1}{2g_{\omega_{\mathbf{a}}}(0)}$. Since $g_\omega(0)$ is strictly decreasing in ω, $g_{\omega_{\mathbf{a}}}$ is minimized at $\omega_{\mathbf{a}} = \bar{\omega}$. Therefore, the subgame equilibrium price is maximized at $\omega_{\mathbf{a}} = \bar{\omega}$, which implies that there must exist an SPE with maximal obfuscation.[2]

Next, consider case (ii). We first prove the following two lemmas.

Lemma B.1. *If $supp(g_0)$ is bounded, then* $\lim_{\omega\to+\infty} \ g_\omega(0) = 0$.

PROOF. Since $supp(g_0)$ is bounded, we must have $supp(g_0) \subset [-\omega, \omega]$ for sufficiently large ω. As a result,

$$\lim_{\omega\to+\infty} \ g_\omega(0) = \lim_{\omega\to+\infty} \int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega} d\varepsilon = \lim_{\omega\to+\infty} \int_{supp(g_0)} \frac{g_0(\varepsilon)}{2\omega} d\varepsilon = 0. \qquad \blacksquare$$

Lemma B.2. *If $supp(g_0)$ is bounded, then $g_\omega(0)$ is strictly decreasing in ω for all $\omega > \sup supp(g_0)$.*

PROOF. Since $supp(g_0)$ is bounded, we must have $\sup supp(g_0) < +\infty$, and $g_0(\omega) = 0$ for all $\omega > \sup supp(g_0)$. It then follows that, for every $\omega > \sup supp(g_0)$,

$$\frac{\partial g_\omega(0)}{\partial \omega} = \frac{g_0(\omega)}{\omega} - \int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega^2} d\varepsilon = -\int_{-\omega}^{\omega} \frac{g_0(\varepsilon)}{2\omega^2} d\varepsilon < 0. \qquad \blacksquare$$

Now consider any preference distribution that is polarized on $supp(g_0)$. By definition, g_0 is strictly decreasing on $(\inf supp(g_0), 0]$ and is strictly increasing on $[0, \sup supp(g_0))$. This implies that the support of g_0 must be bounded, as otherwise we would have

$$\int_{supp(g_0)} g_0(x)dx = \int_{-\infty}^{+\infty} g_0(x)dx \geq \int_{-\infty}^{+\infty} g_0(0)dx = +\infty,$$

contradicting the definition of g_0 as a density function. Applying Lemmas B.1 and B.2, we can conclude that $\lim_{\omega\to+\infty} g_\omega(0) = 0$ and that $g_\omega(0)$ is strictly decreasing in ω on $(\sup supp(g_0), +\infty)$. Hence, using $g_0(0) > 0$ there must exist $\hat{\omega} > 0$, such that if $\omega \geq \hat{\omega}$, then $g_\omega(0) \leq g_0(0)$. Moreover, $g_\omega(0)$ can be arbitrarily small for sufficiently large ω. Therefore, if $\bar{\omega}$ is sufficiently large, the subgame equilibrium price is maximized at $\omega_{\mathbf{a}} = \bar{\omega}$, which implies that there must exist an SPE with maximal obfuscation. $\square$

B.5 Proof of Proposition 2.1

Let $\eta \equiv \sup supp\,(g_0)$. We can write the expected welfare loss from mismatch as a function of the degree of confusion:

$$L(\omega) = 2\int_0^{\min\{\omega,\eta\}} \left[x \cdot \frac{-x+\omega}{2\omega} \cdot g_0(x)\right] dx = \int_0^{\min\{\omega,\eta\}} \left[x\left(1 - \frac{x}{\omega}\right) g_0(x)\right] dx.$$

[2] By Theorem 2.2, this SPE will be unique if we further assume that $\omega_{\mathbf{a}} = \varphi(a_1, a_2) \ \forall \mathbf{a} \in \mathcal{A}$, where $\varphi : \mathcal{A} \to \mathbb{R}_+$ is strictly increasing in both a_1 and a_2.

Taking the first derivative, we obtain

$$L'(\omega) = \int_0^{\eta} \left[\frac{x^2}{\omega^2}\right] dG_0(x)$$

if $\omega \geq \eta$, and

$$L'(\omega) = \int_0^{\omega} \left[\frac{x^2}{\omega^2}\right] dG_0(x) + \omega \left(1 - \frac{\omega}{\omega}\right) g_0(\omega) = \int_0^{\omega} \left[\frac{x^2}{\omega^2}\right] dG_0(x)$$

if $\omega < \eta$. Since by assumption G_0 is a non-degenerate distribution, $L'(\omega) > 0\ \forall \omega > 0$. Hence, the expected welfare loss is strictly increasing in ω. □

B.6 Proof of Proposition 2.2

We start by arguing that if $\alpha \leq \hat{\alpha}$, then (A2.1.2) holds, that is, G is log-concave on its support. To show this, we will make use of the Lemma B.3 below, which states that H is log-concave on its support if $\alpha \leq \hat{\alpha}$.

Lemma B.3. *If $\alpha \leq \hat{\alpha}$, then H is log-concave on $[-\lambda, \lambda]$.*

Proof. If $\alpha \leq 0$, the statement of the lemma immediately follows because in these cases the density function h is log-concave, which is sufficient (but not necessary) for the distribution function H to be log-concave on $[-\lambda, \lambda]$.

Suppose now that $\alpha \in (0, \hat{\alpha}]$. We will show that H remains to be log-concave despite that the density function h is actually log-convex. By continuity, it suffices to show that H is log-concave on the open interval $(-\lambda, \lambda)$. Since h is differentiable on $(-\lambda, \lambda)$, H is log-concave on this interval if and only if for all $\theta \in (-\lambda, \lambda)$,

$$\begin{aligned}
& h'(\theta)H(\theta) - (h(\theta))^2 \leq 0 \\
\iff & 2\alpha\theta \cdot \left(\frac{1}{3}\alpha\theta^3 + \beta\theta + \frac{1}{2}\right) \leq \left(\alpha\theta^2 + \beta\right)^2 \\
\iff & \frac{2}{3}\alpha^2\theta^4 + 2\alpha\beta\theta^2 + \alpha\theta \leq \alpha^2\theta^4 + \beta^2 + 2\alpha\beta\theta^2 \\
\iff & -\frac{1}{3}\alpha^2\theta^4 + \alpha\theta \leq \left(\frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right)^2.
\end{aligned} \tag{B.3}$$

Given that $\alpha > 0$, the inequality obviously holds when $\theta \leq 0$. But given that $\theta > 0$, the LHS of (B.3) is increasing in θ on $[0, \lambda]$, since

$$\left(\frac{1}{3}\alpha^2\theta^4 + \alpha\theta\right)' = -\frac{4}{3}\alpha^2\theta^3 + \alpha \geq -\frac{4}{3}\alpha^2\lambda^3 + \alpha > 0,$$

where the last inequality holds as $\hat{\alpha} < 3/(4\lambda^3)$. Hence, inequality (2.3) holds for all $\theta \in (-\lambda, \lambda)$ if and only if

$$-\frac{1}{3}\alpha^2\lambda^4 + \alpha\lambda \leq \frac{1}{4\lambda^2} + \frac{\alpha^2\lambda^4}{9} - \frac{\alpha\lambda}{3}$$

$$\begin{aligned}
&\iff -\frac{4}{9}\alpha^2\lambda^4 + \frac{4}{3}\alpha\lambda \le \frac{1}{4\lambda^2} \\
&\iff -\alpha^2\lambda^4 + 3\alpha\lambda \le \frac{9}{16\lambda^2} \\
&\iff \left(\alpha\lambda^2 - \frac{3}{2\lambda}\right)^2 \ge \frac{27}{16\lambda^2}.
\end{aligned} \tag{B.4}$$

Since $\lambda > 0$ and $\hat{\alpha} \le 3/(2\lambda^3)$, (B.4) is further equivalent to $\alpha \le \frac{6-3\sqrt{3}}{4\lambda^3} = \hat{\alpha}$. ■

Since $G(x) = \Pr(4\lambda\theta \le x) = \Pr\left(\theta \le \frac{x}{4\lambda}\right) = H\left(\frac{x}{4\lambda}\right)\ \forall x \in \mathbb{R}$, and the function $t(x) = x/(4\lambda)$ is increasing and concave in x, G is log-concave on $[4\lambda a, 4\lambda b]$ if H is log-concave on $[a, b] \subset \mathbb{R}$. Hence, by Lemma B.2, G is log-concave on $[-4\lambda^2, 4\lambda^2]$ provided that $\alpha \le \hat{\alpha}$.

It is straightforward to check that all conditions in Assumption 2.1 are satisfied. Hence, by Lemma 2.1, we can conclude that there exists a unique pure-strategy equilibrium in every pricing subgame, where each firm chooses the same price $p^*_{\mathbf{a}} = \frac{1}{2g_{\mathbf{a}}(0)}\ \forall \mathbf{a} \in \mathcal{A}$. The statements of the proposition then immediately follow from Theorems 2.1 and 2.2. □

B.7 Proof of Proposition 2.3

Since $G(x) = H\left(\frac{x}{4\lambda}\right)\ \forall x \in \mathbb{R}$, the density function of G, which we denote as g, is given by

$$g(x) = \frac{1}{4\lambda} h\left(\frac{x}{4\lambda}\right) = \begin{cases} \frac{1}{4\lambda}\cdot\left(\alpha\left(\frac{x}{4\lambda}\right)^2 + \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right) & \text{if } x \in [-4\lambda^2, 4\lambda^2], \\ 0 & \text{otherwise.} \end{cases}$$

The welfare loss can now be written as a function of α:

$$\begin{aligned}
L(\alpha) &= \int_0^{\min\{\omega, 4\lambda^2\}} \left[x\left(1 - \frac{x}{\omega}\right)\cdot\frac{1}{4\lambda}\cdot h_0\left(\frac{x}{4\lambda}\right)\right] dx \\
&= \frac{1}{4\lambda}\int_0^{\min\{\omega, 4\lambda^2\}} \left[x\left(1 - \frac{x}{\omega}\right)\left(\alpha\left(\frac{x}{4\lambda}\right)^2 + \frac{1}{2\lambda} - \frac{\alpha\lambda^2}{3}\right)\right] dx.
\end{aligned}$$

Taking derivative with respect to α, we have

$$L'(\alpha) = \frac{1}{4\lambda}\int_0^{\min\{\omega, 4\lambda^2\}} \left[x\left(1 - \frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2 - \frac{\lambda^2}{3}\right)\right] dx.$$

First, suppose that $\omega \le 4\lambda^2$. In this case, we obtain

$$\begin{aligned}
L'(\alpha) &= \frac{1}{4\lambda}\int_0^{\omega} \left[x\left(1 - \frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2 - \frac{\lambda^2}{3}\right)\right] dx \\
&= \frac{1}{4\lambda}\left[\int_0^{\omega}\left(\frac{x^3}{16\lambda^2} - \frac{\lambda^2 x}{3}\right) dx - \int_0^{\omega}\left(\frac{x^4}{16\lambda^2\omega} - \frac{\lambda^2 x^2}{3\omega}\right) dx\right] \\
&= \frac{1}{4\lambda}\left[\left(\frac{x^4}{64\lambda^2} - \frac{\lambda^2 x^2}{6}\right)\Bigg|_0^{\omega} - \left(\frac{x^5}{80\lambda^2\omega} - \frac{\lambda^2 x^3}{9\omega}\right)\Bigg|_0^{\omega}\right] \\
&= \frac{1}{4\lambda}\left[\frac{\omega^4}{64\lambda^2} - \frac{\omega^4}{80\lambda^2} - \frac{\lambda^2\omega^2}{6} + \frac{\lambda^2\omega^2}{9}\right].
\end{aligned}$$

Hence, provided that $\omega \in (0, 4\lambda^2]$, we further have

$$L'(\alpha) < 0 \iff \left(\frac{1}{64\lambda^2} - \frac{1}{80\lambda^2}\right)\omega^2 < \frac{\lambda^2}{6} - \frac{\lambda^2}{9} \iff \omega < \frac{4\sqrt{10}}{3}\lambda^2.$$

Since $(4\sqrt{10}/3) \approx 4.22 > 4$, it follows that $L'(\alpha) < 0$ whenever $\omega \leq 4\lambda^2$.

Next, consider the case where $\omega > 4\lambda^2$. Expanding the equation $L'(\alpha)$ again, we have

$$\begin{aligned}
L'(\alpha) &= \frac{1}{4\lambda}\int_0^{4\lambda^2}\left[x\left(1-\frac{x}{\omega}\right)\left(\left(\frac{x}{4\lambda}\right)^2 - \frac{\lambda^2}{3}\right)\right]dx \\
&= \frac{1}{4\lambda}\left[\left(\frac{x^4}{64\lambda^2} - \frac{\lambda^2x^2}{6}\right)\Bigg|_0^{4\lambda^2} - \left(\frac{x^5}{80\lambda^2\omega} - \frac{\lambda^2x^3}{9\omega}\right)\Bigg|_0^{4\lambda^2}\right] \\
&= \frac{1}{4\lambda}\left[\left(\frac{4^4\lambda^8}{64\lambda^2} - \frac{4^2\lambda^6}{6}\right) - \frac{1}{\omega}\left(\frac{4^5\lambda^{10}}{80\lambda^2} - \frac{4^3\lambda^8}{9}\right)\right] \\
&= 4\lambda^5\left[\left(\frac{16}{64} - \frac{1}{6}\right) - \frac{\lambda^2}{\omega}\left(\frac{64}{80} - \frac{4}{9}\right)\right].
\end{aligned}$$

Hence, provided that $\omega > 4\lambda^2$, we further have

$$L'(\alpha) > 0 \iff \frac{\lambda^2}{\omega}\left(\frac{4}{5} - \frac{4}{9}\right) < \frac{1}{4} - \frac{1}{6} \iff \omega > \frac{64}{15}\lambda^2.$$

Note that $64/15 \approx 4.27 > 4$. We can now conclude that $L'(\alpha) < 0$ whenever $\omega < \hat{\omega} \equiv 64\lambda^2/15$, and $L'(\alpha) > 0$ whenever $\omega > \hat{\omega}$. □

B.8 Proof of Proposition 2.4

The demand function of firm 1 is given by

$$\begin{aligned}
D(p_1, p_2) &= \int \Pr\left(\tilde{v}_1^k - p_1 \geq \max\{\tilde{v}_2^k - p_2, 0\}\right) d\Gamma_{\mathbf{a}} \\
&= \int \Pr\left(v \geq p_1 - p_2 - \varepsilon,\ v \geq 2p_1 - m - \varepsilon\right) d\Gamma_{\mathbf{a}} \\
&= \int \min\left\{\Pr\left(v \geq p_1 - p_2 - \varepsilon\right), \Pr\left(v \geq 2p_1 - m - \varepsilon\right)\right\} d\Gamma_{\mathbf{a}} \\
&= 1 - \int \max\left\{G_0\left(p_1 - p_2 - \varepsilon\right), G_0\left(2p_1 - m - \varepsilon\right)\right\} d\Gamma_{\mathbf{a}}.
\end{aligned}$$

Recall that, for all $x \in \mathbb{R}$, we write

$$G_{\mathbf{a}}(x) = \int G_0(x-\varepsilon)d\Gamma_{\mathbf{a}}, \text{ and } g_{\mathbf{a}}(x) = \int g_0(x-\varepsilon)d\Gamma_{\mathbf{a}}$$

Note that

$$D(p,p) = \begin{cases} \frac{1}{2} & \text{if } p \leq \frac{m}{2}, \\ 1 - G_{\mathbf{a}}(2p-1) & \text{if } p > \frac{m}{2}. \end{cases}$$

Let $\Pi_1(p_1, p_2) = p_1 D(p_1, p_2)$. For every $p_2 > 0$, function Π_1 is differentiable in p_1 almost everywhere. In particular, if $p_1 < m - p_2$, we have

$$\frac{\partial \Pi_1(p_1, p_2)}{\partial p_1} = 1 - G_{\mathbf{a}}(p_1 - p_2) - p_1 g_{\mathbf{a}}(p_1 - p_2),$$

which is also the left derivative of $\Pi_1(p_1, p_2)$ at $p_1 = m - p_2$. Similarly, if $p_1 > m - p_2$, we have

$$\frac{\partial \Pi_1(p_1, p_2)}{\partial p_1} = 1 - G_{\mathbf{a}}(2p_1 - m) - 2p_1 g_{\mathbf{a}}(2p_1 - m),$$

which is also the right derivative of $\Pi_1(p_1, p_2)$ at $p_1 = m - p_2$.

Finally, let $p_{\mathbf{a}}^M$ be the solution to the monopoly problem

$$\max_{p \geq 0} \ \Pi_{\mathbf{a}}^M(p) \equiv p\left(1 - G_{\mathbf{a}}(2p - m)\right).$$

Note that the log-concavity of G_0 and $\Gamma_{\mathbf{a}}$ implies that the above objective function is strictly quasi-concave. Therefore, a unique $p_{\mathbf{a}}^M$ exists for every $\mathbf{a} \in \mathcal{A}$.

Since log-concavity is preserved under convolution, the function $G_{\mathbf{a}}$ is log-concave on its support *supp*$(g_{\mathbf{a}})$. In addition, since $G_{\mathbf{a}}$ is a distribution function, its log-concavity also holds on $[0, +\infty)$. Hence, for all $p_2 > 0$ and $\mathbf{a} \in \mathcal{A}$, the demand function D must be log-concave in p_1 on both $[0, m - p_2]$ and $[m - p_2, +\infty)$. Note that we are not claiming that D is log-concave in p_1 on the entire interval $[0, +\infty)$. In what follows, we will show that although the global log-concavity of the demand function is not guaranteed, Assumption 2.1 is still sufficient to guarantee the existence of a unique symmetric equilibrium in every pricing subgame.

First, suppose that $g_{\mathbf{a}}(0) > \frac{1}{m}$. Suppose also that firm 2 is choosing $p_2 = \frac{1}{2g_{\mathbf{a}}(0)} < \frac{m}{2}$. In this case, the whole market is guaranteed to be covered (i.e., every consumer will buy from one of the firms) if $p_1 \leq m/2$. In addition, since

$$\left.\frac{\partial \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1 = p_2 = \frac{1}{2g_{\mathbf{a}}(0)} < \frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - \frac{1}{2g_{\mathbf{a}}(0)} \cdot g_{\mathbf{a}}(0) = 0,$$

and the function $\Pi_1\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ is strictly quasi-concave in p_1 on $\left[0, m - \frac{1}{2g_{\mathbf{a}}(0)}\right]$, and $g_{\mathbf{a}}(0) > \frac{1}{m}$ implies that $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ is a maximum of the function $\Pi_1\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ over the range $\left[0, m - \frac{1}{2g_{\mathbf{a}}(0)}\right]$. We now argue that, in addition,

$$\Pi_1\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right) \leq \Pi_1\left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right) \ \forall p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}.$$

To see this, note that $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ implies that the market will no longer be fully covered and, in particular, there will be consumers who choose to stick to their outside options even though they prefer firm 1 over firm 2. Therefore, a deviation to $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ must be less profitable than it would have been in the case without outside option. But then, as we have shown in Theorem 2.1, in the absence of the outside option, choosing $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ actually maximizes firm 1's expected profits over $[0, +\infty)$ given that its competitor plays $p_2 = \frac{1}{2g_{\mathbf{a}}(0)}$. This implies that deviating to $p_1 > m - \frac{1}{2g_{\mathbf{a}}(0)}$ cannot be profitable either in the presence of the outside option. Therefore, $p_1 = \frac{1}{2g_{\mathbf{a}}(0)}$ must be a global maximum of the function $\Pi_1\left(p_1, \frac{1}{2g_{\mathbf{a}}(0)}\right)$, and

$(p_1, p_2) = \left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ indeed constitutes an equilibrium in the pricing subagme. It is easy to see that this is the only symmetric equilibrium with a price strictly less than $\frac{m}{2}$. In addition, since

$$\left.\frac{\partial \Pi^M(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) < \frac{1}{2} - 1 < 0$$

and $\Pi^M(p)$ is strictly quasi-concave, even a monopoly will not choose a price $p \geq \frac{m}{2}$. Hence, when $g_{\mathbf{a}}(0) > 1/m$, there cannot be any symmetric equilibrium in which both firms choose a price larger than $\frac{m}{2}$. As a result, $(p_1, p_2) = \left(\frac{1}{2g_{\mathbf{a}}(0)}, \frac{1}{2g_{\mathbf{a}}(0)}\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) > 1/m$.

Next, consider the case $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$. Taking $p_2 = \frac{m}{2}$ as given, we will show that $p_1 = \frac{m}{2}$ is a best response for firm 1. As mentioned, the profit function $\Pi_1(p_1, p_2)$ is semi-differentiable at the point $p_1 = m - p_2$. In particular, we have

$$\left.\frac{\partial^- \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - \frac{m}{2} \cdot g_{\mathbf{a}}(0) = \frac{1}{2} - \frac{m}{2} \cdot g_{\mathbf{a}}(0) \geq 0,$$

and

$$\left.\frac{\partial^+ \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = 1 - G_{\mathbf{a}}(0) - g_{\mathbf{a}}(0) = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) \leq 0.$$

Since $\Pi_1\left(p_1, \frac{m}{2}\right)$ is strictly quasi-concave on both $\left[0, \frac{m}{2}\right]$ and $[\frac{m}{2}, +\infty)$, the above inequalities show that $p_1 = \frac{m}{2}$ is a maximum of $\Pi_1\left(p_1, \frac{1}{2}\right)$ on each of these two intervals. This immediately implies that $p_1 = \frac{m}{2}$ is a global maximum of $\Pi_1\left(p_1, \frac{m}{2}\right)$ on $[0, +\infty)$. Hence, if $g_{\mathbf{a}}(0) \in [\frac{1}{2m}, \frac{1}{m}]$, the pricing subgame admits a symmetric equilibrium with $p_1 = p_2 = \frac{m}{2}$. Since $\frac{1}{2g_{\mathbf{a}}(0)} \geq \frac{m}{2}$, a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$ cannot exist. In addition, because

$$\left.\frac{\partial \Pi^M(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) \leq \frac{1}{2} - \frac{1}{2} = 0$$

and $\Pi^M(p)$ is strictly quasi-concave, even a monopoly will not choose a price strictly higher than $\frac{m}{2}$. Hence, no symmetric equilibrium with $p_1 = p_2 > \frac{m}{2}$ can exist either. As a result, $(p_1, p_2) = \left(\frac{m}{2}, \frac{m}{2}\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) \in \left[\frac{1}{2m}, \frac{1}{m}\right]$.

Finally, suppose that $g_{\mathbf{a}}(0) < \frac{m}{2}$. Observe that in this case, we must have $p_{\mathbf{a}}^M > \frac{m}{2}$, since

$$\left.\frac{\partial \Pi^M(p)}{\partial p}\right|_{p=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) > 0$$

and $\Pi^M(p)$ is strictly quasi-concave on $[0, +\infty)$. Now suppose that firm 2 plays $p_2 = p_{\mathbf{a}}^M$, and consider firm 1's profit function $\Pi_1(p_1, p_{\mathbf{a}}^M)$. Given the formula of the demand function and $p_{\mathbf{a}}^M > m - p_{\mathbf{a}}^M$, we have

$$\Pi_1(p_{\mathbf{a}}^M, p_{\mathbf{a}}^M) = \Pi^M(p_{\mathbf{a}}^M) \geq \Pi^M(p_1) \geq \Pi_1(p_1, p_{\mathbf{a}}^M) \; \forall p_1 \in [0, +\infty),$$

which further implies that $p_1 = p_{\mathbf{a}}^M$ is a best response for firm 1. Hence, $(p_1, p_2) = (p_{\mathbf{a}}^m, p_{\mathbf{a}}^m)$ indeed constitutes an equilibrium in the pricing subgame where $g_{\mathbf{a}}(0) < \frac{m}{2}$. Moreover, given $\frac{1}{2g_{\mathbf{a}}(0)} > \frac{m}{2}$, there cannot exist a symmetric equilibrium with $p_1 = p_2 < \frac{m}{2}$. Since

$$\left.\frac{\partial^- \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} > \left.\frac{\partial^+ \Pi_1(p_1, p_2)}{\partial p_1}\right|_{p_1=p_2=\frac{m}{2}} = \frac{1}{2} - m \cdot g_{\mathbf{a}}(0) > 0,$$

$p_1 = p_2 = \frac{m}{2}$ does not constitute an equilibrium either. In conclusion, $(p_1, p_2) = \left(p_{\mathbf{a}}^M, p_{\mathbf{a}}^M\right)$ is the unique symmetric pure-strategy equilibrium when $g_{\mathbf{a}}(0) < \frac{m}{2}$. □

N

B.9 Competition for Voters

Given $\mathbf{a} \in \mathcal{A}$ an interior symmetric second stage equilibrium necessarily solves

$$\frac{\partial \Pi_j(s, s; \mathbf{a})}{\partial s_j} = 0$$

with solution $s(\mathbf{a})$. Finally, we suppose that for any $\mathbf{a} \in \mathcal{A}$ the payoff function $\Pi_j(s_1, s_2; \mathbf{a})$ is strictly quasiconcave in s_j, such that the above condition is also sufficient, and assures that $(s_1, s_2) = (s(\mathbf{a}), s(\mathbf{a}))$ is a symmetric equilibrium in any second stage subgame induced by $\mathbf{a} \in \mathcal{A}$.[3]

Proposition B.1. *Consider the political economy application.*

(i) If there exists $\delta > 0$ with supp$(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ such that preferences are weakly δ-polarized, then there exists an SPE without voter confusion.

(ii) If there exists $\delta > 0$ with supp$(\gamma_{\mathbf{a}}) \subset [-\delta, \delta]$ $\forall \mathbf{a} \in \mathcal{A}$ such that preferences are δ-indecisive, then no SPE without voter confusion exists.[4]

Proof. Part (i): As the commitment case is isomorphic to the oligopoly model, the result follows by direct application of Theorem 2.1. (ii) Now consider the case

$$\Pi_j(s_1, s_2; \mathbf{a}) = \Pr(\tilde{v}_\Delta^i \le s_\Delta^i + \varepsilon_{\mathbf{a}}) - c(s_i).$$

Given that $\Pi_i(s_1, s_2; \mathbf{a})$ is strictly quasiconcave in s_1 for each $\mathbf{a} \in \mathcal{A}$, the second stage symmetric equilibrium is described by

$$\int g_0(e)\gamma_{\mathbf{a}}(e)de = c'(s_i), \tag{B.5}$$

with equilibrium profits

$$\Pi_i^* = 1/2 - c(s_i).$$

[3] In the commitment example, strict quasiconcavity follows from the convexity assumptions imposed on $c(\cdot)$. In the advertising example, this may additionally require that c is sufficiently convex.

[4] In the knife-edge case where g_0 is constant on $[-\delta, \delta]$, obfuscation has no effect and there are SPE both with and without voter confusion.

Hence each firm benefits from an equilibrium which involves a lower s. By the proof of Theorem 2.1,

$$\int g_0(e)\gamma_{\mathbf{a}}(e)de > g_0(0)$$

with weakly δ-polarized preferences whenever $\varepsilon_{\mathbf{a}}$ is non-degenerate. Hence the choice of $\mathbf{a}$ such that $\varepsilon_{\mathbf{a}}$ is degenerate constitutes a SPE.

The proof for Part (ii) of the proposition is omitted because it is analogous. □

B.10 Marketing Decisions

In this section, we provide a more technical discussion of the idea that our framework provides a suitable reduced-form approach which is consistent with various forms of confusion marketing that have been recognized by the marketing literature (Kasabov, 2015; Walsh et al., 2007) (see Section 2.4.2).

For convenience, we illustrate the following arguments by means of a general random utility model. Suppose that the (real-valued) utilities that the consumers associate with a product $j \in \{1, 2\}$, net of prices, is determined by a general random utility model $U_j = V_j + \varepsilon_j(\mathbf{a})$, where V_j is the distribution of true valuations for product j over the consumer population, and $\varepsilon_j(\mathbf{a})$ is a random variable for any $\mathbf{a} \in \mathcal{A}$. Absent a binding outside option, the difference distribution $U_\Delta \equiv U_2 - U_1$ alone is relevant for the firm pricing decisions, where

$$U_\Delta = V_\Delta + \varepsilon_{\mathbf{a}}$$

with $\varepsilon_{\mathbf{a}} \equiv \varepsilon_2(\mathbf{a}) - \varepsilon_1(\mathbf{a})$. This decomposition of $\varepsilon_{\mathbf{a}}$ is suggested by some but not all examples we elaborate below. In this respect, it is helpful to realize that expressing the effects of marketing activities in terms of $\varepsilon_{\mathbf{a}}$ directly, instead indirectly via $\varepsilon_2(\mathbf{a}) - \varepsilon_1(\mathbf{a})$, is without loss of generality in thus that any zero-symmetric random variable can always be expressed as the sum of two zero-symmetric random variables (Rubin and Sellke, 1986).

For easier reference, we let $\hat{\mathcal{H}}$ denote the set of all random variables with a 0-symmetric and log-concave density function. As before, O denotes the constant random variable that yields $x = 0$ with probability one, and $\mathcal{H} \equiv \hat{\mathcal{H}} \cup O$.

B.10.1 Product complexity and information overload

To tie the above discussion of product complexity as quantified by the number of attributes into the model, suppose that each firm can choose on a number of features to implement in the marketing process. Each feature needs separate information processing, which results in a noisy attribution of the product's valuation. Assuming that each such feature has an iid effect on consumer evaluation of the product, determined by a random variable $Z_s \in \hat{\mathcal{H}}$, the number of features n_j implemented corresponds to the marketing activity of firm j. If firm j implements n_j features, the perception noise is determined as

$$\varepsilon_j(n_j) = \sum\nolimits_{s=0}^{n_j} Z_s, \tag{B.6}$$

where $\varepsilon_j(n_j) \in \mathcal{H}$ if $n_j > 0$,[5] and $\varepsilon_j(0) = O$. The intuition captured by (B.6) is that a more complex product, in the sense of coming with more features, increases the difficulty of evaluation and hence also the spread of opinions across consumers. If $\varepsilon_1, \varepsilon_2$ are determined in the above way, we have $\varepsilon(n_1, n_2) \equiv \varepsilon_1(n_1) - \varepsilon_2(n_2) \in \mathcal{H}$, and $\varepsilon(n_1, n_2) \in \hat{\mathcal{H}}$ iff $n_1 + n_2 > 0$. Because $\varepsilon(n_1', n_2')$ is a mean-preserving spread of $\varepsilon(n_1, n_2)$ whenever $n_1' + n_2' > n_1 + n_2$, the type of obfuscation process captured by (B.6) has the additional feature that the resulting distributions ε can be ordered in the sense of second-order stochastic dominance.[6]

B.10.2 Heterogeneous effects and interactions

The simple i.i.d model (B.6) does not capture the main observation stated above that different features may affect consumer perception differentially, possibly with dependencies. However, the model can be readily generalized to such a case. Let $Z = \{Z_1, ...Z_k\}$ denote a a set of random variables, where the random vector $(Z_1, ..., Z_k)$ has a joint density function $f(z_1, ..., z_k)$ that is coordinate-wise symmetric, meaning that

$$f(z_1, ..., z_s, ..., z_k) = f(z_1, ..., -z_s, ..., z_k), \quad \forall (z_1, ..., z_k) \in supp(f), \forall s = 1, ..., k.$$

The non-empty selection $M_j \subset Z$ corresponds to the set of features implemented by firm j, which affects the product valuation according to

$$\varepsilon_j(M_j) = \sum_{s \in M_j} Z_s. \tag{B.7}$$

Formulation (B.7) can capture that the combination of different features affect consumer perceptions differentially, or that some features are more confusing than others. The set of marketing strategies A corresponds to all possible selections, i.e., any marketing activity a_j belongs to the power set of Z, where $a_j = \emptyset$ means that no feature is chosen and ε_j is degenerate. If the density of Z is log-concave and coordinate-wise symmetric, so is the density of any non-empty selection M_j, meaning that $\varepsilon_j(M_j)$ is symmetric and log-concave as well. Assuming independent product evaluation implies that $\varepsilon(a_1, a_2) = \varepsilon_1(a_1) - \varepsilon_2(a_2) \in \mathcal{H}$ and $\varepsilon(a_1, a_2) \in \hat{\mathcal{H}}$ iff $a_j \neq \emptyset$ for some $j \in \{1, 2\}$. Finally, the ordering between distributions with the above marketing technology is such that $\varepsilon(a_1', a_2')$ is a mean-preserving spread of $\varepsilon(a_1, a_2)$ whenever $a_j \subsetneq a_j'$, $j = 1, 2$.

It is easy to verify that (B.6) is a special case of (B.7). The model (B.7) could be further extended, e.g., to capture that confusion occurs only once a sufficient number of features have been implemented. This could be achieved by introducing a threshold value $k \in \mathbb{N}$ such that $\varepsilon_j(M_j) = \sum_{s \in M_j} Z_s$ if $|M_j| > k$ and $\varepsilon_j(M_j) = O$ if $|M_j| \leq k$.

[5] This follows because an independent sum of random variables with log-concave and symmetric densities again produces a random variable with symmetric and log-concave density.

[6] Note that (B.6) also allows for a non-cognitive explanation, where the "features" are perfectly perceived but of heterogeneous valuations to consumers, where some consumers like certain features that others dislike in a way that these features do not correspond to an increase in product quality on average.

C Appendix: Chapter 3

C.1 Proof of Proposition 3.1

We prove a more general version of Proposition 3.1 by allowing the prior distribution of the state to be non-uniform and the accuracy of the private signals to be state-dependent. Specifically, we assume $\Pr(\theta = 0) = 1 - \Pr(\theta = 1) = \pi \in (0, 1)$, and each of the private signals is independently drawn according to the conditional probability distribution characterized by $\Pr(s_i = 0|\theta = 0) = \alpha_0$ and $\Pr(s_i = 1|\theta = 1) = \alpha_1$, where $\alpha_0, \alpha_1 \in (1/2, 1)$. The results in the main text will then follow by letting $\pi = 1/2$ and $\alpha_0 = \alpha_1 = \alpha$.

For every signal profile $s = (s_1, ..., s_n)$, let $m_s = \sum_{i=1}^n s_i$. As an auxiliary result, note that conditional on observing the whole profile of private signals and the public signal, the posterior belief $\pi(s, s_p)$ that a Bayesian agent would assign to the event $\theta = 1$ is given by:

$$\begin{aligned}\pi(s, s_p) =& \frac{\Pr(s, s_p|\theta = 1)\Pr(\theta = 1)}{\Pr(s, s_p|\theta = 1)\Pr(\theta = 1) + \Pr(s, s_p|\theta = 0)\Pr(\theta = 0)} \\ =& \frac{\alpha_1^{m_s}(1-\alpha_1)^{n-m_s}\beta^{\mathbb{1}_{s_p=1}}(1-\beta)^{\mathbb{1}_{s_p=0}}(1-\pi)}{\alpha_1^{m_s}(1-\alpha_1)^{n-m_s}\beta^{\mathbb{1}_{s_p=1}}(1-\beta)^{\mathbb{1}_{s_p=0}}(1-\pi) + (1-\alpha_0)^{m_s}\alpha_0^{n-m_s}(1-\beta)^{\mathbb{1}_{s_p=1}}\beta^{\mathbb{1}_{s_p=0}}\pi} \\ =& \frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^{m_s}\left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-m_s}\left(\frac{1-\beta}{\beta}\right)^{\mathbb{1}_{s_p=1}}\left(\frac{\beta}{1-\beta}\right)^{\mathbb{1}_{s_p=0}}\left(\frac{\pi}{1-\pi}\right)},\end{aligned}$$

where the first equality follows from Bayes rule and the second equality follow from the independence assumption of the signals.

We will show that under a given k-voting rule g^k, the informative voting equilibrium exists if and only if

$$\forall i \in \mathcal{I},\ q_i \in \left[\frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^{k-1}\left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k+1}\left(\frac{1-\beta}{\beta}\right)\left(\frac{\pi}{1-\pi}\right)}, \frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^{k}\left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k}\left(\frac{\beta}{1-\beta}\right)\left(\frac{\pi}{1-\pi}\right)}\right].$$

Suppose all agents $j \neq i$ play $\sigma_j(s_j, s_p) = s_j$. Firstly, note that if $\sigma_i(1, 0) = 1$ is rational for agent i, so is $\sigma_i(1, 1) = 1$; similarly, if $\sigma_i(0, 1) = 0$ is rational for agent i, so is $\sigma_i(0, 0) = 0$. Hence, we only need to consider the optimality of the informative voting strategy in the cases where $s_i \neq s_p$. Secondly, agent i is decisive when and only when there are $k - 1$ agents who observe a positive signal ($s_j = 1$) and each of the remaining $n - k$ agents observes an opposite signal ($s_j = 0$). Therefore, given $s_i = 1, s_p = 0$ and being pivotal, the posterior probability that agent i assigns to the event $\theta = 1$ is:

$$\pi_1^0 = \frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^{k}\left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k}\left(\frac{\beta}{1-\beta}\right)\left(\frac{\pi}{1-\pi}\right)}.$$

Similarly, given $s_i = 0, s_p = 1$ and being pivotal, the posterior probability that agent i assigns to the event $\theta = 1$ is:

$$\pi_0^1 = \frac{1}{1 + \left(\frac{1-\alpha_0}{\alpha_1}\right)^{k-1} \left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k+1} \left(\frac{1-\beta}{\beta}\right) \left(\frac{\pi}{1-\pi}\right)}.$$

Hence, to have informative voting as an equilibrium, it is both necessary and sufficient to have $\forall i \in \mathcal{I},\ q_i \in [\pi_0^1, \pi_1^0]$. By letting $\pi = 1/2$ and $\alpha_0 = \alpha_1 = \alpha$, we immediately obtain condition (3.2). □

C.2 Proof of Corollary 3.1

Note that the interval $[\pi_0^1, \pi_1^0]$ as defined in the proof of Proposition 3.1 is non-empty if and only if

$$\left(\frac{1-\alpha_0}{\alpha_1}\right)^{k-1} \left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k+1} \left(\frac{1-\beta}{\beta}\right) \left(\frac{\pi}{1-\pi}\right) \geq \left(\frac{1-\alpha_0}{\alpha_1}\right)^{k} \left(\frac{\alpha_0}{1-\alpha_1}\right)^{n-k} \left(\frac{\beta}{1-\beta}\right) \left(\frac{\pi}{1-\pi}\right),$$

which is equivalent to

$$\left(\frac{\alpha_0}{1-\alpha_0}\right) \left(\frac{\alpha_1}{1-\alpha_1}\right) \geq \left(\frac{\beta}{1-\beta}\right)^2. \tag{C.1}$$

If the accuracy of the private signals is state-independent, i.e., $\alpha_0 = \alpha_1 = \alpha$, (C.1) is further equivalent to $\alpha \geq \beta$. □

C.3 Proof of Corollary 3.2

Suppose that $\pi = 1/2$, $\alpha_0 = \alpha_1 = \alpha$ and there exists a k-voting rule under which the informative voting equilibrium exists. According to the proof of Proposition 3.1, the preferences of agents i and j must satisfy

$$q_i, q_j \in \left[\frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2} \frac{1-\beta}{\beta}}, \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n} \frac{\beta}{1-\beta}}\right]. \tag{C.2}$$

Moreover, (C.2) and $q_i < \frac{1}{1+\frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}}$ implies

$$\frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n-2} \frac{1-\beta}{\beta}} < \frac{1}{1 + \frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta}} \iff k < \frac{n+1}{2}. \tag{C.3}$$

Similarly, (C.2) and $q_j > \frac{1}{1+\frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}}$ implies

$$\frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{2k-n} \frac{\beta}{1-\beta}} > \frac{1}{1 + \frac{1-\alpha}{\alpha}\frac{\beta}{1-\beta}} \iff k > \frac{n+1}{2}. \tag{C.4}$$

Clearly, (C.3) and (C.4) are mutually exclusive. Hence, we can conclude that the informative voting equilibrium does not exist under any k-voting rule. $\square$

C.4 Proof of Proposition 3.2

Given that each agent i observes $\hat{s}_i = s_i + s_p$, we let $\hat{S}_i \equiv \{0, 1, 2\}$ be each agent's signal space. Therefore, agent i's strategy is now a mapping $\sigma_i : \hat{S}_i \to [0, 1]$, where $\sigma_i(\hat{s}_i)$ denotes the probability that agent i will vote $v_i = 1$ when observing $\hat{s}_i \in \hat{S}_i$.

Fix an arbitrary sequence of preference profiles $\{\mathbf{q}^n\}_{n\in\mathbb{N}}$. Suppose, in contradiction, that there exists a sequence of k-voting rules $\{g^{k_n}\}_{n\in\mathbb{N}}$ that aggregates information asymptotically. Let $\{g^{k_{n(\tau)}}\}_{\tau\in\mathbb{N}}$ and $\{\sigma^{k_{n(\tau)}}\}_{\tau\in\mathbb{N}}$ be the corresponding convergent subsequences of voting rules and equilibria, where $\lim_{\tau\to\infty} k_{n(\tau)}/n(\tau) = \kappa \in [0, 1]$. Since for any agent i her posterior belief about the state being 1 is strictly increasing in $\hat{s}_i$, we must have for all $i = 1, ..., n(\tau)$ and for all $\tau \in \mathbb{N}$, $\sigma_i^{k_{n(\tau)}}(0) \leq \sigma_i^{k_{n(\tau)}}(1) \leq \sigma_i^{k_{n(\tau)}}(2)$. Let $Y^\tau \equiv \sum_{i=1}^{n(\tau)} v_i$. For each $\tau \in \mathbb{N}$, we can decompose the conditional expectation of Y^τ as follows:

$$\begin{aligned}\mathbb{E}\left[Y^\tau|\theta, s_p\right] =& \Pr\left(Y^\tau \geq k_{n(\tau)}|\theta, s_p\right)\mathbb{E}\left[Y^\tau|Y^\tau \geq k_{n(\tau)}, \theta, s_p\right] \\ &+ \Pr\left(Y^\tau < k_{n(\tau)}|\theta, s_p\right)\mathbb{E}\left[Y^\tau|Y^\tau < k_{n(\tau)}, \theta, s_p\right].\end{aligned}$$

Now consider the case $\theta = 1$ and $s_p = 0$, which occurs with probability $(1 - \beta)/2 > 0$. In this case, along the sequence of the equilibria $\sigma^{k_{n(\tau)}}$ agent i will cast the right vote ($v_i = 1$) with probability $\alpha\sigma_i^{k_{n(\tau)}}(1) + (1 - \alpha)\sigma_i^{k_{n(\tau)}}(0)$. Therefore, given the sequences of equilibria and voting rules, we have

$$\mathbb{E}\left[Y^\tau|\theta = 1, s_p = 0\right] = \sum_{i=1}^{n(\tau)}\left[\alpha\sigma_i^{k_{n(\tau)}}(1) + (1 - \alpha)\sigma_i^{k_{n(\tau)}}(0)\right].$$

Thus, for the probability of reaching the right decision ($d = 1$) converging to 1 along this path, that is $\Pr\left(Y^\tau \geq k_{n(\tau)}|\theta = 1, s_p = 0\right) \to 1$, we must have

$$\lim_{\tau\to\infty} \mathbb{E}\left[Y^\tau|\theta = 1, s_p = 0\right] = \lim_{\tau\to\infty} \mathbb{E}\left[Y^\tau|Y^\tau \geq k_{n(\tau)}, \theta = 1, s_p = 0\right].$$

Together with the monotonicity condition $\sigma_i^{k_{n(\tau)}}(0) \leq \sigma_i^{k_{n(\tau)}}(1)$, this further implies that

$$\lim_{\tau\to\infty} \frac{\sum_{i=1}^{n(\tau)} \sigma_i^{k_{n(\tau)}}(1)}{n(\tau)} \geq \lim_{\tau\to\infty} \frac{k_{n(\tau)}}{n(\tau)} = \kappa. \tag{C.5}$$

Next, consider the case $\theta = 0$ and $s_p = 1$, which also occurs with probability $(1 - \beta)/2 > 0$. In this case, along the sequence of the equilibria $\sigma^{k_{n(\tau)}}$ agent i will cast the right vote ($v_i = 0$) with probability $\alpha(1 - \sigma_i^{k_{n(\tau)}}(1)) + (1 - \alpha)(1 - \sigma_i^{k_{n(\tau)}}(2))$. Hence, similar the previous case, for the probability of reaching the right decision ($d = 0$) converging to 1 along this path, that is,

$\Pr\left(Y^\tau < k_{n(\tau)}|\theta = 0, s_p = 1\right) \to 1$, it is necessary to have

$$\lim_{\tau\to\infty} \frac{\sum_{i=1}^{n(\tau)}(1-\sigma_i^{k_{n(\tau)}}(1))}{n(\tau)} > \lim_{\tau\to\infty}\left(1-\frac{k_{n(\tau)}}{n(\tau)}\right) = 1-\kappa \iff \lim_{\tau\to\infty}\frac{\sum_{i=1}^{n(\tau)}\sigma_i^{k_{n(\tau)}}(1)}{n(\tau)} < \kappa. \quad \text{(C.6)}$$

Clearly, (C.5) and (C.6) cannot hold simultaneously. We thus reach a contradiction. Therefore, ex ante there must be some non-vanishing probability that the committee will reach a wrong decision even its size goes to arbitrarily large. In other words, no sequence of k-voting rules can aggregate information asymptotically. □

C.5 Proof of Proposition 3.3

Recall that we write $\pi(s, s_p)$ as the posterior belief that a Bayesian agent will assign to the event $\theta = 1$ when she knows that the actual signal profile is (s, s_p). Since the function $\pi(s, s_p)$ is clearly symmetric in every private signal, we also use $\pi(m, s_p)$ to denote the posterior belief of a Bayesian agent when observing any signal profile (s, s_p) with $m_s \equiv \sum_{i=1}^n s_i = m$.

We first establish a lemma that fully characterizes ex post incentive compatibility.

Lemma C.1. *A mechanism f is ex post incentive compatible if and only if* $\forall(s_{-i}, s_p) \in S_{-i} \times S_p$ *and* $\forall i \in \mathcal{I}$,

(i) $f(1, s_{-i}, s_p) \geq f(0, s_{-i}, s_p)$*, and*

(ii) $\pi(0, s_{-i}, s_p) > q_i$ *or* $\pi(1, s_{-i}, s_p) < q_i \implies f(0, s_{-i}, s_p) = f(1, s_{-i}, s_p)$.

Proof of Lemma C.1. Given the specification of the agents' payoff functions, the ex post incentive compatibility constraints can be equivalently rewritten as follows:

$$(f(s_i, s_{-i}, s_p) - f(s_i', s_{-i}, s_p))\,(\pi(s_i, s_{-i}, s_p) - q_i) \geq 0, \quad \text{(C.7)}$$

for all $s_i, s_i' \in S_i$, $s_{-i} \in S_{-i}$, $s_p \in S_p$ and $i \in \mathcal{I}$. The sufficiency part of the lemma then immediately follows.

Let us now prove the necessity part. For (i), suppose, in contradiction, that there exist some $i \in \mathcal{I}$, $s_{-i} \in S_{-i}$ and $s_p \in S_p$ such that $f(0, s_{-i}, s_p) > f(1, s_{-i}, s_p)$. By (C.7), we must have

$$\pi(1, s_{-i}, s_p) - q_i \leq 0 \leq \pi(0, s_{-i}, s_p) - q_i,$$

which contradicts to $\pi(1, s_{-i}, s_p) > \pi(0, s_{-i}, s_p)$. Hence, $f(1, s_{-i}s_p) \geq f(0, s_{-i}, s_p)$ $\forall s_i, s_i' \in S_i$, $s_{-i} \in S_{-i}$, $s_p \in S_p$ and $i \in \mathcal{I}$. As for (ii), note that together with (C.7) either $\pi(0, s_{-i}, s_p) > q_i$ or $\pi(1, s_{-i}, s_p) < q_i$ would imply that $f(0, s_{-i}, s_p) \geq f(1, s_{-i}, s_p)$. By implication (i) of EPIC, we further have $f(0, s_{-i}, s_p) = f(1, s_{-1}, s_p)$. ■

For every $m \in \{1, ..., n\}$ and $s_p \in S_p$, let us define

$$\mathcal{I}(m, s_p) \equiv \{i \in \mathcal{I} : \pi(m, s_p) \geq q_i \geq \pi(m-1, s_p)\}.$$

The next lemma shows that in an EPIC mechanism, if two agents have very different preferences, then their pivotality cannot depend on each other's report.

Lemma C.2. *Suppose that f is EPIC. $\forall s_p \in S_p$ and $i, j \in \mathcal{I}$, if there does not exist $m \in \{1, ..., n\}$ such that $\{i, j\} \subseteq \mathcal{I}(m, s_p)$, then $\forall s_{-i,-j} \in S_{-i,-j}$, we have either*

$$f(1, s_j, s_{-i,-j}, s_p) > f(0, s_j, s_{-i,-j}, s_p) \ \forall s_j \in S_j,$$

or

$$f(1, s_j, s_{-i,-j}, s_p) = f(0, s_j, s_{-i,-j}, s_p) \ \forall s_j \in S_j.$$

Proof of Lemma C.2. We shall prove the lemma by contraposition. Consider any $s_p \in S_p$ and $i, j \in \mathcal{I}$. Suppose that there exists $s_{-i,-j} \in S_{-i,-j}$ such that

$$f(1, s_j, s_{-i,-j}, s_p) > f(0, s_j, s_{-i,-j}, s_p), \text{ and } f(1, s'_j, s_{-i,-j}, s_p) = f(0, s'_j, s_{-i,-j}, s_p),$$

where $s_j, s'_j \in \{0, 1\}$ and $s_j \neq s'_j$. Consider first the case where $s_j = 1$ and $s'_j = 0$. By Lemma C.1(i), f must be monotone in every agent's report, and we thus have

$$f(1, 1, s_{-i,-j}, s_p) > f(0, 1, s_{-i,-j}, s_p) \geq f(0, 0, s_{-i,-j}, s_p) = f(1, 0, s_{-i,-j}, s_p).$$

Since $f(1, 1, s_{-i,-j}, s_p) > f(0, 1, s_{-i,-j}, s_p)$ and $f(1, 1, s_{-i,-j}, s_p) > f(1, 0, s_{-i,-j}, s_p)$, by Lemma C.1(ii) we must have

$$\begin{aligned} &\pi(1, 1, s_{-i,-j}, s_p) \geq q_i \geq \pi(0, 1, s_{-i,-j}, s_p), \text{ and} \\ &\pi(1, 1, s_{-i,-j}, s_p) \geq q_j \geq \pi(1, 0, s_{-i,-j}, s_p). \end{aligned}$$

It then follows that $\{i, j\} \subseteq \mathcal{I}(m, s_p)$, where $m = m_{(1,1,s_{-i,-j})} \in \{1, ..., n\}$. For the case where $s_j = 0$ and $s'_j = 1$, the proof is analogous. ■

For every $s_p \in S_p$, let

$$\mathcal{I}^*(s_p) \equiv \{i \in \mathcal{I} : \exists s_{-i} \in S_{-i}, \text{ s.t. } f(1, s_{-i}, s_p) > f(0, s_{-i}, s_p)\}$$

be the set of agents who can possibly be pivotal when the public signal is s_p. The next lemma states that in any EPIC mechanism, pivotality can only be granted to a subset of agents who share similar preferences.

Lemma C.3. *Suppose that f is EPIC. Then, $\forall s_p \in S_p$, $\exists m \in \{1, ..., n\}$ such that $\mathcal{I}^*(s_p) \subseteq \mathcal{I}(m, s_p)$.*

Proof of Lemma C.3. Consider any $s_p \in S_p$. Without loss of generality, assume that $\exists i, j \in \mathcal{I}^*(s_p)$ such that $q_i > q_j$. We claim that there must exist $m \in \{1, ..., n\}$, such that $\{i, j\} \subseteq \mathcal{I}(m, s_p)$.

Suppose that such an integer m does not exist. Together with the assumption that $i, j \in \mathcal{I}^*(s_p)$, Lemma C.2 implies that there must exist $s_{-i,-j}, s'_{-i,-j} \in S_{-i,-j}$ such that

$$f(1, s_j, s_{-i,-j}, s_p) > f(0, s_j, s_{-i,-j}, s_p) \; \forall s_j \in S_j, \text{ and}$$
$$f(s_i, 1, s'_{-i,-j}, s_p) > f(s_i, 0, s'_{-i,-j}, s_p) \; \forall s_i \in S_i.$$

By Lemma C.1(ii), this further implies that

$$\pi(1,1,s_{-i,-j},s_p) \geq q_i \geq \pi(1,0,s_{-i,-j},s_p), \quad \pi(1,0,s_{-i,-j},s_p) \geq q_i \geq \pi(0,0,s_{-i,-j},s_p), \text{ and}$$
$$\pi(1,1,s'_{-i,-j},s_p) \geq q_j \geq \pi(1,0,s'_{-i,-j},s_p), \quad \pi(0,1,s'_{-i,-j},s_p) \geq q_j \geq \pi(0,0,s'_{-i,-j},s_p).$$

It then immediately follows that $q_i = \pi(1,0,s_{-i,-j},s_p)$ and $q_j = \pi(1,0,s'_{-i,-j},s_p)$. Since $q_i > q_j$ and there does not exist $m \in \{1, ..., n\}$ such that $\{i, j\} \subseteq \mathcal{I}(m, s_p)$, we must have $m_{(1,0,s_{-i,-j})} > m_{(1,0,s'_{-i,-j})} + 1$.

We discuss two possible cases separately. First, suppose that $m_{(1,0,s_{-i,-j})} > m_{(1,0,s'_{-i,-j})} + 2$. Consider the profile $s''_{-i,-j}$, which is obtained as follows: $\forall \ell \neq i, j$ such that $\pi(1,1,s_{-i,-j}) \geq q_\ell \geq \pi(0,0,s_{-i,-j})$, we let $s''_\ell = s_\ell$. Otherwise, we let $s''_\ell = s'_\ell$. By construction, whenever $s''_\ell \neq s_\ell$, there cannot exist $m \in \{1, ..., n\}$ such that $\{i, \ell\} \subseteq \mathcal{I}(m, s_p)$. It then follows from Lemma C.2 that

$$f(1, s_j, s''_{-i,-j}, s_p) > f(0, s_j, s''_{-i,-j}, s_p) \; \forall s_j \in S_j.$$

Similarly, given that $m_{(1,0,s_{-i,-j})} > m_{(1,0,s'_{-i,-j})} + 2$, we have if $s''_\ell \neq s'_\ell$, then there does not exist $m \in \{1, ..., n\}$ such that $\{j, \ell\} \subseteq \mathcal{I}(m, s_p)$. It again follows from Lemma C.2 that

$$f(s_i, 1, s''_{-i,-j}, s_p) > f(s_i, 0, s''_{-i,-j}, s_p) \; \forall s_i \in S_i.$$

But then, we must have $q_i = q_j = \pi(1,0,s''_{-i,-j},s_p)$. We thus reach a contradiction.

Next, suppose that $m_{(1,0,s_{-i,-j})} = m_{(1,0,s'_{-i,-j})} + 2$. Since n is odd, there must exist $\ell \neq i, j$, such that $s_\ell = 0$. Together with $q_j < q_i = \pi(1,0,s_\ell,s_{-i,-j,-\ell},s_p)$, Lemma C.1(ii) implies that

$$\begin{aligned} f(1,1,0,s_{-i,-j,-\ell},s_p) &= f(1,0,0,s_{-i,-j,-\ell},s_p) \\ &> f(0,1,0,s_{-i,-j,-\ell},s_p) = f(0,0,0,s_{-i,-j,-\ell},s_p). \end{aligned}$$

If $q_\ell \leq \pi(0,0,0,s_{-i,-j,-\ell},s_p)$, Lemma C.1(ii) will further imply that

$$f(1,1,0,s_{-i,-j,-\ell},s_p) = f(1,1,1,s_{-i,-j,-\ell},s_p)$$

and

$$f(0,1,0,s_{-i,-j,-\ell},s_p) = f(0,1,1,s_{-i,-j,-\ell},s_p).$$

Hence, we have $f(1,1,1,s_{-i,-j,-\ell},s_p) > f(0,1,1,s_{-i,-j,-\ell},s_p)$. This means that agent i is pivotal given the public signal is s_p and the other agents' signals are $s_{-i} = (1,1,s_{-i,-j,-\ell})$. However, this contradicts to that f being EPIC because $q_i = \pi(1,0,0,s_{-i,-j,-\ell},s_p) < \pi(0,1,1,s_{-i,-j,-\ell},s_p)$.

If $q_\ell > \pi(0,0,0,s_{-i,-j,-\ell},s_p)$, we can consider the signal profile $s'_{-i,-j} = (s'_\ell, s'_{-i,-j,-\ell})$ instead. Since $m_{(1,0,s_{-i,-j})} = m_{(1,0,s'_{-i,-j})} + 2$ and $s_\ell = 0$, we must have $\pi(0,0,0,s_{-i,-j,-\ell},s_p) = \pi(1,1,s'_\ell, s'_{-i,-j,-\ell},s_p)$. Given that $q_\ell > \pi(1,1,s'_\ell,s'_{-i,-j,-\ell},s_p)$, if $s'_\ell = 1$, we can simply replicate the previous argument and reach a contradiction to the incentive compatibility of f. We thus focus on the case where $s'_\ell = 0$. Together with $q_i > q_j = \pi(1,0,s'_\ell,s'_{-i,-j,-\ell},s_p)$, Lemma C.1(ii) implies that

$$\begin{aligned} f(1,1,s'_\ell,s'_{-i,-j,-\ell},s_p) &= f(0,1,s'_\ell,s'_{-i,-j,-\ell},s_p) \\ &> f(1,0,s'_\ell,s_{-i,-j,-\ell},s_p) = f(0,0,s'_\ell,s_{-i,-j,-\ell},s_p). \end{aligned}$$

Since $q_\ell > \pi(1,1,s'_\ell,s'_{-i,-j,-\ell},s_p)$, Lemma C.1(ii) further implies that

$$f(1,1,1,s'_{-i,-j,-\ell},s_p) = f(1,1,0,s'_{-i,-j,-\ell},s_p)$$

and

$$f(1,0,1,s'_{-i,-j,-\ell},s_p) = f(1,0,0,s'_{-i,-j,-\ell},s_p).$$

Hence, we have $f(1,1,1,s'_{-i,-j,-\ell},s_p) > f(1,0,1,s'_{-i,-j,-\ell},s_p)$. This again contradicts to the incentive compatibility of f, because $s'_\ell = 0$ and $q_j = \pi(1,0,0,s'_{-i,-j,-\ell},s_p) < \pi(1,0,1,s'_{-i,-j,-\ell},s_p)$.

In sum, we have shown that $\forall i,j \in \mathcal{I}^*(s_p)$, there must exist $m \in \{1,..,n\}$ such that $\{i,j\} \subseteq \mathcal{I}(m,s_p)$, because otherwise there will be a contradiction to the assumption that f being EPIC. Since for each $i \in \mathcal{I}^*(s_p)$ there can be *at most* one pair of integers $m', m'' \in \{1,...,n\}$ such that $i \in \mathcal{I}(m',s_p) \cap \mathcal{I}(m'',s_p)$ and, provided they exist, m' and m'' must be adjacent, we can conclude that there must exist $m \in \{1,...,n\}$ such that $\mathcal{I}^*(s_p) \subseteq \mathcal{I}(m,s_p)$. ■

Note that the no-indifference condition is not needed for Lemmas C.1 - C.2. Our last lemma below shows if this condition is further assumed, then any EPIC mechanism must be symmetric respect to the reports of the agents.

Lemma C.4. *Suppose that $\forall i \in \mathcal{I}$, $\nexists (s,s_p) \in S \times S_p$ such that $q_i = \pi(s,s_p)$. Then, for any EPIC mechanism f, there exists $\hat{f} : \{1,...,n\} \times S_p \to [0,1]$, such that $f(s,s_p) = \hat{f}(m_s,s_p)$ $\forall (s,s_p) \in S \times S_p$.*

Proof of Lemma C.4. Consider any $s_p \in S_p$. We first argue that we must have either $\mathcal{I}^*(s_p) = \emptyset$ or $\mathcal{I}^*(s_p) = \mathcal{I}$. Suppose, in contradiction, that there exist $i \in \mathcal{I}^*(s_p)$ and $j \in \mathcal{I} \setminus \mathcal{I}^*(s_p)$. By definition, there must exist $s_{-i,-j} \in S_{-i,-j}$, such that $f(1,s_j,s_{-i,-j},s_p) > f(0,s_j,s_{-i,-j},s_p) \,\forall s_j \in S_j$. Lemma C.1(ii) then implies that we must have $q_i = \pi(1,0,s_{-i,-j},s_p)$, which contradicts to the assumption of the lemma.

If $\mathcal{I}^*(s_p) = \emptyset$, then $f(s,s_p) = f(s',s_p)$ $\forall s,s' \in S$, and the statement of the lemma trivially holds. Let us now consider the case where $\mathcal{I}^*(s_p) = \mathcal{I}$. By Lemma C.3, there exists $m \in \{1,...,n\}$ such that $\mathcal{I}^*(s_p) \subseteq \mathcal{I}(m,s_p)$. Because $\mathcal{I}^*(s_p) = \mathcal{I}$ and $\mathcal{I}(m,s_p) \subseteq \mathcal{I}$, we further have $\mathcal{I}(m,s_p) = \mathcal{I}$. In addition, the no-difference condition assumed in the lemma guarantees that $\mathcal{I}(m',s_p) \cap \mathcal{I}(m'',s_P) = \emptyset$ $\forall m',m'' \in \{1,...,n\}$, and hence the above integer m is unique.

Therefore, there cannot exist $i \in \mathcal{I}$ and $s_{-i} \in S_{-i}$ such that $m_{(1,s_{-i})} \neq m$ and $f(1, s_{-i}, s_p) > f(0, s_{-i}, s_p)$. That is, an agent i can be pivotal only if there are exactly $m-1$ other agents are reporting $s_j = 1$.

Since $\mathcal{I}^*(s_p) = \mathcal{I}$, for every $i \in \mathcal{I}$ there must exist $s_{-i} \in S_{-i}$ such that $f(1, s_{-i}, s_p) > f(0, s_{-i}, s_p)$ and $m_{(1,s_{-i})} = m$. We claim that for all $s'_{-i} \in S_{-i}$ such that $m_{(1,s'_{-i})} = m$, we must have

$$f(1, s_{-i}, s_p) = f(1, s'_{-i}, s_p) > f(0, s'_{-i}, s_p) = f(0, s_{-i}, s_p). \tag{C.8}$$

The claim is trivial if $m = n$, so let us assume that $m < n$. This implies that in the profile s_{-i}, there must exist $j, \ell \in \mathcal{I}$, such that $s_j = 1$ and $s_\ell = 0$. We have

$$f(1, 1, 0, s_{-i,-j,-\ell}, s_p) = f(1, 1, 1, s_{-i,-j,-\ell}, s_p) = f(1, 0, 1, s_{-i,-j,-\ell}, s_p),$$

where the equalities follow that no agent can be pivotal when there are m other agents reporting strictly positive signals. Similarly, we also have

$$f(0, 1, 0, s_{-i,-j,-\ell}, s_p) = f(0, 0, 0, s_{-i,-j,-\ell}, s_p) = f(0, 0, 1, s_{-i,-j,-\ell}, s_p).$$

In addition, since $m_{(1,s_{-i})} = m_{(1,s'_{-i})}$, $s'_{-i,-j}$ can be obtained by swapping the 0 and 1 signals in the profile $s_{-i,-j}$. We can therefore conclude that (C.8) must hold.

The above claim shows that if an agent i is pivotal at some profile s_{-i}, then it must be pivotal whenever the profile of other agents' signals is equal to a permutation of s_{-i}. Moreover, conditional on agent i is pivotal, the probability of choosing $d = 1$ can only depend on her report. Since $\mathcal{I}^*(s_p) = \mathcal{I}(m, s_p) = \mathcal{I}$, this further implies that

$$f(s, s_p) = f(\hat{s}, s_p) > f(s', s_p) = f(\hat{s}', s_p)$$

for all $s, s', \hat{s}, \hat{s}' \in S$ that satisfy $m_s = m_{\hat{s}} = m$ and $m_{s'} = m_{\hat{s}'} = m - 1$.

Finally, take any s, s' such that $m_s = m$ and $m_{s'} = m - 1$. Since no agent can be pivotal at any profile $\tilde{s} \in S$ such that $m_{\tilde{s}} \notin \{m, m-1\}$, we must have $f(\tilde{s}, s_p) = f(s, s_p)$ if $m_{\tilde{s}} \geq m$, and $f(\tilde{s}, s_p) = f(s', s_p)$ if $m_{\tilde{s}} < m$. The statement of the lemma then immediately follows. ■

Given Lemma C.4, let us use $f(m, s_p)$ to denote the allocation rule for an EPIC mechanism, where m is the number of agents who report $s_i = 1$. Lemma C.1 can then be restated as follows: a mechanism $f : \{1, ..., n\} \times S_p \to [0, 1]$ is ex post incentive compatible if and only if $\forall m \in \{1, ..., n\}$ and $\forall s_p \in S_p$, we have (i) $f(m, s_p)$ being non-decreasing in m, and (ii) $f(m, s_p) = f(m-1, s_p)$ if either $\pi(m-1, s_p) > \max_{i \in \mathcal{I}} q_i$ or $\pi(m, s_p) < \min_{i \in \mathcal{I}} q_i$. In addition, given the no-indifference condition, for every EPIC mechanism f one can find a unique threshold $m_{s_p} \in \{0, 1, ..., n, n+1\}$ for every $s_p \in S_p$, such that $\forall m, m' \in \{1, ..., n\}$, $f(m, s_p) > f(m', s_p)$ if and only if $m \geq m_{s_p} > m'$.

Now consider the set of optimal EPIC mechanisms, $\mathcal{F}^*$. We first argue that, provided that it is non-empty, $\mathcal{F}^*$ must contain a deterministic mechanism, i.e., $\exists f^* \in \mathcal{F}^*$, such that $f^*(m, s_p) \in \{0, 1\}$ $\forall m \in \{1, ..., n\}$ and $\forall s_p \in S_p$. To see this, take any $f^* \in \mathcal{F}^*$ and consider any

$s_p \in S_p$. If $m_{s_p} \in \{0, n+1\}$, then we must have $f(m, s_p) = f(m', s_p)\ \forall m, m' \in \{1, ..., n\}$, i.e., f^* does not make use of the agents' private information at all given the public signal s_p. Thus, in this case for f^* to be optimal we must have $f^*(m, s_p) = 1\ \forall m \in \{1, ..., n\}$ if $\Pr(\theta = 1|s_p) > \frac{1}{2}$, and $f^*(m, s_p) = 0\ \forall m \in \{1, ..., n\}$ if $\Pr(\theta = 1|s_p) < \frac{1}{2}$. If $\Pr(\theta = 1|s_p) = \frac{1}{2}$ (which would be the case if $\beta = \frac{1}{2}$), then all mechanisms that only depend on the public signal will lead to the same precision of the collective decision, and hence we can assume without loss of generality that either $f^*(m, s_p) = 1 \forall m \in \{1, ..., n\}$ or $f^*(m, s_p) = 0 \forall m \in \{1, ..., n\}$.

Next, suppose that in an optimal EPIC mechanism f^* we have $m_{s_p} \in \{1, ..., n\}$. Since $\alpha \geq \frac{1}{2}$, we have $\Pr(\theta = 1|m_s \geq m_{s_p}, s_p) \geq \Pr(\theta = 1|m_s < m_{s_p}, s_p)$. If $\Pr(\theta = 1|m_s \geq m_{s_p}, s_p) \geq \Pr(\theta = 1|m_s < m_{s_p}, s_p) \geq \frac{1}{2}$. In this case, conditional on the public signal s_p, the mechanism f^* will not perform strictly better than a mechanism f' with $f'(m, s_p) = 1\ \forall m \in \{1, ..., n\}$. Similarly, If $\frac{1}{2} \geq \Pr(\theta = 1|m_s \geq m_{s_p}, s_p) \geq \Pr(\theta = 1|m_s < m_{s_p}, s_p)$, the mechanism f^* will not perform strictly better than a mechanism f' with $f'(m, s_p) = 0\ \forall m \in \{1, ..., n\}$. Finally, suppose that $\Pr(\theta = 1|m_s \geq m_{s_p}, s_p) > \frac{1}{2} > \Pr(\theta = 1|m_s < m_{s_p}, s_p)$. In this case, we must have $f^*(m, s_p) = 1\ \forall m \geq m_{s_p}$ and $f^*(m, s_p) = 0\ \forall m < m_{s_p}$. Otherwise, we can either further increase $f^*(m, s_p)\ \forall m \geq m_{s_p}$ or further decrease $f^*(m, s_p)\ \forall m < m_{s_p}$, which will strictly increase $\Pr(d = \theta \mid f^*)$ without violating any incentive compatibility constraint. Clearly, this contradicts to that $f^* \in \mathcal{F}^*$.

We thus have shown that in the search of optimal EPIC mechanisms, it is without loss to focus on mechanisms that are deterministic. Since the set of feasible deterministic mechanisms is finite, an optimal EPIC mechanism exists.

To complete the proof, let us fix a preference profile $\mathbf{q}$ and consider any optimal EPIC mechanism f^* that is deterministic and is characterized by the two threshold values m_0, $m_1 \in \{0, 1, ..., n, n+1\}$. Let g^{k_0,k_1} be the contingent k-voting rule with $k_0 = m_0$ and $k_1 = m_1$. It is straightforward to check that g^{k_0,k_1} implements informative voting, since the corresponding direct mechanism f^* is EPIC. The fact that $k_0 = m_0$ and $k_1 = m_1$ makes sure that the informative voting equilibrium under g^{k_0,k_1} will achieve the same $\Pr(d = \theta \mid s, s_p)$ for every $(s, s_p) \in S \times S_p$ as the truth-telling equilibrium under mechanism f^*. The construction of k_0 and k_1 also makes it clear that g^{k_0,k_1} is responsive if and only if f^* is responsive. ■□

C.6 Proof of Proposition 3.4

To prove (i), fix a conflict-preserving sequence $\{\mathbf{q}^\tau\}_{\tau \in \mathbb{N}}$ and pick any element $\mathbf{q}^\tau$ from it. From Proposition 3.5, we know that for the preference profile $\mathbf{q}^\tau$, there exists a responsive contingent k-voting rule that implements informative voting if and only if there exists a pair of integers $k_0^\tau, k_1^\tau \in \{1, ..., n+2\tau\}$ such that

$$k_0^\tau \in K_0^\tau = \left[(\pi_1^0)_\tau^{-1}(\bar{q}^\tau), (\pi_0^0)_\tau^{-1}(\underline{q}^\tau)\right], \text{ and } k_1^\tau \in K_1^\tau = \left[(\pi_1^1)_\tau^{-1}(\bar{q}^\tau), (\pi_0^1)_\tau^{-1}(\underline{q}^\tau)\right],$$

where

$$(\pi_0^0)_\tau^{-1}(\underline{q}^\tau) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau + 2 + r\right), \quad (\pi_1^0)_\tau^{-1}(\bar{q}^\tau) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau + r\right),$$

$$(\pi_0^1)_\tau^{-1}(\underline{q}^\tau) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau + 2 - r\right), \quad (\pi_1^1)_\tau^{-1}(\bar{q}^\tau) = \frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau - r\right).$$

Let $\mathbf{q}^0 \equiv \mathbf{q}$ and suppose that there exists $k_0 \in \{1, ..., n\} \cap K_0^0$. Since $\ln\left(\frac{1-\underline{q}}{\underline{q}}\right) \leq \ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right) \leq \ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right) \leq \ln\left(\frac{1-\bar{q}}{\bar{q}}\right)$, we have $k_0 + \tau \in \{1, ..., n+2\tau\} \cap K_0^\tau$. Similarly, if there exists $k_1 \in \{1, ..., n\} \cap K_1^0$, then $k_1 + \tau \in \{1, ..., n+2\tau\} \cap K_1^\tau$. Moreover, since

$$(\pi_0^0)_\tau^{-1}(\underline{q}^\tau) - (\pi_1^0)_\tau^{-1}(\bar{q}^\tau) = \frac{\ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right) - \ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right)}{2\ln\left(\frac{1-\alpha}{\alpha}\right)} + 1, \text{and}$$

$$(\pi_0^1)_\tau^{-1}(\underline{q}^\tau) - (\pi_1^1)_\tau^{-1}(\bar{q}^\tau) = \frac{\ln\left(\frac{1-\underline{q}^\tau}{\underline{q}^\tau}\right) - \ln\left(\frac{1-\bar{q}^\tau}{\bar{q}^\tau}\right)}{2\ln\left(\frac{1-\alpha}{\alpha}\right)} + 1,$$

it is clear that both $(\pi_0^0)_\tau^{-1}(\underline{q}^\tau) - (\pi_1^0)_\tau^{-1}(\bar{q}^\tau)$ and $(\pi_0^1)_\tau^{-1}(\underline{q}^\tau) - (\pi_1^1)_\tau^{-1}(\bar{q}^\tau)$ are strictly less than one if $\underline{q}^\tau < \bar{q}^\tau$. This implies that whenever $\underline{q}^\tau < \bar{q}^\tau$, both the intervals K_0^τ and K_1^τ can contain *at most* one integer. Hence, in this case the contingent k-voting rule that can be used to implement informative voting is unique.[1]

We now proceed to prove (ii). Consider the threshold values

$$k_0^\tau = \left[\frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau + r\right)\right]^+, \quad k_1^\tau = \left[\frac{1}{2}\left(\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2\tau - r\right)\right]^+,$$

where $[x]^+$ denotes the smallest integer that is larger or equal to $x \in \mathbb{R}$. Since for the preference profile $\mathbf{q}^0 = \mathbf{q}$ there exists a responsive contingent-k voting rule that implements informative voting, from our analysis for (i) we can conclude that for every $\tau \in \mathbb{N}$ and preference profile $\mathbf{q}^\tau$, the voting rule $g^{k_0^\tau, k_1^\tau}$ implements informative voting. By Proposition 3.6, for an efficiency-maximizing social planner who can observe all the $n + 2\tau$ private signals and the public signal, it would be optimal to implement $d = 1$ if either $s_p = 0$ and there is more than $k_0^{\tau*} = \frac{n+2\tau+1}{2} + \left[\frac{r-1}{2}\right]^+$ private signals equal to 1, or $s_p = 1$ and there is more than $k_1^{\tau*} = \frac{n+2\tau++1}{2} - \left[\frac{r-1}{2}\right]^+$ private signals equal to 1. Otherwise implementing $d = 0$ would be optimal. Now consider the

[1] The intervals K_0^τ and K_1^τ will contain at least one integer (and at most two) if $\bar{q}^\tau = \underline{q}_\tau = q$. In particular, K_0^τ will contain exactly two integers if and only if $(\pi_1^0)_\tau^{-1}(q)$ is an integer. Similarly, there will be two integers in K_1^τ if and only if $(\pi_1^1)_\tau^{-1}(q)$ is an integer.

differences

$$\Delta_\tau^0 \equiv \frac{k_0^\tau - k_0^{\tau *}}{n+2\tau} = \frac{\left[\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{2\ln\left(\frac{1-\alpha}{\alpha}\right)} + \frac{r-1}{2}\right]^+ - \left[\frac{r-1}{2}\right]^+}{n+2\tau},$$

and

$$\Delta_\tau^1 \equiv \frac{k_1^\tau - k_1^{\tau *}}{n+2\tau} = \frac{\left[\frac{\ln\left(\frac{1-\bar{q}}{\bar{q}}\right)}{2\ln\left(\frac{1-\alpha}{\alpha}\right)} - \frac{r+1}{2}\right]^+ + \left[\frac{r-1}{2}\right]^+}{n+2\tau}.$$

Both Δ_τ^0 and Δ_τ^1 are decreasing in τ, and we have $\lim_{\tau\to\infty}\Delta_\tau^0 = \lim_{\tau\to\infty}\Delta_\tau^1 = 0$. This implies that as τ increases and goes to ∞, the ex ante probability that the collective decision made in the informative voting equilibria under $g^{k_0^\tau, k_0^\tau}$ coincide with the social planner's choice is increasing and converges to 1. Finally, for any preference profile $\mathbf{q}^\tau$ in the sequence, whenever a non-responsive contingent k-voting rule $g^{k_0^\tau, k_1^\tau}$ with $\{k_0^\tau, k_1^\tau\} \cap \{0, n+2\tau+1\} \neq \emptyset$ is used all the private information in the committee would be entirely disregarded for at least some realization of the public signal. This implies that the efficiency of the informative voting equilibrium under any non-responsive contingent k-voting rule would be strictly dominated by the social planner's solution.[2] Hence, for sufficiently large τ, it will also be dominated by the informative voting equilibrium under the responsive contingent k-voting rule that we constructed above. Since the preference profile $\mathbf{q}$ is assumed to be generic, we can apply Proposition 3.3 to conclude that the above responsive contingent k-voting rule is equivalent to an optimal EPIC mechanism. Therefore, the threshold $\tau^* \in \mathbb{N}$ described in the current proposition must exist.

Finally, we prove (iii). Note that $\forall q \in (0,1)$,

$$\lim_{\tau\to\infty} \frac{(\pi_1^0)_\tau^{-1}(q)}{n+2\tau} = \lim_{\tau\to\infty} \frac{(\pi_0^0)_\tau^{-1}(q)}{n+2\tau} = \lim_{\tau\to\infty} \frac{(\pi_1^1)_\tau^{-1}(q)}{n+2\tau} = \lim_{\tau\to\infty} \frac{(\pi_0^1)_\tau^{-1}(q)}{n+2\tau} = \frac{1}{2}.$$

Hence, after adding sufficiently many members to the committee, the probability that the collective decisions made in the informative voting equilibria under the corresponding responsive contingent k-voting rules coincide with that in the informative voting equilibrium under the simple majority rule becomes arbitrarily close to 1. Since the informative voting equilibrium under the simple majority rule is asymptotically efficient when $\alpha > 1/2$, so are the informative voting equilibria under a responsive contingent k-voting rules. □

C.7 Proof of Proposition 3.5

We start from establishing a lemma that characterizes when informative voting can be implemented by a *given* contingent k-voting rule with $k_0, k_1 \in \{1, ..., n\}$, which is in fact a counterpart to Proposition 3.1.

Lemma C.5. *A contingent k-voting rule with $k_0, k_1 \in \{1, ..., n\}$ implements informative voting*

[2]The assumption of the proposition implies that the public signal would not be so precise that it would be optimal for the social planner to always follow the public signal. Otherwise, for the preference profile $\mathbf{q}$ there cannot be a responsive contingent k-voting rule that implements informative voting.

if and only if

$$\forall i \in \mathcal{I},\ q_i \in \left[\max\{\pi_0^0, \pi_0^1\}, \min\{\pi_1^0, \pi_1^1\}\right], \tag{C.9}$$

where

$$\pi_0^0 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2}\frac{\beta}{1-\beta}}, \quad \pi_1^0 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n}\frac{\beta}{1-\beta}},$$
$$\pi_0^1 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2}\frac{1-\beta}{\beta}}, \quad \pi_1^1 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n}\frac{1-\beta}{\beta}}.$$

Proof of Lemma C.5. Suppose $s_p = 1$. Given a responsive contingent k-voting rule g^{k_0,k_1}, the threshold value for choosing $d = 1$ is $k_1 \in \{1, ..., n\}$. Assume all agents $j \neq i$ are playing the informative voting strategy. Conditional on being pivotal, the posterior probability that agent i would assign to the event $\theta = 1$ if $s_i = 0$ or $s_i = 1$ are, respectively:

$$\pi_0^1 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{k_1-1}\left(\frac{\alpha}{1-\alpha}\right)^{n-k_1+1}\frac{1-\beta}{\beta}} \quad \text{and} \pi_1^1 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{k_1}\left(\frac{\alpha}{1-\alpha}\right)^{n-k_1}\frac{1-\beta}{\beta}}$$
$$= \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n-2}\frac{1-\beta}{\beta}} \qquad\qquad = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_1-n}\frac{1-\beta}{\beta}}.$$

Now suppose $s_p = 0$. Under the contingent k-voting rule g^{k_0,k_1}, the threshold value for choosing the decision $d = 1$ is $k_0 \in \{1, ..., n\}$. Assume all agents $j \neq i$ are playing the informative voting strategy. Conditional on being pivotal, the posterior probability that agent i would assign to the event $\theta = 1$ if $s_i = 0$ or $s_i = 1$ are, respectively:

$$\pi_0^0 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n-2}\frac{\beta}{1-\beta}} \quad \text{and} \quad \pi_1^0 = \frac{1}{1+\left(\frac{1-\alpha}{\alpha}\right)^{2k_0-n}\frac{\beta}{1-\beta}}.$$

Hence, the voting rule g^{k_0,k_1} implements informative voting if and only if $\forall i \in \mathcal{I}$, $q_i \geq \max\{\pi_0^0, \pi_0^1\}$ and $q_i \leq \min\{\pi_1^0, \pi_1^1\}$. □

Lemma C.5 implies that for the existence of a responsive contingent k-voting rule that implements informative voting, it is necessary and sufficient that there exist $k_0, k_1 \in \{1, ..., n\}$ satisfying (C.9). To check whether such integers k_0 and k_1 exist, we first invert the functions π_0^0 and π_1^0 of k_0 and the functions π_0^1 and π_1^1 of k_1 that are defined in the above lemma. This is feasible because all these are strictly increasing functions. We then apply the inverse functions $(\pi_0^0)^{-1}$ and $(\pi_0^1)^{-1}$ to $\bar{q}$ and $(\pi_1^0)^{-1}$ and $(\pi_1^1)^{-1}$ to $\underline{q}$. It is straightforward to check that if there exist $k_0, k_1 \in \{1, ..., n\}$ such that $k_0 \in K_0 \equiv [(\pi_1^0)^{-1}(\bar{q}), (\pi_0^0)^{-1}(\underline{q})]$ and $k_1 \in K_1 \equiv [(\pi_1^1)^{-1}(\bar{q}), (\pi_0^1)^{-1}(\underline{q})]$, then k_0 and k_1 will also satisfy condition (C.9). □

C.8 Proof of Corollary 3.3

From Proposition 3.5, we know that for a given preference profile $\mathbf{q}$, there exists a responsive contingent k-voting rule g^{k_0,k_1} that implements informative voting if and only if there exist

$k_0 \in \{1,...,n\} \cap K_0$ and $k_0 \in \{1,...,n\} \cap K_1$. When $\bar{q} = \underline{q} = q \in (0,1)$, we have $(\pi_0^0)^{-1}(\underline{q}) - (\pi_1^0)^{-1}(\bar{q}) = (\pi_0^1)^{-1}(\underline{q}) - (\pi_1^1)^{-1}(\bar{q}) = 1$. Thus, in this case both the intervals K_0 and K_1 will contain *at least* one integer. It remains to show that for sufficiently large n, it is guaranteed that $\{1,...,n\} \cap K_0 \neq \emptyset$ and $\{1,...,n\} \cap K_1 \neq \emptyset$. This is not trivial because the intervals K_0 and K_1 actually also depend on n. Given the remark in footnote 1 and since $(\pi_1^1)^{-1}(q) \leq (\pi_1^0)^{-1}(q)$ and $(\pi_0^1)^{-1}(q) \leq (\pi_1^1)^{-1}(q)$ for all $q \in (0,1)$ and $r \geq 0$, the intersections $\{1,...,n\} \cap K_0$ and $\{1,...,n\} \cap K_1$ are non-empty if

$$(\pi_1^1)^{-1}(q) \geq 0 \Longleftrightarrow \frac{1}{2}\left(\frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n - r\right) \geq 0 \Longleftrightarrow n \geq r - \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}$$

and

$$(\pi_0^0)^{-1}(q) \leq n+1 \Longleftrightarrow \frac{1}{2}\left(\frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)} + n + 2 + r\right) \leq n+1 \Longleftrightarrow n \geq r + \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}.$$

Let

$$\bar{n}(q) = \left[\max\left\{r - \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}, r + \frac{\ln\left(\frac{1-q}{q}\right)}{\ln\left(\frac{1-\alpha}{\alpha}\right)}\right\}\right]^+.$$

We can now conclude that when agents' preference are perfectly aligned, there exists a threshold value $\bar{n}(q)$, such that for all $n \geq \bar{n}(q)$, there exists a responsive contingent k-voting rule that implements informative voting. □

C.9 Proof of Corollary 3.4

Plugging $k_0 = (n+1)/2 + [(r-1)/2]^+$ in the formulas of π_0^0 and π_1^0 that we obtained in Lemma C.5, one can easily verify that for all $r \geq 0$,

$$\max\{\pi_0^0, \pi_1^0\} = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{|r-2[(r-1)/2]^+|-1}}.$$

Similarly, with $k_1 = (n+1)/2 - [(r-1)/2]^+$, we have for all $r \geq 0$,

$$\min\{\pi_1^0, \pi_1^1\} = \frac{1}{1 + \left(\frac{1-\alpha}{\alpha}\right)^{-|r-2[(r-1)/2]^+|+1}}.$$

The result of the corollary thus immediately follows Lemma C.5. □

C.10 Proof of Proposition 3.6

Consider a social planner who observes the whole profile of private signals $s = (s_1, ..., s_n)$ and the public signal s_p. Suppose the public signal is r-times as informative as the private signal, where $r \geq 0$. Again we let $m_s = \sum_{i=1}^n s_i$. To maximize the accuracy of his decision, the social planner would choose the following optimal decision rule:

$$d^*(s, s_p) = \begin{cases} 1 & \text{if } m_s - (n - m_s) + r\mathbb{1}_{s_p=1} - r\mathbb{1}_{s_p=0} > 0, \\ \{0,1\} & \text{if } m_s - (n - m_s) + r\mathbb{1}_{s_p=1} - r\mathbb{1}_{s_p=0} = 0, \\ 0 & \text{if } m_s - (n - m_s) + r\mathbb{1}_{s_p=1} - r\mathbb{1}_{s_p=0} < 0. \end{cases}$$

Under the contingent majority rule, $k(s_p) = \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+$ if $s_p = 1$ and $k(s_p) = \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+$ if $s_p = 0$. First, suppose that $r > n$. In this case, the public signal is so precise that the social planner would find it optimal to always follow it and entirely ignore the private signals, i.e., $d^*(s, s_p) = s_p\ \forall s \in S$ and $s_p \in S_p$. Meanwhile, we have $k(1) > n$ and $k(0) < 1$, which means that the agents' votes would never count and the contingent majority rule simply replicates the outcome of the obedient voting equilibrium. The statement of the proposition then immediately follows.

Next, suppose $r \leq n$. When $s_p = 1$, in the informative voting equilibrium, $d = 1$ if and only if

$$m_s \geq \frac{n+1}{2} - \left[\frac{r-1}{2}\right]^+ \iff (n - m_s) - m_s \leq 2\left[\frac{r-1}{2}\right]^+ - 1 \equiv R_1,$$

while when $s_p = 0$, $d = 1$ if and only if

$$m_s \geq \frac{n+1}{2} + \left[\frac{r-1}{2}\right]^+ \iff m_s - (n - m_s) \geq 2\left[\frac{r-1}{2}\right]^+ + 1 \equiv R_0.$$

There are four possible scenarios:

1. r is an even integer: $R_1 = r - 1 < r + 1 = R_0$.

2. r is an odd integer: $R_1 = r - 2 < r = R_0$.

3. r is not an integer and $[r]^+$ is even: $R_1 = [r]^+ - 1 < [r]^+ + 1 = R_0$.

4. r is not an integer and $[r]^+$ is odd: $R_1 = [r]^+ - 2 < [r]^+ = R_0$.

Since $|m_s - (n - m_s)|$ is odd, the above four inequalities jointly show that the decision achieved by the contingent majority rule always coincides with the planner's choice. □

C.11 Proof of Proposition 3.7

Consider the following strategy profile of the agents in the two-stage voting game. In the first stage, all agents vote for k_0 if $s_p = 0$, and they all vote for k_1 if $s_p = 1$. In the second stage, if either $s_p = 0$ and $k^* = k_0$, or $s_p = 1$ and $k^* = k_1$, then all agents vote informatively. Since an unilateral deviation from the above first-stage voting strategy will not change the threshold k^* that will be selected to be used in the second stage, we need not specify the agents' contingent strategies for any other case.

We thus need only to check whether the agents indeed have the incentive to vote informatively whenever $(s_p, k^*) \in \{(0, k_0), (1, k_1)\}$. This is the case because the informative voting strategy profile actually constitutes an ex post Nash equilibrium under g^{k_0,k_1}, which implies that no agent would have the incentive to unilaterally deviate from informative voting regardless of how she updates her beliefs after observing the first stage voting outcome. Hence, in the two-stage voting game there must exist a Perfect Bayesian Nash equilibrium in which the agents first vote to agree on choosing either k_0 or k_1 (depending on whether $s_p = 0$ or $s_p = 1$), and then they all vote informatively in the second stage. $\square$

C.12 Proof of Proposition 3.8

Since the signal s_p is only observed to the controller and the vote takes place after the agents receive the message from the controller, we have a dynamic game of incomplete information. We look for sequential equilibria, which require the beliefs and the strategies of the players to be sequentially rational and consistent (Fudenberg and Tirole, 1991). Note that since the biased of the controller is not (directly) payoff-relevant to the agents, we need to keep track of agents' beliefs about the state only.

First, consider the scenario where $m = s_p$ is sent. No agent would have the incentive to deviate from obedient voting given all other agents are following the public signal revealed by the controller. This is always the case regardless of the relative precision of the signals.[3]

Now consider the information set where $m = \emptyset$ is sent and the controller's disclosure policy is to withhold his information if and only if he observes $s_p = 1$. Conditional all other agents are voting informatively, voting informatively is optimal for agent i if and only if

$$\begin{aligned} \Pr(\theta = 1 | s_i = 1, m = \emptyset) &= \frac{\frac{1}{2}\alpha(\lambda + (1-\lambda)(1-\beta))}{\frac{1}{2}\alpha(\lambda + (1-\lambda)(1-\beta)) + \frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)\beta)} \\ &\geq \frac{\frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)\beta)}{\frac{1}{2}\alpha(\lambda + (1-\lambda)(1-\beta)) + \frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)\beta)} \\ &= \Pr(\theta = 0 | s_i = 1, m = \emptyset) \end{aligned}$$

and

$$\begin{aligned} \Pr(\theta = 0 | s_i = 0, m = \emptyset) &= \frac{\frac{1}{2}\alpha(\lambda + (1-\lambda)\beta)}{\frac{1}{2}\alpha(\lambda + (1-\lambda)\beta) + \frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)(1-\beta))} \\ &\geq \frac{\frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)(1-\beta))}{\frac{1}{2}\alpha(\lambda + (1-\lambda)\beta) + \frac{1}{2}(1-\alpha)(\lambda + (1-\lambda)(1-\beta))} \\ &= \Pr(\theta = 1 | s_i = 0, m = \emptyset). \end{aligned}$$

Since $\alpha, \beta \geq 1/2$ and $\lambda > 0$, the second inequality always holds. It can be also checked that the first inequality holds if and only if $(\beta - (1-\alpha))\lambda \geq \beta - \alpha$. Hence, whenever $\lambda \geq \hat{\lambda}$, the informative voting strategy profile and the beliefs that are formed according to Bayes rule are sequentially rational for the agents at the information set $m = \emptyset$. By the same token, if the controller only reveals $s_p = 0$ to the agents, no agent can profitably deviate from the proposed strategy profile as long as $\lambda \geq \hat{\lambda}$ and beliefs are formed according to Bayes rule.

[3]In contrast, as implied by Corollary 3.1 informative voting does not constitute an equilibrium in these subgames whenever $\beta > \alpha$.

Given the strategies of the agents, suppose the controller observes $s_p = 1$. By revealing this information to the agents, his expected payoff is given by $U_c^r(1) = -q_c(1-\beta)$. On the other hand, withholding this information from the agents yields him an expected payoff of $U_c^{nr}(1) = -q_c(1-\beta)P - (1-q_c)\beta P$, where

$$P = \sum_{k=\frac{n+1}{2}}^{n} C_n^k (1-\alpha)^k \alpha^{n-k}$$

is the probability that the committee reaches a wrong decision when all agents vote informatively. Similarly, by revealing $s_p = 0$ to the agents, the controller's expected payoff is $U_c^r(0) = -(1-q_c)(1-\beta)$, while concealing this information yields him an expected payoff of $U_c^{nr}(0) = -q_c\beta P - (1-q_c)(1-\beta)P$. Hence, the controller would find it optimal to reveal $s_p = 1$ and withhold $s_p = 0$ if $U_c^r(1) \geq U_c^{nr}(1)$ and $U_c^{nr}(0) \geq U_c^r(0)$, which, after some rearrangements, are equivalent to

$$q_c \leq \hat{q} = \min\left\{\frac{\beta P}{(1-\beta)(1-P)+\beta P}, \frac{(1-\beta)(1-P)}{(1-\beta)(1-P)+\beta P}\right\}.$$

Similarly, revealing $s_p = 0$ and withholding $s_p = 1$ is optimal for the controller if

$$q_c \geq 1-\hat{q} = \max\left\{\frac{\beta P}{(1-\beta)(1-P)+\beta P}, \frac{(1-\beta)(1-P)}{(1-\beta)(1-P)+\beta P}\right\}.$$

The threshold value $\hat{q}$ achieves its supremum at $P = 1-\beta$, which equals to $1/2$. Also, since $\alpha > 1/2$, it is straightforward to check that $P < 1/2$ and, hence, $\hat{q} \geq \min\{\beta, 1-\beta\} = 1-\beta$.

In conclusion, if $\lambda \geq \hat{\lambda}$ and $q_c \leq \hat{q}$ (or $q_c \geq 1-\hat{q}$), the strategy profile stated in the proposition together with the beliefs formed according to Bayes rule constitute a Perfect Bayesian Equilibrium. Since the signal s_p is verifiable and the information set with $m = \emptyset$ can be reached with positive probability, it is also a sequential equilibrium. $\square$

C.13 Proof of Proposition 3.9

First, consider the scenario where $m = s_p$ is sent. By the same reasoning as in Corollary 3.4, no agent would have the incentive to deviate from informative voting given all other agents are voting informatively.

Next, consider the information set where $m = \emptyset$ is sent and suppose the strategy of controller is such that he never withholds information. In this case, the controller's message is not informative at all and given that the agents are unbiased and the voting threshold corresponds to the simple majority rule, the informative voting strategy profile along with the beliefs formed according to Bayes rule are clearly sequentially rational for the agents. Now suppose the controller's strategy is such that he will reveal his information to the agents if and only if $s_p = 1$ (or $s_p = 0$). By Proposition 3.8, we know that in this case no agent can profitably deviate from informative voting provided that $\lambda \geq \hat{\lambda}$.

Given that the agents will always vote informatively, suppose that the controller observes $s_p = 1$. By withholding the signal, the controller obtains an expected payoff of $U_c^{nr}(1) =$

$-q_c(1-\beta)P - (1-q_c)\beta P$, while revealing yields $U_c^r(1) = -q_c(1-\beta)P' - (1-q_c)\beta\tilde{P}$, where

$$P' = \sum_{k=\frac{n+1}{2}-\left[\frac{r-1}{2}\right]^+}^{n} C_n^k(1-\alpha)^k\alpha^{n-k}, \;\; \tilde{P} = \sum_{k=\frac{n+1}{2}+\left[\frac{r-1}{2}\right]^+}^{n} C_n^k(1-\alpha)^k\alpha^{n-k}.$$

Similarly, by concealing $s_p = 0$ from the agents, the controller's expected payoff is given by $U_c^{nr}(0) = -q_c\beta P - (1-q_c)(1-\beta)P$, while revealing yields him an expected payoff of $U_c^r(0) = -q_c\beta\tilde{P} - (1-q_c)(1-\beta)P'$. Hence, the controller would find it optimal to always share his information with the agents if $U_c^r(1) \geq U_c^{nr}(1)$ and $U_c^r(0) \geq U_c^{nr}(0)$. Note that these inequalities trivially hold for all $q_c \in [0,1]$ if $[(r-1)/2]^+ = 0$ or, equivalently, $\beta \leq \alpha$. Now suppose $[(r-1)/2]^+ \geq 1$. Then, it is straightforward to check that these two inequalities are satisfied if and only if $q_c \in [q^*, 1-q^*]$, where

$$q^* = \frac{(1-\beta)(P'-P)}{\beta(P-\tilde{P}) + (1-\beta)(P'-P)} \geq 0.$$

Since $\alpha \geq 1/2$ and $C_n^k = C_n^{n-k}$, we have

$$\frac{P-\tilde{P}}{P'-P} = \frac{\sum_{k=\frac{n+1}{2}}^{\frac{n+1}{2}+\left[\frac{r-1}{2}\right]^+-1} C_n^k(1-\alpha)^k\alpha^{n-k}}{\sum_{k=\frac{n+1}{2}-\left[\frac{r-1}{2}\right]^+}^{\frac{n+1}{2}-1} C_n^k(1-\alpha)^k\alpha^{n-k}} \leq 1$$

and, hence, $q^* \leq 1-\beta$. Clearly, together with the beliefs formed according to Bayes rule, the proposed strategies for the controller (always share his information) and the agents (always vote informatively) constitute a Perfect Bayesian equilibrium. It is also a sequential equilibrium since all information set can be reached with positive probability in equilibrium. We thus have proven the first part of the proposition. The proof of the second part of the proposition is analogous, and we omit it here to avoid repetition. $\square$

D Appendix: Chapter 4

D.1 Proof of Proposition 4.1

Part (i) First, consider agent i's incentives in the decision-making stage. Taking (e_i, s_i, m_i, m_j) as given, in the decision-making stage agent i solves:

$$\max_{y_i \in \mathbb{R}} \; q\left(K - \mathbb{E}\left[(y_i - \theta_i)^2 \,|s_i\right] - \delta \mathbb{E}\left[(y_i - y_j^d(e_j, s_j, m_i, m_j))^2 | m_i, m_j\right]\right).$$

Sequential rationality then implies that agent i's should take the following action:

$$y_i = \frac{\mathbb{E}\left[\theta_i|s_i\right] + \delta \mathbb{E}\left[y_j^d(e_j, s_j, m_i, m_j)|m_i, m_j\right]}{1+\delta}.$$

Note that the best response of the agent does not depend on his sunk effort e_i. From now on, we drop (e_i, e_j) from the functions (y_i^d, y_j^d) as they play no role. Solving the best response functions through repeated substitution, we obtain the following decision rules which must be satisfied in any equilibrium:

$$y_i^d(s_i, m_i, m_j) = \frac{\mathbb{E}\left[\theta_i|s_i\right]}{1+\delta} + \frac{\delta^2 \mathbb{E}[\theta_i|m_i]}{(1+\delta)(1+2\delta)} + \frac{\delta \mathbb{E}[\theta_j|m_j]}{1+2\delta}, \; \forall i, j = 1, 2, i \neq j. \tag{D.1}$$

where the conditional expectations $\mathbb{E}[\theta_i|s_i]$ and $\mathbb{E}[\theta_j|m_j]$ ($\mathbb{E}[\theta_j|s_j]$ and $\mathbb{E}[\theta_i|m_i]$, resp.) are taken according to the agent i's (agent j's, resp.) posterior beliefs about the local states.

Now suppose that agent i anticipates that agent j will exert some arbitrary effort $e_j \in [0, 1]$, communicate his finding truthfully according to the strategy m_j^d specified in the proposition, and choose his action according to the mapping y_j^d specified in (D.1). Taking the the sequentially rational decision rule y_i^d as given, we consider agent i's incentive in the communication stage. Since by construction (m_i^d, m_j^d) are effort-independent, we drop the variables (e_i, e_j) from them. To ease notation, we also assume without loss of generality that $m^{s_i} = s_i \; \forall s_i \in \Theta$.

Let $s_i \in \mathcal{S}$ be the signal received by agent i. For any message $m_i \in \mathcal{M}(s_i)$, we have

$$\begin{aligned}
EL_a^d(s_i, m_i) =& \mathbb{E}_{s_j}\left[\mathbb{E}\left[\left(y_i^d(s_i, m_i, m_j^d(s_j)) - \theta_i\right)^2 \Big| s_i\right]\right] \\
=& \mathbb{E}_{s_j}\left[\mathbb{E}\left[\left(\frac{\mathbb{E}[\theta_i|s_i]}{1+\delta} + \frac{\delta^2 \mathbb{E}[\theta_i|m_i]}{(1+\delta)(1+2\delta)} + \frac{\delta \mathbb{E}[\theta_j|m_j^d(s_j)]}{1+2\delta} - \theta_i\right)^2 \Bigg| s_i\right]\right] \\
=& \mathbb{E}\left[\left(\frac{\mathbb{E}[\theta_i|s_i]}{1+\delta} + \frac{\delta^2 \mathbb{E}[\theta_i|m_i]}{(1+\delta)(1+2\delta)} - \theta_i\right)^2 \Bigg| s_i\right] + \mathbb{E}_{s_j}\left[\left(\frac{\delta \mathbb{E}[\theta_j|s_j]}{1+2\delta}\right)^2\right],
\end{aligned} \tag{D.2}$$

where the last equality follows that $\mathbb{E}_{s_j}\left[\mathbb{E}[\theta_j|s_j]\right] = \mathbb{E}[\theta_j] = 0$.

Similarly, for the expected loss of mis-coordination resulted by any message m_i, we have

$$\begin{aligned}
&EL_c^d(s_i, m_i)\\
=&\mathbb{E}_{s_j}\left[\mathbb{E}[(y_i^d(s_i, m_i, m_j^d(s_j)) - y_j^d(s_j, m_i, m_j^d(s_j)))^2|s_i]\right]\\
=&\mathbb{E}_{s_j}\left[\mathbb{E}\left[\left(\frac{\mathbb{E}[\theta_i|s_i]}{1+\delta} - \frac{\delta\mathbb{E}[\theta_i|m_i]}{(1+\delta)(1+2\delta)} - \frac{\mathbb{E}[\theta_j|s_j]}{1+\delta} + \frac{\delta\mathbb{E}[\theta_j|m_j^d(s_j)]}{(1+\delta)(1+2\delta)}\right)^2\middle|s_j\right]\right]\\
=&\mathbb{E}\left[\left(\frac{\mathbb{E}[\theta_i|s_i]}{1+\delta} - \frac{\delta\mathbb{E}[\theta_i|m_i]}{(1+\delta)(1+2\delta)}\right)^2\middle|s_i\right] + \mathbb{E}_{s_j}\left[\left(\frac{\mathbb{E}[\theta_j|s_j]}{1+\delta} - \frac{\delta\mathbb{E}[\theta_j|s_j]}{(1+\delta)(1+2\delta)}\right)^2\right]\\
=&\mathbb{E}\left[\left(\frac{(1+\delta)\mathbb{E}[\theta_i|s_i] + \delta(\mathbb{E}[\theta_i|s_i] - \mathbb{E}[\theta_i|m_i])}{(1+\delta)(1+2\delta)}\right)^2\middle|s_i\right] + \mathbb{E}_{s_j}\left[\left(\frac{\mathbb{E}\left[\theta_j|s_j\right]}{1+2\delta}\right)^2\right].
\end{aligned} \tag{D.3}$$

Since for every $(s_i, m_i) \in \mathcal{S} \times \mathcal{M}(s_i)$ the (interim) expected payoff of agent i is given by

$$\hat{\Pi}_i^d(s_i, m_i) = q\left(K - EL_a^d(s_i, m_i) - \delta EL_c^d(s_i, m_i)\right),$$

communicating according to m_i^d is incentive compatible for agent i *if* under some consistent posterior beliefs of agent j, we have

$$EL_a^d(s_i, s_i) \le EL_a^d(s_i, m_i) \text{ and } EL_c^d(s_i, s_i) \le EL_c^d(s_i, m_i),\ \forall m_i \in \mathcal{M}(s_i), \forall s_i \in \mathcal{S}. \tag{D.4}$$

To construct the required posterior beliefs, for every $m_i \in \mathcal{M}$ we let agent j assign probability one to that agent i's type is $\underline{s}^{m_i} \in \arg\min_{s_i \in \mathcal{S}^{m_i}} |\mathbb{E}[\theta_i|s_i]|$, i.e., $\mu_j^i(\{\underline{s}^{m_i}|m_i\}) = 1$. If $\emptyset \in \mathcal{S}^{m_i}$, the existence of $\underline{s}^{m_i}$ is trivial. If $\emptyset \notin \mathcal{S}^{m_i}$, the existence of $\underline{s}^{m_i}$ is guaranteed by the assumption that $\mathcal{S}^{m_i}$ is closed. This is because $\min_{s_i \in \mathcal{S}^{m_i}} |\mathbb{E}[\theta_i|s_i]| = \min_{s_i \in \mathcal{S}^{m_i} \cap [-|s_i'|, |s_i'|]} |\mathbb{E}[\theta_i|s_i]|$, where s_i' is any element of $\mathcal{S}^{m_i}$, and the set $\mathcal{S}^{m_i} \cap [-|s_i'|, |s_i'|]$ is compact. In addition, by construction we have $\underline{s}^{s_i} = s_i\ \forall s_i \in \mathcal{S}$. Given the constructed beliefs, we have $\mathbb{E}[\theta_i|\emptyset] = \mathbb{E}[\theta_i|m_i] = 0$ $\forall m_i \in \mathcal{M}(\emptyset)$, $\mathbb{E}[\theta_i|s_i] = s_i \ge \mathbb{E}[\theta_i|m_i]$ $\forall s_i \ge 0$ and $m_i \in \mathcal{M}(s_i)$, and $\mathbb{E}[\theta_i|s_i] = s_i \le \mathbb{E}[\theta_i|m_i]$ $\forall s_i < 0$ and $m_i \in \mathcal{M}(s_i)$. It is then straightforward to check that (D.4) is satisfied.

To complete the construction of a fully revealing PBE, we finally consider the information acquisition stage. Given the communication strategies (m_1^d, m_2^d), the decision rules (y_1^d, y_2^d), and any pair of efforts $(e_1, e_2) \in E^2$, agent i's expected payoff is

$$\begin{aligned}
&U_i^d(e_i, e_j)\\
=&q\Bigg(K - (1-e_i)\left[(1-e_j) + e_j\left(\frac{\delta^2+\delta}{(1+2\delta)^2} + 1\right)\right]\sigma_\theta^2\\
&- e_i\left[(1-e_j)\left(\frac{\delta^2+\delta}{(1+2\delta)^2}\right) + e_j\left(\frac{2\delta^2+2\delta}{(1+2\delta)^2}\right)\right]\sigma_\theta^2\Bigg) - c(e_i)\\
=&q\left(K - (1-e_i)\left[1 + e_j\left(\frac{\delta^2+\delta}{(1+2\delta)^2}\right)\right]\sigma_\theta^2 - e_i(1+e_j)\left(\frac{\delta^2+\delta}{(1+2\delta)^2}\right)\sigma_\theta^2\right) - c(e_i)\\
=&q\left(K - \left[1 + e_j\left(\frac{\delta^2+\delta}{(1+2\delta)^2}\right)\right]\sigma_\theta^2 + e_i\left(1 - \frac{\delta^2+\delta}{(1+2\delta)^2}\right)\sigma_\theta^2\right) - c(e_i).
\end{aligned}$$

Differentiating with respect to e_i, we obtain the following first-order condition:

$$\frac{\partial U_i^d(e_i, e_j)}{\partial e_i} = \left(1 - \frac{\delta^2 + \delta}{(1 + 2\delta)^2}\right) q\sigma_\theta^2 - c'(e_i) = 0. \tag{D.5}$$

When the cost function c is strictly increasing, and satisfies $\lim_{e \to 0} c'(e) < \left(1 - \frac{\delta^2+\delta}{(1+2\delta)^2}\right) q\sigma_\theta^2 < c'(1)$, (D.5) will admit a unique interior solution $e_i^d \in (0, 1)$, which is given by

$$e_i^d = e^d \equiv (c')^{-1}\left(\left(1 - \frac{\delta^2 + \delta}{(1 + 2\delta)^2}\right) q\sigma_\theta^2\right). \tag{D.6}$$

In addition, when c is strictly convex, the function U_i^d will be strictly concave in e_i, and thus the solution $e_i = e^d$ is also the unique global maximizer of $U_i^d(e_i, e_j)$, $\forall e_j \in [0, 1]$. We have assumed that the cost function c satisfies all these properties (see Section 4.3).

Similarly, choosing $e_j = e^d$ also maximizes the expected payoff of agent j independent of the effort choice of agent i. We can therefore conclude that, together with the "conservative" beliefs that we construct above for the agents, the symmetric strategy profile $((e^d, m_1^d, y_1^d), (e^d, m_2^d, y_2^d))$ constitutes a fully revealing PBE. ■

Part (ii) Let $((e_1^*, m_1^*, y_1^*), (e_2^*, m_2^*, y_2^*))$ be an equilibrium strategy profile under decentralization. Consider any $s_i \in \mathcal{S} \setminus \{0, \emptyset\}$. Repeating the calculations of (D.1), (D.2) and (D.3), it can be checked that agent i would strictly prefer the type-revealing message $m_i = m^{s_i}$ than the proposed equilibrium message $m_i^*(e_i^*, s_i)$ *if* both of the following two inequalities hold:

$$\begin{aligned}
&\mathbb{E}\left[\left(\frac{\mathbb{E}[\theta_i|s_i]}{1+\delta} + \frac{\delta^2\mathbb{E}[\theta_i|m_i^*(e_i^*, s_i)]}{(1+\delta)(1+2\delta)} - \theta_i\right)^2 \middle| s_i\right] \\
> &\mathbb{E}\left[\left(\frac{\mathbb{E}[\theta_i|s_i]}{1+\delta} + \frac{\delta^2\mathbb{E}[\theta_i|m_i^*(e_i^*, s_i)]}{(1+\delta)(1+2\delta)} - \theta_i\right)^2 \middle| s_i\right].
\end{aligned} \tag{D.7}$$

and

$$\begin{aligned}
&\mathbb{E}\left[\left(\frac{(1+\delta)\mathbb{E}[\theta_i|s_i] + \delta(\mathbb{E}[\theta_i|s_i] - \mathbb{E}[\theta_i|m_i^*(e_i^*, s_i)])}{(1+\delta)(1+2\delta)}\right)^2 \middle| s_i\right] \\
> &\mathbb{E}\left[\left(\frac{(1+\delta)\mathbb{E}[\theta_i|s_i] + \delta(\mathbb{E}[\theta_i|s_i] - \mathbb{E}[\theta_i|m^{s_i}])}{(1+\delta)(1+2\delta)}\right)^2 \middle| s_i\right].
\end{aligned} \tag{D.8}$$

Note that for any $s_i \neq \emptyset$, (D.7) is further equivalent to

$$\left(\frac{\delta(1+\delta)s_i + \delta^2(s_i - \mathbb{E}[\theta_i|m_i^*(e_i^*, s_i)])}{(1+\delta)(1+2\delta)}\right)^2 > \left(\frac{\delta(1+\delta)s_i + \delta^2(s_i - \mathbb{E}[\theta_i|m^{s_i}])}{(1+\delta)(1+2\delta)}\right)^2. \tag{D.9}$$

From (D.8) and (D.9), it is clear that if $s_i > 0$, then deviating to m^{s_i} is not profitable for agent i only if $s_i \leq \mathbb{E}[\theta_i|m_i^*(e_i^*, s_i)]$. Similarly, if $s_i < 0$, then deviating to m^{s_i} is not profitable for agent i only if $s_i \geq \mathbb{E}[\theta_i|m_i^*(e_i^*, s_i)]$. These arguments also imply that we must have $m_i^*(e_i^*, s_i) \neq m_i^*(e_i^*, s_i')$ $\forall s_i, s_i' \in \mathcal{S} \setminus \{0, \emptyset\}$ such that $s_i \cdot s_i' < 0$.

Next, suppose, in contradiction to Proposition 4.1(ii), that there exist $i \in \{1,2\}$ and a non-null subset $\hat{\mathcal{S}} \subseteq \mathcal{S} \setminus \{0, \emptyset\}$ with respect to Γ, such that $\mu_j^i(\{\hat{s}_i\}|m_i^*(e_i^*, \hat{s}_i)) < 1 \; \forall \hat{s}_i \in \hat{\mathcal{S}}$.[1] Since the beliefs must be consistent in equilibrium, we have $m_i^*(e_i^*, \hat{s}_i) \neq m^{\hat{s}_i} \; \forall \hat{s}_i \in \hat{\mathcal{S}}$. For every on-path equilibrium message $\hat{m}_i^*$ that is sent by some $\hat{s}_i \in \hat{\mathcal{S}}$, define $\hat{\mathcal{S}}(\hat{m}_i^*) = \{s_i \in \hat{\mathcal{S}} : m_i^*(e_i^*, s_i) = \hat{m}_i^*\}$. Let $\hat{\mathcal{M}}^*$ be the set of all such messages $\hat{m}_i^*$.

We claim that the set $\hat{\mathcal{S}}(\hat{m}_i^*)$ is null with respect to Γ for all $\hat{m}_i^* \in \hat{\mathcal{M}}^*$. This is because if $\hat{\mathcal{S}}(\hat{m}_i^*)$ is non-null with respect to Γ for some $\hat{m}_i^*$, the condition $\mu_j^i(\{\hat{s}_i\}|m_i^*(e_i^*, \hat{s}_i)) < 1 \, \forall \hat{s}_i \in \hat{\mathcal{S}}$ would imply that there exists $s_i \in \hat{\mathcal{S}}(\hat{m}_i^*)$ such that either $s_i > \max\{0, \mathbb{E}[\theta_i|\hat{m}_i^*]\}$ or $s_i < \min\{0, \mathbb{E}[\theta_i|\hat{m}_i^*]\}$ holds. This is not possible given our analysis of (D.8) and (D.9).

Since $\hat{\mathcal{S}}(\hat{m}_i^*)$ is null with respect to Γ for all $\hat{m}_i^* \in \hat{\mathcal{M}}^*$, Bayes' rule implies that for every $\hat{m}_i^* \in \hat{\mathcal{M}}^*$ there must exist an atom $s^{\hat{m}_i^*} \in \mathcal{S}$ in the distribution Γ, such that $m_i^*(e_i^*, s^{\hat{m}_i^*}) = \hat{m}_i^*$ and $\mu_j^i(\{s^{\hat{m}_i^*}\}|\hat{m}_i^*) = 1$. Note that by construction, each $\hat{m}_i^* \in \hat{\mathcal{M}}^*$ is associated with a different atom. However, since $\hat{\mathcal{S}} = \cup_{\hat{m}_i^*} \hat{\mathcal{S}}(\hat{m}_i^*)$ is non-null with respect to Γ, the set $\hat{\mathcal{M}}^*$ must be uncountable, and this would imply that the distribution Γ admits uncountably many atoms. We thus reach a contradiction. ■□

D.2 Proof of Proposition 4.2

Let $((e_1^*, m_1^*, y_1^*), (e_2^*, m_2^*, y_2^*))$ be a fully revealing equilibrium under decentralization. Given the full revelation, we have $\mathbb{E}[\theta_i|m_i^*(e_i^*, s_i)] = \mathbb{E}[\theta_i|s_i] \; \forall s_i \in \mathcal{S}$ and $\forall i = 1, 2$. Then, (D.1) implies that the on-path equilibrium decision rules are uniquely pinned down by Bayes' rule and sequential rationality, and they are the exactly ones given by the proposition. As we have shown in Proposition 4.1(i), given the equilibrium decisions are taken according to $(y_1^d(\mathbf{s}), y_1^d(\mathbf{s}))$, e^d is the unique expected-payoff-maximizing effort level for both agents. Hence, we must have $e_i^* = e^d$ and $y_i^*(e_i^*, s_i, m_i^*(e_i^*, s_i), m_j^*(e_j^*, s_j)) = y_i^d(s_i, s_j)$, $\forall (s_i, s_j) \in \mathcal{S}^2$, $i = 1, 2$. □

D.3 Proof of Proposition 4.3

Part (i) First, consider the principal's incentive in the decision-making stage. Taking (m_1, m_2) and (η_1, η_2) as given, in the decision-making stage the principal solves:

$$\max_{y_1, y_2 \in \mathbb{R}} (\eta_1 + \eta_2) \left(K - \delta(y_1 - y_2)^2 \right) - \eta_1 \mathbb{E}\left[(y_1 - \theta_1)^2 | m_1 \right] - \eta_2 \mathbb{E}\left[(y_2 - \theta_2)^2 | m_2 \right].$$

The first-order conditions imply that at optimum the principal's actions (y_1, y_2) must solve the following system of equations:

$$\begin{aligned} &-\delta(\eta_1 + \eta_2)(y_1 - y_2) - \eta_1 \left(y_1 - \mathbb{E}\left[\theta_1|m_1\right] \right) = 0, \\ &-\delta(\eta_1 + \eta_2)(y_2 - y_1) - \eta_1 \left(y_2 - \mathbb{E}\left[\theta_2|m_2\right] \right) = 0. \end{aligned}$$

[1] Formally, we say that a set $\hat{S} \subseteq \mathcal{S}$ is non-null with respect to Γ if $\int_{\mathcal{S}} \mathbb{1}_{\{s \in \hat{S}\}} d\Gamma > 0$, and it is null with respect to Γ if $\int_{\mathcal{S}} \mathbb{1}_{\{s \in \hat{S}\}} d\Gamma = 0$.

Solving the above equations, we obtain the following decision rules which must be satisfied in any equilibrium:

$$y_i^c(\boldsymbol{m},\boldsymbol{\eta}) = \frac{\frac{\eta_i}{\eta_i+\eta_j}\cdot\left(\frac{\eta_j}{\eta_i+\eta_j}+\delta\right)\mathbb{E}\left[\theta_i|m_i\right]+\frac{\delta\eta_j}{\eta_i+\eta_j}\mathbb{E}[\theta_j|m_j]}{\frac{\eta_i}{\eta_i+\eta_j}\cdot\frac{\eta_j}{\eta_i+\eta_j}+\delta} \quad \forall i=1,2, \tag{D.10}$$

where the conditional expectations $\mathbb{E}[\theta_i|m_i]$ and $\mathbb{E}[\theta_j|m_j]$ are taken according to the principal posterior beliefs about the local states.

Next, we take the above decision rules (y_1^c, y_2^c) of the principal as given and consider the agents' incentives in the communication stage. We will only verify the equilibrium incentives of agent 1, as for agent 2 the problem is analogous. Suppose that agent 1 anticipates that agent 2 will exert some arbitrary effort $e_2 \in [0,1]$ and communicate his finding truthfully according to the strategy m_2^c specified in the proposition. Since by construction (m_1^c, m_2^c) are effort-independent, we drop the variables (e_1, e_2) from them. To ease notation, we also assume without loss of generality that $m^s = s \; \forall s \in \Theta$.

Let $s_1 \in \mathcal{S}$ be the signal received by agent 1. Letting $\lambda = \eta_1/(\eta_1+\eta_2)$, for every message $m_1 \in \mathcal{M}(s_1)$ and every $\boldsymbol{\eta} \in \left[\underline{\eta}, \bar{\eta}\right]^2$ we have

$$\begin{aligned}
&EL_a^c(s_1, m_1, \boldsymbol{\eta}) \\
&= \mathbb{E}_{s_2}\left[\mathbb{E}\left[(y_1^c(m_1, m_2^c(s_2), \boldsymbol{\eta}) - \theta_1)^2 \middle| s_1\right]\right] \\
&= \mathbb{E}_{s_2}\left[\mathbb{E}\left[\left(\frac{\lambda\left(1-\lambda+\delta\right)\mathbb{E}\left[\theta_1|m_1\right]+\delta(1-\lambda)\mathbb{E}[\theta_2|m_2^c(s_2)]}{\lambda(1-\lambda)+\delta}-\theta_1\right)^2 \middle| s_1\right]\right] \\
&= \mathbb{E}\left[\left(\frac{(\lambda(1-\lambda)+\lambda\delta)\,\mathbb{E}\left[\theta_1|m_1\right]}{\lambda(1-\lambda)+\delta}-\theta_1\right)^2 \middle| s_1\right] + \mathbb{E}_{s_2}\left[\left(\frac{\delta(1-\lambda)\mathbb{E}[\theta_2|s_2]}{\lambda(1-\lambda)+\delta}\right)^2\right],
\end{aligned} \tag{D.11}$$

where the last equality follows that $\mathbb{E}_{s_2}\left[\mathbb{E}[\theta_2|s_2]\right] = \mathbb{E}[\theta_2] = 0$.

Similarly, for the expected loss of mis-coordination, we have for every $m_1 \in \mathcal{M}(s_1)$ and every $\boldsymbol{\eta} \in \left[\underline{\eta}, \bar{\eta}\right]^2$,

$$\begin{aligned}
EL_c^c(s_1, m_1, \boldsymbol{\eta}) &= \mathbb{E}_{s_2}\left[\mathbb{E}[(y_1^c(m_1, m_2^c(s_2), \boldsymbol{\eta}) - y_2^c(m_1, m_2^c(s_2)))^2|s_1]\right] \\
&= \mathbb{E}_{s_2}\left[\left(\frac{\lambda(1-\lambda)\mathbb{E}[\theta_1|m_1]-\lambda(1-\lambda)\mathbb{E}[\theta_2|m_2^c(s_2)]}{\lambda(1-\lambda)+\delta}\right)^2\right] \\
&= \left(\frac{\lambda(1-\lambda)\mathbb{E}[\theta_1|m_1]}{\lambda(1-\lambda)+\delta}\right)^2 + \mathbb{E}_{s_2}\left[\left(\frac{\lambda(1-\lambda)\mathbb{E}[\theta_2|s_2]}{\lambda(1-\lambda)+\delta}\right)^2\right].
\end{aligned} \tag{D.12}$$

where the last equality follows that $\mathbb{E}_{s_2}\left[\mathbb{E}[\theta_2|s_2]\right] = \mathbb{E}[\theta_2] = 0$.

Since for every $(s_1, m_1) \in \mathcal{S} \times \mathcal{M}(s_1)$, the interim expected payoff of agent 1 is given by

$$\hat{\Pi}_i^c(s_1, m_1) = \mathbb{E}_{\boldsymbol{\eta}}\left[q\left(K - EL_a^c(s_1, m_1, \boldsymbol{\eta}) - \delta EL_c^c(s_1, m_1, \boldsymbol{\eta})\right)\right],$$

communicating according to m_1^c is incentive compatible for agent 1 *if* under some consistent posterior beliefs of the principal, we have

$$EL_a^c(s_1, s_1, \boldsymbol{\eta}) + \delta EL_c^c(s_1, s_1, \boldsymbol{\eta}) \le EL_a^c(s_1, m_1, \boldsymbol{\eta}) + \delta EL_c^c(s_1, m_1, \boldsymbol{\eta}), \tag{D.13}$$

for all $s_1 \in \mathcal{S}$, $m_1 \in \mathcal{M}(s_1)$, $\boldsymbol{\eta} \in \left[\underline{\eta}, \bar{\eta}\right]^2$. We note that after some rearrangement, (D.13) is equivalent to

$$\begin{aligned}
&\left(\frac{(\lambda(1-\lambda)+\lambda\delta)\mathbb{E}[\theta_1|\emptyset]}{\lambda(1-\lambda)+\delta}\right)^2 + \delta\left(\frac{\lambda(1-\lambda)\mathbb{E}[\theta_1|\emptyset]}{\lambda(1-\lambda)+\delta}\right)^2 \\
\le &\left(\frac{(\lambda(1-\lambda)+\lambda\delta)\mathbb{E}[\theta_1|m_1]}{\lambda(1-\lambda)+\delta}\right)^2 + \delta\left(\frac{\lambda(1-\lambda)\mathbb{E}[\theta_1|m_1]}{\lambda(1-\lambda)+\delta}\right)^2
\end{aligned} \tag{D.14}$$

if $s_1 = \emptyset$. If $s_1 \neq \emptyset$, then (D.13) is equivalent to

$$\begin{aligned}
&\left(\frac{(\lambda(1-\lambda)+\lambda\delta)s_1}{\lambda(1-\lambda)+\delta} - s_1\right)^2 + \delta\left(\frac{\lambda(1-\lambda)s_1}{\lambda(1-\lambda)+\delta}\right)^2 \\
\le &\left(\frac{(\lambda(1-\lambda)+\lambda\delta)\mathbb{E}[\theta_1|m_1]}{\lambda(1-\lambda)+\delta} - s_1\right)^2 + \delta\left(\frac{\lambda(1-\lambda)\mathbb{E}[\theta_1|m_1]}{\lambda(1-\lambda)+\delta}\right)^2.
\end{aligned} \tag{D.15}$$

Since $\mathbb{E}[\theta_1|\emptyset] = 0$, (D.14) always holds regardless of the principal's beliefs. To show (D.15), we construct the following consistent beliefs for the principal: for every $m_1 \in \mathcal{M}$ we let the principal assign probability one to that agent 1's type is $\underline{s}^{m_1} \in \arg\min_{s_1 \in \mathcal{S}^{m_1}} |\mathbb{E}[\theta_1|s_1]|$, i.e., $\mu_p^1(\{\underline{s}^{m_1}|m_1\}) = 1$. The existence of $\underline{s}^{m_1}$ is guaranteed by the assumption that $\mathcal{S}^{m_1}$ is closed. Also, by construction $\underline{s}^{s_1} = s_1\ \forall s_1 \in \mathcal{S}$. Given the constructed beliefs, we have $\mathbb{E}[\theta_1|\emptyset] = \mathbb{E}[\theta_1|m_1] = 0\ \forall m_1 \in \mathcal{M}(\emptyset)$, $\mathbb{E}[\theta_1|s_1] = s_i \ge \mathbb{E}[\theta_1|m_1]\ \forall s_1 \ge 0$ and $m_1 \in \mathcal{M}(s_1)$, and $\mathbb{E}[\theta_1|s_1] = s_1 \le \mathbb{E}[\theta_1|m_1]\ \forall s_1 < 0$ and $m_1 \in \mathcal{M}(s_1)$.

Next, note that the RHS of (D.15) can be rewritten as the sum of the following two terms:

$$\frac{(\lambda(1-\lambda)+\lambda\delta)^2}{(\lambda(1-\lambda)+\delta)^2}\left(\mathbb{E}[\theta_1|m_1] - s_1\right)^2 + \frac{(1-\lambda)^2\delta^2}{(\lambda(1-\lambda)+\delta)^2}s_1^2 \tag{D.16}$$

and

$$-\frac{2\delta(1-\lambda)(\lambda(1-\lambda)+\delta)}{(\lambda(1-\lambda)+\delta)^2}\left(\mathbb{E}[\theta_1|m_1] - s_1\right)s_1 + \frac{\delta\lambda^2(1-\lambda)^2}{(\lambda(1-\lambda)+\delta)^2}\mathbb{E}[\theta_1|m_1]^2. \tag{D.17}$$

We claim that $\forall s_1 \in \Theta$, $\lambda \in [0,1]$ and $\delta > 0$, both of these two terms are minimized when $\mathbb{E}[\theta_1|m_1] = s_1$, which is in turn sufficient for (D.15) to hold for all $s_1 \neq \emptyset$. For the first term (D.16), this is straightforward. For the second term (D.17), we note that

$$\begin{aligned}
&-2\delta(1-\lambda)(\lambda(1-\lambda)+\delta)\mathbb{E}[\theta_1|m_1]s_1 + \delta\lambda^2(1-\lambda)^2\mathbb{E}[\theta_1|m_1]^2 \\
= &-\delta\lambda(1-\lambda)^2\left(2s_1 - \lambda\mathbb{E}[\theta_1|m_1]\right)\mathbb{E}[\theta_1|m_1] - 2\delta^2(1-\lambda)\mathbb{E}[\theta_1|m_1]s_1,
\end{aligned}$$

and that the function $v(x) = -\delta\lambda(1-\lambda)^2 \cdot (2s_1 - \lambda x)x$ is decreasing in x when $x \le \bar{x}$ and $s_1 \ge 0$, and it is increasing if $x \ge s_1$ and $s_1 < 0$. Hence, given the beliefs we construct for the principal, (D.17) is also minimized when $\mathbb{E}[\theta_1|m_1] = s_1$.

In sum, we have shown that the truthful-telling constraint (D.13) holds for every pair

$\boldsymbol{\eta} \in \left[\underline{\eta}, \bar{\eta}\right]^2$. In other words, it is a best response for agent 1 to reveal his type even when the distribution of $\boldsymbol{\eta}$ is *deterministic*. Therefore, the same must also hold for arbitrary non-deterministic distribution of $\boldsymbol{\eta}$.

To complete the construction of a fully revealing PBE, we finally consider the information acquisition stage. Given the communication strategies (m_1^c, m_2^c), the decision rules (y_1^c, y_2^c), and any pair of efforts $(e_1, e_2) \in E^2$, agent 1's expected payoff is

$$\begin{aligned}
&U_1^c(e_1, e_2) \\
&= q\Bigg(K - (1-e_1)\left((1-e_2) + e_2\left(\mathbb{E}_\lambda\left[\frac{(1-\lambda)^2\left(\delta^2+\delta\lambda^2\right)}{(\lambda(1-\lambda)+\delta)^2}\right]+1\right)\right)\sigma_\theta^2 \\
&\quad - e_1\left((1-e_2)\mathbb{E}_\lambda\left[\frac{(1-\lambda)^2\left(\delta^2+\delta\lambda^2\right)}{(\lambda(1-\lambda)+\delta)^2}\right] + e_2\mathbb{E}_\lambda\left[\frac{2(1-\lambda)^2\left(\delta^2+\delta\lambda^2\right)}{(\lambda(1-\lambda)+\delta)^2}\right]\right)\sigma_\theta^2\Bigg) - c(e_1) \\
&= q\Bigg(K - \left(1 + e_2\mathbb{E}_\lambda\left[\frac{(1-\lambda)^2\left(\delta^2+\delta\lambda^2\right)}{(\lambda(1-\lambda)+\delta)^2}\right]\right)\sigma_\theta^2 \\
&\quad - e_1\left(1 - \mathbb{E}_\lambda\left[\frac{(1-\lambda)^2\left(\delta^2+\delta\lambda^2\right)}{(\lambda(1-\lambda)+\delta)^2}\right]\right)\sigma_\theta^2\Bigg) - c(e_1).
\end{aligned}$$

Differentiating with respect to e_1, we obtain the following first-order condition:

$$\frac{\partial U_1^c(e_1, e_2)}{\partial e_1} = \left(1 - \mathbb{E}_\lambda\left[\frac{(1-\lambda)^2\left(\delta^2+\delta\lambda^2\right)}{(\lambda(1-\lambda)+\delta)^2}\right]\right) q\sigma_\theta^2 - c'(e_1) = 0. \tag{D.18}$$

When the cost function c is strictly increasing and satisfies

$$\lim_{e\to 0} c'(e) < \left(1 - \mathbb{E}_\lambda\left[\frac{(1-\lambda)^2\left(\delta^2+\delta\lambda^2\right)}{(\lambda(1-\lambda)+\delta)^2}\right]\right) q\sigma_\theta^2 < c'(1),$$

(D.18) will admit a unique interior solution $e_1^c \in (0,1)$, which is given by

$$e_1^c = (c')^{-1}\left(\left(1 - \mathbb{E}_\lambda\left[\frac{(1-\lambda)^2\left(\delta^2+\delta\lambda^2\right)}{(\lambda(1-\lambda)+\delta)^2}\right]\right) q\sigma_\theta^2\right). \tag{D.19}$$

In addition, when c is strictly convex, the function U_1^c will be strictly concave in e_1, and thus the solution e_1^c is also the unique global maximizer of $U_1^c(e_1, e_2)$ $\forall e_2 \in [0,1]$. These properties of the cost function have all been assumed in Section 3.

By analogous arguments, one can show that choosing

$$e_2^c = (c')^{-1}\left(\left(1 - \mathbb{E}_\lambda\left[\frac{\lambda^2\left(\delta^2+\delta(1-\lambda)^2\right)}{(\lambda(1-\lambda)+\delta)^2}\right]\right) q\sigma_\theta^2\right)$$

will maximize the expected payoff of agent 2 regardless of the effort choice of agent 1. Further, since the distribution F is symmetric in its arguments (η_1, η_2), λ must be symmetrically distributed around $1/2$. Exploiting this symmetry, we obtain

$$e_1^c = e_2^c = e_F^c \equiv (c')^{-1}\left(\left(1 - \mathbb{E}_\lambda\left[\frac{\delta^2(\lambda^2+(1-\lambda)^2) + 2\delta\lambda^2(1-\lambda)^2}{2(\lambda(1-\lambda)+\delta)^2}\right]\right) q\sigma_\theta^2\right).$$

We can now conclude that, together with the degenerate posterior beliefs $\mu_p^i(e_i, m_i) = (e_F^c, m_i)$ (i.e., the principal assigns probability one to $e_i = e_F^c$ and $s_i = m_i$) $\forall i = 1, 2$, the strategy profile $((e_F^c, m_1^c), (e_F^c, m_2^c), (y_1^c, y_2^c))$ constitutes a fully revealing PBE. ■

Part (ii) Let $((e_1^*, m_1^*, y_1^*), (e_2^*, m_2^*, y_2^*))$ be an equilibrium strategy profile under centralization. Without loss of generality, we focus on agent 1 and consider any $s_1 \in \mathcal{S} \setminus \{0, \emptyset\}$. Repeating the calculations of (D.10), (D.11) and (D.12), it can be checked that agent 1 would strictly prefer the type-revealing message $m_1 = m^{s_1}$ than the proposed equilibrium message $m_1^*(e_1^*, s_1)$ if *both* of the following two inequalities hold:

$$\begin{aligned}
&\frac{(\lambda(1-\lambda)+\lambda\delta)^2}{(\lambda(1-\lambda)+\delta)^2}\left(\mathbb{E}[\theta_1|m_1^*(e_1^*, s_1)] - s_1\right)^2 + \frac{(1-\lambda)^2\delta^2}{(\lambda(1-\lambda)+\delta)^2}s_1^2 \\
&> \frac{(\lambda(1-\lambda)+\lambda\delta)^2}{(\lambda(1-\lambda)+\delta)^2}\left(\mathbb{E}[\theta_1|m^{s_1}] - s_1\right)^2 + \frac{(1-\lambda)^2\delta^2}{(\lambda(1-\lambda)+\delta)^2}s_1^2,
\end{aligned} \tag{D.20}$$

and

$$\begin{aligned}
&-\frac{2\delta(1-\lambda)(\lambda(1-\lambda)+\delta)}{(\lambda(1-\lambda)+\delta)^2}\left(\mathbb{E}[\theta_1|m_1^*(e_1^*, s_1)] - s_1\right)s_1 + \frac{\delta\lambda^2(1-\lambda)^2}{(\lambda(1-\lambda)+\delta)^2}\mathbb{E}[\theta_1|m_1^*(e_1^*, s_1)]^2 \\
&> -\frac{2\delta(1-\lambda)(\lambda(1-\lambda)+\delta)}{(\lambda(1-\lambda)+\delta)^2}\left(\mathbb{E}[\theta_1|m^{s_1}] - s_1\right)s_1 + \frac{\delta\lambda^2(1-\lambda)^2}{(\lambda(1-\lambda)+\delta)^2}\mathbb{E}[\theta_1|m^{s_1}]^2.
\end{aligned} \tag{D.21}$$

Since $|\mathbb{E}[\theta_1|m_1^*(e_1^*, s_1)] - s_1| \geq 0\ \forall s_1 \in \mathcal{S}\setminus\{0, \emptyset\}$, (D.20) always holds. In addition, similar to what we have shown for (D.17), (D.21) will also hold if $\mathbb{E}[\theta_1|m^{s_1}] = s_1 > \max\{\mathbb{E}[\theta_1|m_1^*(e_1^*, s_1)], 0\}$ or $\mathbb{E}[\theta_1|m^{s_1}] = s_1 < \min\{\mathbb{E}[\theta_1|m_1^*(e_1^*, s_1)], 0\}$. Hence, for the proposed strategy profile to constitute an equilibrium, it is necessary that $\forall s_1 \in \mathcal{S}\{0, \emptyset\}$, either $\mathbb{E}[\theta_1|m^{s_1}] = 0 < s_1 \leq \mathbb{E}[\theta_1|m_1^*(e_1^*, s_1)]$ or $\mathbb{E}[\theta_1|m^{s_1}] = 0 < s_1 \leq \mathbb{E}[\theta_1|m_1^*(e_1^*, s_1)]$ must hold.

By replacing "the beliefs of agent j" (μ_j^i) with "the beliefs of the principal (μ_p^1)", the rest of the proof follows exactly the same steps as in the case of decentralization (see the proof of Proposition 4.1(ii)). ■□

D.4 Proof of Proposition 4.4

Analogous to the proof of Proposition 4.2. □

D.5 Comparative Statics of e^d and e_F^c

In this section, we will formally show that the equilibrium effort levels under decentralization and centralization (e^d and e_F^c) are both decreasing in δ. Let us define

$$D(\delta) \equiv \frac{\delta^2 + \delta}{(1+2\delta)^2} \tag{D.22}$$

and, for every $\lambda \in (0, 1)$,

$$C(\delta, \lambda) \equiv \frac{\delta^2(\lambda^2 + (1-\lambda)^2) + 2\delta\lambda^2(1-\lambda)^2}{2(\lambda(1-\lambda)+\delta)^2}. \tag{D.23}$$

Differentiating with respect to δ, we have

$$D'(\delta) = \frac{(2\delta+1)(1+2\delta) - 4(\delta^2+\delta)}{(1+2\delta)^3} = \frac{1}{(1+2\delta)^3} > 0, \tag{D.24}$$

and

$$\begin{aligned}
&\frac{\partial C(\delta,\lambda)}{\partial \delta} \\
=& \frac{[2\delta(\lambda^2+(1-\lambda)^2) + 2\lambda^2(1-\lambda)^2]\cdot(\lambda(1-\lambda)+\delta) - 2[\delta^2(\lambda^2+(1-\lambda)^2) + 2\delta\lambda^2(1-\lambda)^2]}{2(\lambda(1-\lambda)+\delta)^3} \\
=& \frac{\lambda^3(1-\lambda)^3 + \delta\lambda(1-\lambda)(\lambda^2+(1-\lambda)^2 - \lambda(1-\lambda))}{(\lambda(1-\lambda)+\delta)^3} \\
=& \frac{\lambda^3(1-\lambda)^3 + \delta\lambda(1-\lambda)((2\lambda-1)^2 + \lambda(1-\lambda))}{(\lambda(1-\lambda)+\delta)^3} \\
>& \, 0.
\end{aligned} \tag{D.25}$$

Thus, both functions $D(\delta)$ and $C(\delta,\lambda)$ are increasing in δ, for all $\lambda \in (0,1)$. This further implies that both e^d and e_F^c are decreasing in δ, because

$$e^d = (c')^{-1}\left((1-D(\delta))\, q\sigma_\theta^2\right), \quad e_F^c = (c')^{-1}\left((1-\mathbb{E}_\lambda\left[C(\delta,\lambda)\right])\, q\sigma_\theta^2\right), \tag{D.26}$$

and the cost function c is strictly increasing and convex.

D.6 Proof of Theorem 4.1

First, suppose that $corr(\eta_1,\eta_2) = 1$. Since the distribution F is symmetric in η_1 and η_2, for the global states to be perfectly and positively correlated, we must have $\Pr(\eta_1 = \eta_2) = 1$, and thus $\Pr\left(\lambda = \frac{1}{2}\right) = 1$, where $\lambda = \eta_1/(\eta_1+\eta_2)$. In this case, the RHS of condition (4.4) becomes

$$C_F(\delta) = \frac{\delta^2\left(\frac{1}{4}+\frac{1}{4}\right) + 2\delta\cdot\frac{1}{4}\cdot\frac{1}{4}}{2\left(\frac{1}{4}+\delta\right)^2} = \frac{4\delta^2+\delta}{(1+4\delta)^2} = \frac{\delta}{1+4\delta}.$$

$\forall\delta > 0$, we have

$$\begin{aligned}
\frac{\delta^2+\delta}{(1+2\delta)^2} > \frac{\delta}{1+4\delta} &\iff \frac{(1+\delta)(1+4\delta)}{(1+2\delta)^2} > 1 \\
&\iff \frac{1+5\delta+4\delta^2}{1+4\delta+4\delta^2} > 1,
\end{aligned}$$

which always holds. Therefore, when $corr(\eta_1,\eta_2) = 1$, we have $D(\delta) > C_F(\delta)$ $\forall\delta > 0$, i.e., condition (4.4) is always violated. From the arguments in the main text, this immediately implies that $e^d < e_F^c$ $\forall\delta > 0$.

Next, consider the case $corr(\eta_1,\eta_2) < 1$. Taking the limit of both sides of (4.4) with respect to δ, we obtain

$$\lim_{\delta\to+\infty} D(\delta) = \lim_{\delta\to+\infty} \frac{1+\frac{1}{\delta}}{\left(\frac{1}{\delta}+2\right)^2} = \frac{1}{4},$$

and

$$\lim_{\delta\to+\infty} C_F(\delta) = \lim_{\delta\to+\infty} \mathbb{E}\left[\frac{(\lambda^2+(1-\lambda)^2)+\frac{2\lambda^2(1-\lambda)^2}{\delta}}{2\left(\frac{\lambda(1-\lambda)}{\delta}+1\right)^2}\right] = \mathbb{E}\left[\frac{\lambda^2+(1-\lambda)^2}{2}\right] = \mathbb{E}\left[\lambda^2\right],$$

where the last equality follows that the distribution of λ must be symmetric around 1/2. Since $corr(\eta_1, \eta_2) < 1$, the distribution of λ cannot be degenerate, and thus by Jensen's inequality we further have

$$\lim_{\delta\to+\infty} C_F(\delta) > (\mathbb{E}[\lambda])^2 = \frac{1}{4} = \lim_{\delta\to+\infty} D(\delta).$$

Therefore, by continuity there must exist $\bar{\delta}_1 < +\infty$, such that $D(\delta) < C_F(\delta)\ \forall\delta > \bar{\delta}_1$. Since $e^d > e_F^C \iff D(\delta) < C_F(\delta)$, it immediately follows that $e^d > e_F^c\ \forall\delta > \bar{\delta}_1$.

To show that the effort difference $e^d - e_F^c$ is increasing in δ for sufficiently large δ, note that

$$\frac{\partial(e^d - e_F^c)}{\partial\delta} = \frac{-D'(\delta)q\sigma_\theta^2}{c''\left((c')^{-1}\left((1-D(\delta))\,q\sigma_\theta^2\right)\right)} - \frac{-C_F'(\delta)q\sigma_\theta^2}{c''\left((c')^{-1}\left((1-C_F(\delta))\,q\sigma_\theta^2\right)\right)}.$$

Since the cost function c is strictly convex, and both $D'(\delta)$ and $C_F'(\delta)$ are strictly positive (see Section D.5), the above partial derivative is strictly positive if and only if

$$\frac{C_F'(\delta)}{D'(\delta)} > \frac{c''\left((c')^{-1}\left((1-C_F(\delta))\,q\sigma_\theta^2\right)\right)}{c''\left((c')^{-1}\left((1-D(\delta))\,q\sigma_\theta^2\right)\right)}. \tag{D.27}$$

For the RHS of (D.27), we have

$$\lim_{\delta\to+\infty} \frac{c''\left((c')^{-1}\left((1-C_F(\delta))\,q\sigma_\theta^2\right)\right)}{c''\left((c')^{-1}\left((1-D(\delta))\,q\sigma_\theta^2\right)\right)} = \frac{c''\left((c')^{-1}\left((1-\mathbb{E}[\lambda^2])\,q\sigma_\theta^2\right)\right)}{c''\left((c')^{-1}\left(\frac{3}{4}\cdot q\sigma_\theta^2\right)\right)} < +\infty.$$

Using the calculation results from Section D.5 (see (D.24) and (D.25)), we also have

$$\begin{aligned}
\lim_{\delta\to+\infty} \frac{C_F'(\delta)}{D'(\delta)} &= \lim_{\delta\to+\infty} \mathbb{E}\left[\frac{\lambda(1-\lambda)(1+2\delta)^3}{(\lambda(1-\lambda)+\delta)^3}\cdot\left(\lambda^2(1-\lambda)^2+\delta((2\lambda-1)^2+\lambda(1-\lambda)\right))\right] \\
&= \lim_{\delta\to+\infty} \mathbb{E}\left[\frac{\lambda(1-\lambda)(\frac{1}{\delta}+2)^3}{\left(\frac{\lambda(1-\lambda)}{\delta}+1\right)^3}\cdot\left(\lambda^2(1-\lambda)^2+\delta((2\lambda-1)^2+\lambda(1-\lambda)\right))\right] \\
&= \lim_{\delta\to+\infty} \mathbb{E}\left[8\lambda(1-\lambda)\cdot\left(\lambda^2(1-\lambda)^2+\delta((2\lambda-1)^2+\lambda(1-\lambda)\right))\right] \\
&= \mathbb{E}\left[8\lambda^3(1-\lambda)^3\right] + \mathbb{E}\left[\lambda(1-\lambda)(2\lambda-1)^2+\lambda^2(1-\lambda)^2\right]\cdot\lim_{\delta\to+\infty}\delta \\
&= +\infty.
\end{aligned}$$

Therefore, by continuity, there must exist $\bar{\delta}_2 < +\infty$, such that (D.27) holds for all $\delta > \bar{\delta}_2$. Equivalently, the effort difference $e^d - e^c_F$ must be increasing in δ for all $\delta > \bar{\delta}_2$.

Finally, we complete the proof of the theorem by letting $\bar{\delta} \equiv \max\{\bar{\delta}_1, \bar{\delta}_2\}$. □

D.7 Proof of Theorem 4.2

To simplify the algebra, let us define

$$\alpha \equiv \frac{\eta_1 \eta_2}{(\eta_1+\eta_2)^2} = \lambda(1-\lambda),\ \beta \equiv \frac{\eta_1^2+\eta_2^2}{(\eta_1+\eta_2)^2} = \lambda^2 + (1-\lambda)^2 \tag{D.28}$$

and

$$\Delta_F(\delta) \equiv C_F(\delta) - D(\delta) = \mathbb{E}\left[\frac{\delta^2\beta + 2\delta\alpha^2}{2(\alpha+\delta)^2}\right] - \frac{\delta^2+\delta}{(1+2\delta)^2}.$$

From (D.24) and (D.25), we have

$$\Delta'_F(\delta) = \mathbb{E}\left[\frac{\alpha^3 + \delta\alpha(\beta-\alpha)}{(\alpha+\delta)^3}\right] - \frac{1}{(1+2\delta)^3}.$$

Further, the second derivative of $\Delta_F(\delta)$ is given by

$$\Delta''_F(\delta) = \mathbb{E}\left[\frac{\alpha^2\beta - 4\alpha^3 - 2\alpha(\beta-\alpha)\delta}{(\alpha+\delta)^4}\right] + \frac{6}{(1+2\delta)^4}.$$

Therefore,

$$\Delta_F(0) = 0,\ \Delta'_F(0) = \mathbb{E}\left[\frac{\alpha^3}{\alpha^3}\right] - 1 = 0,$$

and

$$\begin{aligned}
\Delta''_F(0) &= \mathbb{E}\left[\frac{\alpha^2\beta - 4\alpha^3}{\alpha^4}\right] + 6 \\
&= \mathbb{E}\left[\frac{1}{\lambda^2} + \frac{1}{(1-\lambda)^2} - \frac{4}{\lambda(1-\lambda)}\right] + 6 \\
&= \mathbb{E}\left[\left(\frac{1}{\lambda} - \frac{1}{1-\lambda}\right)^2 - \frac{2}{\lambda(1-\lambda)}\right] + 6 \\
&= \mathbb{E}\left[\frac{2}{\lambda^2} - \frac{4}{\lambda(1-\lambda)}\right] + 6,
\end{aligned}$$

where the last equality follows that λ is symmetrically distributed around 1/2. Note that

$$\Delta''_F(0) > 0 \iff \mathbb{E}\left[\frac{1}{\lambda^2}\right] > \mathbb{E}\left[\frac{2}{\lambda(1-\lambda)}\right] - 3.$$

Hence, if the condition of Theorem 2 is satisfied, then $\Delta''_F(0) > 0$. Since $\Delta'_F(0) = 0$, by continuity, there must exists $\tilde{\delta} > 0$ such that $\Delta'_F(\delta) > 0$ for all $\delta \in \left(0, \tilde{\delta}\right)$. Since $\Delta_F(0) = 0$, and

Δ_F is strictly increasing on $\left(0,\tilde{\delta}\right)$, then again by continuity there must exist $\underline{\delta} \in (0,+\infty]$, such that $\Delta_F(\delta) > 0$ for all $\delta \in (0,\underline{\delta})$. This immediately implies that $e^d > e^c_F$ $\forall \delta \in (0,\underline{\delta})$. $\square$

D.8 Proof of Proposition 4.5

By part (i) of Theorem 4.1, we know that if $\omega = 0$, then $e^d < e^c_F$ $\forall\delta > 0$. In this case, we let $\underline{\delta}(\omega) = 0$ and $\bar{\delta}(\omega) = +\infty$.

Now consider the functions $D(\delta)$ and $C_F(\delta)$ as defined in (4.4). For every binary distribution F_ω with $\omega \in [0,1)$, define

$$\Delta(\delta,\omega) \equiv C_{F_\omega}(\delta) - D(\delta) = \frac{\delta^2 \cdot \frac{1+\omega^2}{2} + 2\delta \cdot \left(\frac{1-\omega^2}{4}\right)^2}{2\left(\frac{1-\omega^2}{4}+\delta\right)^2} - \frac{\delta^2+\delta}{(1+2\delta)^2}. \tag{D.29}$$

Since $\Delta(0,\omega) = 0$ for all $\omega \in [0,1)$, the equation $\Delta(\delta,\omega) = 0$ always has a root $\delta = 0$. To ease the exposition of the algebra, we again use the variables defined in (D.28), which are now given by $\alpha = (1-\omega^2)/4$ and $\beta = (1+\omega^2)/2$. Provided that $\delta > 0$, we have for all $\omega \in (0,1)$,

$$\begin{aligned}
&\Delta(\delta,\omega) = 0 \\
\iff\ & \frac{(\beta\delta+2\alpha^2)(1+2\delta)^2 - 2(\delta+1)(\alpha+\delta)^2}{2(\alpha+\delta)^2(1+2\delta)^2} = 0 \\
\iff\ & (\beta\delta+2\alpha^2)(4\delta^2+4\delta+1) - (2\delta+2)(\alpha^2+\delta^2+2\alpha\delta) = 0 \\
\iff\ & (4\beta-2)\delta^3 + (8\alpha^2-4\alpha+4\beta-2)\delta^2 + (6\alpha^2-4\alpha+\beta)\delta = 0 \\
\iff\ & (4\beta-2)\delta^2 + (8\alpha^2-4\alpha+4\beta-2)\delta + (6\alpha^2-4\alpha+\beta) = 0 \\
\iff\ & \left(\delta + \frac{4\alpha^2-2\alpha+2\beta-1}{4\beta-2}\right)^2 = \frac{(4\alpha^2-2\alpha+2\beta-1)^2 - (6\alpha^2-4\alpha+\beta)(4\beta-2)}{(4\beta-2)^2} \\
\iff\ & \left(\delta + \frac{4\alpha^2-2\alpha+2\beta-1}{4\beta-2}\right)^2 = \frac{(1-2\alpha)^2(4\alpha^2-2\beta+1)}{(4\beta-2)^2},
\end{aligned} \tag{D.30}$$

where the fifth equivalence follows that $4\beta - 2 = 2 + 2\omega^2 - 2 = 2\omega^2 > 0$. In addition, we can verify that the RHS of (D.30) is strictly negative if $\omega > \sqrt{2}-1$. This is because $(1-2\alpha)^2 = (1+\omega^2)^2/4 > 0$, and

$$4\alpha^2 - 2\beta + 1 = \frac{(1-\omega^2)^2}{4} - \omega^2 = \left(\frac{1-\omega^2}{2}+\omega\right)\left(\frac{1-\omega^2}{2}-\omega\right),$$

which, given that $\omega \in (0,1)$, will be positive if and only if $1-\omega^2-2\omega \geq 0$, or, equivalently, $\omega \leq \sqrt{2}-1$. Hence, if $\omega > \sqrt{2}-1$, the equation $\Delta(\delta,\omega) = 0$ does not have any non-zero root on $[0,+\infty)$, and Theorem 1 implies that we must have $\Delta(\delta,\omega) > 0$ for all $\delta > 0$. Since $e^d > e^c_F \iff \Delta(\delta,\omega) > 0$, part (ii) of the proposition immediately follows.

Next, suppose that $\omega \in (0,\sqrt{2}-1]$. In this case, the equation $\Delta(\delta,\omega) = 0$ admits the following two non-zero roots

$$\underline{\delta}(\omega) = -\frac{4\alpha^2-2\alpha+2\beta-1}{4\beta-2} - \frac{(1-2\alpha)\sqrt{4\alpha^2-2\beta+1}}{4\beta-2}, \text{ and}$$

$$\bar{\delta}(\omega) = -\frac{4\alpha^2 - 2\alpha + 2\beta - 1}{4\beta - 2} + \frac{(1-2\alpha)\sqrt{4\alpha^2 - 2\beta + 1}}{4\beta - 2}.$$

In addition, we note that

$$4\alpha^2 - 2\alpha + 2\beta - 1 = \frac{(1-\omega^2)^2}{4} - \frac{1-\omega^2}{2} + 1 + \omega^2 - 1 = \frac{\omega^4 + 4\omega^2 - 1}{4},$$

which is clearly increasing in ω, and it is approximately equal to -0.07 when $\omega = \sqrt{2} - 1$. Thus, the term $-(4\alpha^2 - 2\alpha + 2\beta - 1)/(4\beta - 2)$ must be strictly positive for all $\omega \in (0, \sqrt{2} - 1]$. This implies that if $\omega = \sqrt{2} - 1$, the equation $\Delta(\delta, \omega) = 0$ will actually admit two identical and strictly positive roots, i.e., $\bar{\delta}(\omega) = \underline{\delta}(\omega) > 0$. By continuity, we must have $\Delta(\delta, \sqrt{2} - 1) > 0$ (and thus $e^d > e_F^c$) for all $\delta \in (0, \underline{\delta}(\omega)) \cup (\bar{\delta}(\omega), +\infty)$.

If $\omega < \sqrt{2} - 1$, from the above analysis we know that $\bar{\delta}(\omega) > \max\{\underline{\delta}(\omega), 0\}$. Thus, by continuity and part (ii) of Theorem 4.1, it follows that $\Delta(\delta, \omega) > 0$ for all $\delta > \bar{\delta}(\omega)$. In addition, since $\lim_{\omega\to 0} 4\beta - 2 = \lim_{\omega\to 0} \omega^2 = 0$, it is straightforward to verify that $\lim_{\omega\to 0} \bar{\delta}(\omega) = +\infty$. As for the interval $[\max\{0, \underline{\delta}(\omega)\}, \bar{\delta}(\omega)]$, because we have $\bar{\delta}(\omega) > \underline{\delta}(\omega)$, $4\alpha^2 - 2\beta + 1 > 0$, and $\Delta(\delta, \omega) < 0$ if

$$\left(\delta - \bar{\delta}(\omega) + \frac{(1-2\alpha)\sqrt{4\alpha^2 - 2\beta + 1}}{4\beta - 2}\right)^2 < \frac{(2\alpha - 1)^2(4\alpha^2 - 2\beta + 1)}{(4\beta - 2)^2},$$

it is necessarily the case that $\Delta(\delta, \omega) < 0$ for $\delta = \bar{\delta}(\omega) - \varepsilon > 0$, where $\varepsilon > 0$ is sufficiently small. Hence, we must have $\Delta(\delta, \omega) \leq 0$ for all $\delta \in [\max\{0, \underline{\delta}(\omega)\}, \bar{\delta}(\omega)]$.

It remains to show that $\Delta(\delta, \omega) > 0$ for all $\delta \in (0, \max\{0, \underline{\delta}(\omega)\})$. We note that

$$\begin{aligned}
\underline{\delta}(\omega) \leq 0 &\iff -\frac{4\alpha^2 - 2\alpha + 2\beta - 1}{4\beta - 2} \leq \frac{(1-2\alpha)\sqrt{4\alpha^2 - 2\beta + 1}}{4\beta - 2} \\
&\iff (4\alpha^2 - 2\alpha + 2\beta - 1)^2 \leq (1-2\alpha)^2(4\alpha^2 - 2\beta + 1) \\
&\iff 6\alpha^2 - 4\alpha + \beta \leq 0 \\
&\iff \frac{3(1-\omega^2)^2}{8} - \frac{1 - 3\omega^2}{2} \leq 0, \\
&\iff (1-\omega^2)^2 - 4(1-\omega^2) + 4 - \frac{4}{3}\frac{4}{3} \leq 0 \\
&\iff (1 - \omega^2 - 2)^2 - \frac{4}{3} \leq 0 \\
&\iff (1+\omega^2)^2 - \frac{4}{3} \leq 0,
\end{aligned}$$

where the second equivalence holds because, as we have shown above, $4\alpha^2 - 2\alpha + 2\beta - 1 < 0$ and $4\beta - 2 > 0$ for all $\omega \in \left(0, \sqrt{2} - 1\right)$. Clearly, the equation $(1+\omega^2)^2 - 4/3 = 0$ has a unique real root on $(0, 1)$, which is given by

$$\hat{\omega} = \sqrt{\frac{2\sqrt{3}}{3} - 1} \approx 0.393.$$

It is also straightforward to check that $\underline{\delta}(\omega) < 0$ if $\omega < \hat{\omega}$, and $\underline{\delta}(\omega) > 0$ if $\omega \in (\hat{\omega}, \sqrt{2} - 1)$. Thus, the claim that $\Delta(\delta, \omega) > 0$ for all $\delta \in (0, \max\{0, \underline{\delta}(\omega)\})$ holds trivially if $\omega \in (0, \hat{\omega})$. As for the case $\omega \in (\hat{\omega}, \sqrt{2}-1)$, note that the inequality $\Delta(\delta,\omega) > 0$ can be rewritten as

$$\left(\delta - \underline{\delta}(\omega) - \frac{(1-2\alpha)\sqrt{4\alpha^2-2\beta+1}}{4\beta-2}\right)^2 > \frac{(2\alpha-1)^2(4\alpha^2-2\beta+1)}{(4\beta-2)^2}.$$

But then, given that we have shown $1-2\alpha > 0$ and $4\alpha^2-2\beta+1>0$, it immediately follows that we also have $\Delta(\delta,\omega) > 0$ for all $\delta \in (0, \max\{\underline{\delta}(\omega), 0\})$ in this case. □

D.9 Proof of Proposition 4.6

Consider the function $\Delta(\delta,\omega)$ defined in (D.29). For every $\delta > 0$, we have

$$\begin{aligned}
&\frac{\partial\Delta(\delta,\omega)}{\partial\omega}\\
&= \frac{[8\delta^2\omega + \delta(4\omega^3-4\omega)](1-\omega^2+4\delta) - [4\delta^2(1+\omega^2) + 2\delta(\omega^4-2\omega^2+1)](-4\omega)}{(1-\omega^2+4\delta)^3}\\
&\geq \frac{\delta(4\omega^3-4\omega)(1-\omega^2) + 8\delta\omega(\omega^4-2\omega^2+1)}{(1-\omega^2+4\delta)^3} + \frac{4\delta^2(4\omega^3-4\omega)+16\delta^2\omega(1+\omega^2)}{(1-\omega^2+4\delta)^3}\\
&= \frac{4\delta\omega(1-\omega^2)^2}{(1-\omega^2+4\delta)^3} + \frac{32\delta^2\omega^3}{(1-\omega^2+4\delta)^3},
\end{aligned}$$

which is strictly positive for all $\omega \in (0,1)$. Thus, $\Delta(\delta,\omega)$ is strictly increasing in ω. Since

$$\Delta(\delta,1) = \frac{1}{2} - \frac{\delta^2+\delta}{(1+2\delta)^2} > 0 > \frac{4\delta^2+\delta}{(1+4\delta)^2} - \frac{\delta^2+\delta}{(1+2\delta)^2} = \Delta(\delta,0),$$

there must exist a unique cutoff $\hat{\omega}(\delta) \in (0,1)$, such that $\Delta(\delta,\omega) > 0$ if and only if $\omega > \hat{\omega}(\delta)$.

To obtain the exact analytic form of the cutoff, we expand the equation $\Delta(\delta,\omega) = 0$:

$$\begin{aligned}
\Delta(\delta,\omega) = 0 &\Longleftrightarrow 2\omega^2\delta^2 + \frac{\omega^4+4\omega^2-1}{2}\cdot\delta + \frac{3\omega^4+6\omega^2-1}{8} = 0\\
&\Longleftrightarrow (4\delta+3)\omega^4 + (16\delta^2+16\delta+6)\omega^2 - 4\delta - 1 = 0\\
&\Longleftrightarrow \omega^4 + \frac{16\delta^2+16\delta+6}{4\delta+3}\omega^2 - \frac{4\delta+1}{4\delta+3} = 0\\
&\Longleftrightarrow \left(\omega^2 + 2\delta + \frac{2\delta+3}{4\delta+3}\right)^2 = \frac{4\delta+1}{4\delta+3} + \left(2\delta + \frac{2\delta+3}{4\delta+3}\right)^2\\
&\Longleftrightarrow \left(\omega^2 + 2\delta + \frac{2\delta+3}{4\delta+3}\right)^2 = \frac{64\delta^4+128\delta^3+128\delta^2+64\delta+12}{(4\delta+3)^2}\\
&\Longleftrightarrow \left(\omega^2 + 2\delta + \frac{2\delta+3}{4\delta+3}\right)^2 = \frac{(4\delta+2)^2(4\delta^2+4\delta+3)}{(4\delta+3)^2}.
\end{aligned} \tag{D.31}$$

For every $\delta > 0$, equation (D.31) has a unit root on $[0,1]$, which is given by

$$\hat{\omega}(\delta) = \sqrt{\frac{(4\delta+2)\sqrt{4\delta^2+4\delta+3} - 2\delta - 3}{4\delta+3} - 2\delta}.$$

To prove the remaining claims of the theorem, let us denote

$$\begin{aligned}Z(\delta) &= \frac{(4\delta+2)\sqrt{4\delta^2+4\delta+3}-2\delta-3}{4\delta+3}-2\delta\\ &= \left(1-\frac{1}{4\delta+3}\right)\sqrt{4\delta^2+4\delta+3}-\frac{2\delta+3}{4\delta+3}-2\delta,\end{aligned}$$

and thus $\hat{\omega}(\delta)=\sqrt{Z(\delta)}$. Differentiating with respect to δ, we obtain

$$Z'(\delta)=\frac{4\sqrt{4\delta^2+4\delta+3}}{(4\delta+3)^2}+\frac{(4\delta+2)^2}{4\delta+3}\cdot\left(4\delta^2+4\delta+3\right)^{-\frac{1}{2}}+\frac{6}{(4\delta+3)^2}-2,$$

It is easy to verify that $Z'(0)>0$. In addition, using Mathematica, one can also check that there is a unique solution to $Z'(\delta)=0$ on $(0,+\infty)$, which is $\delta^*=\frac{\sqrt{2}}{2}-\frac{1}{2}$, and it satisfies $\hat{\omega}(\delta^*)=\sqrt{2}-1$. Hence, by continuity, we must have $Z'(\delta)>0$ $\forall\delta\in(0,\delta^*)$, and $Z'(\delta)>0$ $\forall\delta\in(\delta^*,+\infty)$. This further implies that the cutoff $\hat{\omega}(\delta)$ must be strictly increasing on $(0,\delta^*)$, and strictly decreasing on $(\delta^*,+\infty)$. It is also straightforward to verify that $\lim_{\delta\to+\infty}Z(\delta)=0$, and thus $\lim_{\delta\to+\infty}\hat{\omega}(\delta)=0$. □

D.10 Proof of Theorem 4.3

Using Propositions 4.1 and 4.2, we can compute the expected performance of each division $i\in\{1,2\}$ in the fully revealing equilibrium under decentralization, which is given by

$$\Pi_i^d(e^d,y_i^d,y_j^d)=K-\sigma_\theta^2+e^d\left(1-\frac{2\delta^2+2\delta}{(1+2\delta)^2}\right)\sigma_\theta^2.$$

Exploiting that F is symmetric in $\boldsymbol{\eta}$ and the decision rules $\mathbf{y}^d=(y_1^d,y_2^d)$ are independent of $\boldsymbol{\eta}$, we then obtain the expected payoff of the principal under decentralization:

$$\begin{aligned}\Pi_P^d &= \mathbb{E}\left[\eta_1\Pi_1^d(e^d,\mathbf{y}^d)+\eta_2\Pi_2^d(e^d,\mathbf{y}^d)\right]\\ &= 2\mathbb{E}[\eta_i]\Pi_i^d(e^d,\mathbf{y}^d)\\ &= 2\mu\left(K-\sigma_\theta^2+e^d\left(1-\frac{2\delta^2+2\delta}{(1+2\delta)^2}\right)\sigma_\theta^2\right),\end{aligned}$$

where $\mu\equiv\mathbb{E}[\eta_i]>0$, $\forall i=1,2$.

We next derive the equilibrium payoff of the principal under centralization, which we will denote as Π_P^c. Under decentralization, each agent invests $e_i=e_F^c$ in acquiring information, and the decision rules are $\mathbf{y}^c=(y_1^c,y_2^c)$ as described in Proposition 4.4. Hence, for each agent i and a given pair of global states (η_1,η_2), the expected performance of the two divisions are

$$\Pi_1^c(e_F^c,y^c,\boldsymbol{\eta})=K-\sigma_\theta^2+e_F^c\left(1-\frac{2\delta^2+2\delta\lambda^2}{\left(\lambda+\frac{\delta}{1-\lambda}\right)^2}\right)\sigma_\theta^2,$$

$$\Pi_2^c(e_F^c, y^c, \boldsymbol{\eta}) = K - \sigma_\theta^2 + e_F^c \left(1 - \frac{2\delta^2 + 2\delta(1-\lambda)^2}{\left(1 - \lambda + \frac{\delta}{\lambda}\right)^2}\right)\sigma_\theta^2,$$

where we recall that $\lambda = \eta_1/(\eta_1 + \eta_2)$. Exploiting the symmetry of F, we have

$$\begin{aligned}
\Pi_P^c &= \mathbb{E}\left[\eta_1 \Pi_1^c(e_F^c, y^c, \boldsymbol{\eta}) + \eta_2 \Pi_2^c(e^c, y^c, \boldsymbol{\eta})\right] \\
&= 2\mu \left[K - \sigma_\theta^2 + e_F^c \left(1 - \frac{1}{\mu}\left(\mathbb{E}\left[\eta_1 \cdot \frac{\delta^2 + \delta\lambda^2}{\left(\lambda + \frac{\delta}{1-\lambda}\right)^2}\right] + \mathbb{E}\left[\eta_2 \cdot \frac{\delta^2 + \delta(1-\lambda)^2}{\left(1 - \lambda + \frac{\delta}{\lambda}\right)^2}\right]\right)\right)\sigma_\theta^2\right] \\
&= 2\mu \left[K - \sigma_\theta^2 + e_F^c \left(1 - \frac{1}{\mu}\mathbb{E}\left[\frac{\delta^2 \cdot \frac{\eta_1\eta_2}{\eta_1+\eta_2} + \delta \cdot \frac{\eta_1^2\eta_2^2}{(\eta_1+\eta_2)^3}}{\left(\frac{\eta_1\eta_2}{(\eta_1+\eta_2)^2} + \delta\right)^2}\right]\right)\sigma_\theta^2\right].
\end{aligned}$$

Therefore, $\Pi_P^d > \Pi_P^c$ if and only if the following inequality holds:

$$\frac{e^d}{e_F^c} > R_F(\delta) \equiv \left(1 - \frac{1}{\mu}\mathbb{E}\left[\frac{\delta^2 \cdot \frac{\eta_1\eta_2}{\eta_1+\eta_2} + \delta \cdot \frac{\eta_1^2\eta_2^2}{(\eta_1+\eta_2)^3}}{\left(\frac{\eta_1\eta_2}{(\eta_1+\eta_2)^2} + \delta\right)^2}\right]\right) \Big/ \left(1 - \frac{2\delta^2 + 2\delta}{(1+2\delta)^2}\right). \tag{D.32}$$

Note that

$$\lim_{\delta \to +\infty} R_F(\delta) = 2 - \frac{2}{\mu}\mathbb{E}\left[\frac{\eta_1\eta_2}{\eta_1 + \eta_2}\right] < 2,$$

and

$$\lim_{\delta \to +\infty} \frac{e^d}{e_F^c} = \lim_{\delta \to +\infty} \frac{(c')^{-1}\left((1 - D(\delta))\, q\sigma_\theta^2\right)}{(c')^{-1}\left((1 - C_F(\delta))q\sigma_\theta^2\right)} = \frac{(c')^{-1}\left(0.75q\sigma_\theta^2\right)}{(c')^{-1}\left(\left(1 - \mathbb{E}\left[\left(\frac{\eta_1}{\eta_1+\eta_2}\right)^2\right]\right)q\sigma_\theta^2\right)} > 1,$$

where $D(\delta)$ and $C_F(\delta)$ are defined in (4.4), and the last inequality follows Theorem 4.1. Thus, the value of $\lim_{\delta \to} e^d/e_F^c$ is strictly large than 1, and it is increasing as the term $\mathbb{E}\left[(\eta_1/(\eta_1+\eta_2))^2\right]$ increases. Let $\varepsilon \equiv \left(\mathbb{E}\left[(\eta_1/(\eta_1+\eta_2))^2\right] - 0.25\right) q\sigma_\theta^2$, which is strictly positive given the assumption $corr(\eta_1, \eta_2) < 1$. Using Taylor's theorem, we obtain

$$(c')^{-1}\left(\left(1 - \mathbb{E}\left[\left(\frac{\eta_1}{\eta_1+\eta_2}\right)^2\right]\right)q\sigma_\theta^2\right) = (c')^{-1}\left(0.75q\sigma_\theta^2\right) - \frac{\varepsilon}{c''((c')^{-1}(0.75q\sigma_\theta^2))} + o\left(\varepsilon^2\right).$$

Since $c''(e) \cdot e < \zeta\ \forall e \in [0,1]$, we further have

$$(c')^{-1}\left(\left(1 - \mathbb{E}\left[\left(\frac{\eta_1}{\eta_1+\eta_2}\right)^2\right]\right)q\sigma_\theta^2\right) \le \frac{\zeta - \varepsilon}{c''((c')^{-1}(0.75q\sigma_\theta^2))} + o\left(\varepsilon^2\right). \tag{D.33}$$

When ε is sufficiently small, the value of the higher order terms in $o(\varepsilon^2)$ can be neglected. Thus, if ζ is sufficiently close to (but still larger than) ε, the LHS of (D.33) becomes arbitrarily close to zero (but it is still strictly positive). Hence, if the bound ζ is sufficiently close to $\varepsilon > 0$, then for the case with small enough ε (i.e., $\varepsilon \le \bar{\varepsilon}$, where $\bar{\varepsilon}$ is some strictly positive cutoff) we must have

$\lim_{\delta\to+\infty} e^d/e_F^c > \lim_{\delta\to+\infty} R_F(\delta)$.[2] But then, because the value of $\lim_{\delta\to+\infty} e^d/e_F^c$ is strictly increasing in $\mathbb{E}[(\eta_1/(\eta_1+\eta_2))^2]$, and thus also in ε, with the same bound ζ we will also have $\lim_{\delta\to+\infty} e^d/e_F^c > \lim_{\delta\to+\infty} R_F(\delta)$ for all $\varepsilon \geq \bar{\varepsilon}$. By continuity, it follows that $e^d/e_F^c > R_F(\delta)$ for sufficiently large δ. We can conclude that if ζ is sufficiently small, then there must exist $\bar{\delta} > 0$, such that $\Pi_P^d > \Pi_P^c$ if $\delta > \bar{\delta}$. □

D.11 Proof of Theorem 4.4

In the proof of Theorem 4.2, it is shown that if the condition $\mathbb{E}\left[\frac{1}{\lambda^2}\right] > \mathbb{E}\left[\frac{2}{\lambda(1-\lambda)}\right] - 3$ is satisfied, then there exists $\underline{\delta} > 0$, such that $C_F(\delta) > D(\delta)\ \forall \delta \in (0, \underline{\delta})$. Using arguments that are analogous to those in the proof of Theorem 4.3, we can further show that, for every of such δ there must exist a cutoff $\zeta(\delta) > 0$, such that if $c''(e) \cdot e < \zeta(\delta)\ \forall e \in [0,1]$, then we will also have

$$\frac{e^d}{e_F^c} = \frac{(c')^{-1}\left((1 - D(\delta))\, q\sigma_\theta^2\right)}{(c')^{-1}\left((1 - C_F(\delta))\, q\sigma_\theta^2\right)} > R_F(\delta),$$

which, according to (D.32), is both necessary and sufficient for $\Pi_P^d > \Pi_P^c$. □

D.12 Proof of Proposition 4.7

Decentralization Let us first consider part (i) of the proposition. We argue that the proposed equilibrium effort profile $(e_1^*, e_2^*) = (\check{e}^d, \check{e}^d)$, the fully revealing communication strategies $(\check{m}_1^d, \check{m}_2^d)$, and the decision rules $(\check{y}_1^d, \check{y}_2^d)$ described below constitute a PBE, where the beliefs of the players will be fully pinned down Bayes' rule. Specifically, the decision rules are similar to the ones in Proposition 4.1, and they are given by

$$\check{y}_i^d\left(e_i, s_i, m_i, m_j\right) = \frac{\mathbb{E}[\theta_i|s_i]}{1+\delta} + \frac{\delta^2 \mathbb{E}[\theta_i|m_i]}{(1+2\delta)(1+\delta)} + \frac{\delta \mathbb{E}[\theta_j|m_j]}{1+2\delta},$$

$\forall e_i \in E$, $\forall s_i, m_i, m_j \in \mathbb{R} \cup \{\emptyset\}$, and $\forall i, j = 1, 2$, where for each $i \in \{1,2\}$ $\mathbb{E}[\theta_i|m_i]$ is the posterior expectation of the local state θ_i conditional on $s_i = m_i/t^d$ if $m_i \neq \emptyset$, and it is conditional on $s_i = \emptyset$ otherwise. To prove Proposition 4.7(i), we will suppose that agent j is playing according to $(\check{e}^d, \check{m}_j^d, \check{y}_j^d)$, and then show that it is a best response for agent i to also adopt the proposed strategy.

Taking the first stage effort e_i, the signal s_i received, and the message m_i sent as given, we can solve agent i's utility-maximizing problem at the decision-making stage as in the proof of Proposition 4.1, and obtain the decision rule $\check{y}_i^d$ as a solution. Thus, given agent j's strategy and the corresponding beliefs, the decision rule $\check{y}_i^d$ is sequentially rational for agent i.

Next, we take the decision rule $\check{y}_i^d$, effort e_i and signal s_i as given, and consider agent i's strategic incentives when communicating his private information with agent j. We start by

[2] The assumption that "the marginal cost is sufficiently small at $e = 0$ ... so that the agents will endogenously choose to be partially informed ($e_i \in (0,1)$ in equilibrium", which is stated in Section 4.3, implicitly restricts that ζ cannot be too small. Otherwise, we may have the RHS of (D.33) being negative, which implies that in the limit the agents would actually choose to be not informed at all under centralization ($\lim_{\delta\to+\infty} e_F^c = 0$). In this case, if we still have $\lim_{\delta\to+\infty} e^d = (c')^{-1}(0.75 q\sigma_\theta^2) > 0$, i.e., the agents would still want to exert some effort under decentralization, then obviously the statement $\lim_{\delta\to+\infty} e^d/e_F^c > \lim_{\delta\to+\infty} R_F(\delta)$ will also hold.

showing that when agent i receives a *non-null* signal $s_i \in \mathbb{R}$, he will prefer to send message $m_i^d(e_i, s_i) = t^d s_i$ than any other message $m_i \in \mathbb{R}$. Note that since agent j will follow the proposed fully revealing communicating strategy, agent i can always infer the realization of agent j's signal s_j (which is equal to m_j/t^d if $m_j \neq \emptyset$, and it is equal to $\emptyset$ otherwise). Thus for every message $m_i \in \mathbb{R}$ sent by agent i (which may or may not equal to $\check{m}_i^d(e_i, s_i)$), sequential rationality implies that the agents will choose the following actions:

$$y_i = \frac{\theta_i}{1+\delta} + \frac{\delta^2(m_i/t^d)}{(1+2\delta)(1+\delta)} + \frac{\delta\mathbb{E}[\theta_j|s_j]}{1+2\delta},\ y_j = \frac{(1+\delta)\mathbb{E}[\theta_j|s_j] + \delta(m_i/t^d)}{1+2\delta},$$

where $\mathbb{E}[\theta_j|s_j] = 0$ if $s_j = \emptyset$, and $\theta_j = s_j$ otherwise. Thus, given any effort $e_j \in E$ chosen by agent j, conditional on sending the message $m_i \in \mathbb{R}$, the expected performance of division i is

$$\tilde{\Pi}_i^d\,(m_i, \theta_i, e_j) = e_j \mathbb{E}_{\theta_j}[\tilde{\pi}_1^d(m_i, \theta_i, \theta_j)] + (1-e_j)\tilde{\pi}_i^d\,(m_i, \theta_i, \emptyset)\,,$$

where

$$\begin{aligned}\tilde{\pi}_i^d(m_i, \theta_i, \theta_j) = K &- \left(\frac{(\delta+2\delta^2)\theta_i - (m_i/t^d)\delta^2}{(1+2\delta)(1+\delta)} - \frac{\delta}{1+2\delta}\cdot\theta_j\right)^2 \\ &- \delta\left(\frac{(1+2\delta)\theta_i - (m_i/t^d)\delta}{(1+2\delta)(1+\delta)} - \frac{1}{1+2\delta}\cdot\theta_j\right)^2\end{aligned}$$

and $\tilde{\pi}_i^d(m_i, \theta_i, \emptyset) = \tilde{\pi}_i^d(m_i, \theta_i, \theta_j)|_{\theta_j=0}$. Differentiating with respect to m_i, we have, $\forall \theta_j \in \mathbb{R}$,

$$\begin{aligned}&\frac{\partial \mathbb{E}_{\theta_j}[\tilde{\pi}_i^d(m_i, \theta_i, \theta_j)]}{\partial m_i} \\ &= -\,\mathbb{E}_{\theta_j}\left[2\left(\frac{(\delta+2\delta^2)\theta_i - (m_i/t^d)\delta^2}{(1+2\delta)(1+\delta)} - \frac{\delta}{1+2\delta}\cdot\theta_j\right)\left(\frac{-\delta^2/t^d}{(1+2\delta)(1+\delta)}\right)\right] \\ &\quad -\delta\mathbb{E}_{\theta_j}\left[2\left(\frac{(1+2\delta)\theta_i - (m_i/t^d)\delta}{(1+2\delta)(1+\delta)} - \frac{1}{1+2\delta}\cdot\theta_j\right)\left(\frac{-\delta/t^d}{(1+2\delta)(1+\delta)}\right)\right] \\ &= \frac{2\delta^2}{(1+2\delta)(1+\delta)}\cdot\left(\frac{(\delta+2\delta^2)\theta_i - (m_i/t)\delta^2 + (1+2\delta)\theta_i - (m_i/t)}{(1+2\delta)(1+\delta)}\right)\cdot\frac{1}{t^d},\end{aligned}$$

which is also equal to $\partial\tilde{\pi}_i^d(m_i, \theta_i, \emptyset)/\partial m_i$. Hence, we further have

$$\begin{aligned}&\frac{\partial\tilde{\Pi}_i^d(m_i, \theta_i, e_j)}{\partial m_i} \\ &= \frac{2\delta^2}{(1+2\delta)(1+\delta)}\cdot\left(\frac{(\delta+2\delta^2)\theta_i - (m_i/t^d)\delta^2 + (1+2\delta)\theta_i - (m_i/t^d)\delta}{(1+2\delta)(1+\delta)}\right)\cdot\frac{1}{t^d},\end{aligned}$$

which is independent of e_j. This implies that the strategic communication incentives of agent i is independent of his belief about the effort exerted by agent j. Thus, from now on we drop the variable e_j from the function $\tilde{\Pi}_i^d$. We distinguish the following two cases:

Case 1: $s_i = \theta_i = 0$. It is straightforward to verify that

$$\left.\frac{\partial[q\tilde{\Pi}_i^d(m_i,\theta_i)]}{\partial m_i}\right|_{m_i=\theta_i=0} = 0.$$

Since $z(\hat{m}_i, 0) \geq z(0,0) = 0 \; \forall \hat{m}_i \in \mathbb{R}$, it immediately follows that

$$q\tilde{\Pi}_i^d(0,0) - z(0,0) \geq q \cdot \tilde{\Pi}_i^d(\hat{m}_i, 0) - z(\hat{m}_i, 0) \, \forall \hat{m}_i \in \mathbb{R}.$$

Hence, when agent i learns that $\theta_i = 0$, he always prefer to send $m_i = 0$ than any other $\hat{m}_i \in \mathbb{R}$.

Case 2: $s_i = \theta_i \neq 0$. Suppose first that $\theta_i > 0$. Consider any message that $m_i \geq \theta_i$. By assumption 4.2, sending this message will incur a cost $z(m_i, \theta_i) = \kappa(m_i - \theta_i)^2$. Note that by construction, we have $t^d \equiv \frac{1}{2} + \sqrt{\frac{q\delta^2}{\kappa(1+2\delta)^2} + \frac{1}{4}}$, which is larger than 1 for all $\delta \geq 0$, and

$$\left.\left[\frac{\partial[q\tilde{\Pi}_i^d(m_i,\theta_i)]}{\partial m_i} - \frac{\partial z(m_i,\theta_i)}{\partial m_i}\right]\right|_{m_i=t^d\theta_i} = 0,$$

i.e., the first-order condition is satisfied exactly at $m_i = t^d\theta_i$. This implies that if agent i learns that the true state is $\theta_i > 0$, he will prefer to send the message $m_i = t^d\theta_i$ than any other messages $\hat{m}_i \in [\theta_i, +\infty)$.

It remains to show that agent i will also prefer $m_i = t^d\theta_i$ than any message $\hat{m}_i \in [0, \theta_i)$. This is indeed the case, because $\frac{\partial\tilde{\Pi}_i^d(m_i,\theta_i)}{\partial m_i} > 0 \, \forall m_i \in [0, \theta_i)$, and, thus, $\forall \theta_i > 0$ and $\hat{m}_i < \theta_i$,

$$\begin{aligned} q\tilde{\Pi}_i^d\left(t^d\theta_i, \theta_i\right) - z(t^d\theta_i, \theta_i) &\geq q\tilde{\Pi}_i^d\left(\theta_i, \theta_i\right) - z(\theta_i, \theta_i) \\ &= q\tilde{\Pi}_i^d\left(\theta_i, \theta_i\right) \\ &> q\tilde{\Pi}_i^d\left(\hat{m}_i, \theta_i\right) \\ &\geq q\tilde{\Pi}_i^d\left(\hat{m}_i, \theta_i\right) - z(\hat{m}_i, \theta_i). \end{aligned}$$

In sum, we have shown that for all $s_i = \theta_i > 0$, agent i would prefer sending $m_i = t^d\theta_i$ than any other message $\hat{m}_i \in \mathbb{R}$. By symmetry, the same conclusion also holds for all $s_i = \theta_i < 0$.

Since the effect on the actions chosen by the agents are the same for $\hat{m}_i = 0$ and $\hat{m}_i = \emptyset$, and we allow the cost $z(m, 0)$ to be fully general, it also follows that agent i will not find it profitable to send $\hat{m}_i = \emptyset$ when $s_i \neq \emptyset$. As for the remaining scenario $s_i = \emptyset$, it is straightforward to verify that the expected performance of division i conditional on agent i sending any message $m_i \in \mathbb{R} \cup \emptyset$ is the same as when he actually receives a non-null signal that $s_i = \theta_i = 0$. Hence, this expected performance is maximized when $m_1 = \emptyset$. Trivially, the communication cost is also minimized at $m_1 = \emptyset$. Therefore, it must be optimal for agent i to send $m_i = \emptyset$ whenever $s_i = \emptyset$ is observed.

Finally, we take both the decision rules $(\check{y}_1^d, \check{y}_2^d)$ and communication strategies $(\check{m}_1^d, \check{m}_2^d)$ as given and consider the information acquisition problem for agent i. Given an arbitrary effort profile (e_1, e_2), the expected payoff of agent i is now given by

$$U_i^d(e_i, e_j) - e_i\mathbb{E}\left[\kappa(t^d\theta_i - \theta_i)^2\right],$$

where $U_i^d(e_i, e_j)$ was defined in the proof of Proposition 4.1. Thus, with costly exaggeration, the first-order condition at the information acquisition stage is

$$\left(1 - \frac{\delta^2 + \delta}{(1+2\delta)^2}\right) q\sigma_\theta^2 - c'(e_i) - \kappa(t^d - 1)^2\sigma_\theta^2 = 0.$$

Solving the above equation, we obtain the proposed equilibrium effort level $\check{e}^d$ as a unique solution. Therefore, given the above-mentioned decision rules and communication strategies, choosing $e_i = \check{e}^d$ is indeed optimal for agent i. ■

Centralization For part (ii) of the proposition, we consider first the principal's incentive at the decision-making stage. Given the communication strategy profile $(\check{m}_1^c, \check{m}_2^c)$, the relevant information held by both agents will be perfectly revealed to the principal. In particular, whenever the principal observes that $m_i \neq \emptyset$, she can infer that agent i has learned about his local state, which is given by $\theta_i = m_i/t^c$. Thus, sequential rationality implies that the decision rules of the principal should be similar to the ones in the proof of Proposition 4.2:

$$\check{y}_i^c(m_i, m_j, \eta_i, \eta_j) = \frac{\frac{\eta_i}{\eta_i+\eta_j} \cdot \left(\frac{\eta_j}{\eta_i+\eta_j} + \delta\right) \mathbb{E}\left[\theta_i | m_i\right] + \frac{\delta\eta_j}{\eta_i+\eta_j}\mathbb{E}[\theta_j|m_j]}{\frac{\eta_i}{\eta_i+\eta_j} \cdot \frac{\eta_j}{\eta_i+\eta_j} + \delta},$$

$\forall m_i, m_j \in \Theta \cup \{\emptyset\}$, $\forall \eta_i, \eta_j \in [\underline{\eta}, \bar{\eta}]$, and $\forall i, j = 1, 2$, where for each $i \in \{1, 2\}$ $\mathbb{E}[\theta_i|m_i]$ is the posterior expectations of the local state θ_i conditional on $s_i = m_i/t^c$ if $m_i \neq \emptyset$, and it is conditional on $s_i = \emptyset$ otherwise.

Next, we take the above decision rules of the principal as given, and consider the strategic incentives of the agents at the communication stage. Suppose that agent 2 plays the fully revealing communication strategy m_2^c. We start by showing that when receiving a non-null signal $s_1 \in \mathbb{R}$, agent 1 will prefer to send $m_1 = t^c s_1$ than any other message $\hat{m}_1 \in \mathbb{R}$. In particular, suppose that agent 1 learns that his local state is $\theta_1 \in \mathbb{R}$, then by sending an arbitrary message $m_1 \in \mathbb{R}$ he will induce the following contingent actions of the principal:

$$y_1^c = \frac{(\lambda(1-\lambda) + \lambda\delta) \cdot (m_1/t^c) + (1-\lambda)\delta\mathbb{E}[\theta_2|s_2]}{\lambda(1-\lambda) + \delta}$$

and

$$y_2^c = \frac{(\lambda(1-\lambda) + (1-\lambda)\delta)\mathbb{E}[\theta_2|s_2] + \lambda\delta \cdot (m_1/t^c)}{\lambda(1-\lambda) + \delta},$$

where $\lambda = \eta_1/(\eta_1 + \eta_2)$, $\mathbb{E}[\theta_2|s_2] = 0$ if $s_2 = \emptyset$, and $\theta_2 = s_2$ (and thus $\mathbb{E}[\theta_2|s_2] = s_2$) otherwise. Note that we can write the action of the principal as a function of agent 2's private signal because agent 2 s communication strategy is fully revealing.

Given any effort $e_2 \in E$ chosen by agent 2, and any realization of the profitability conditions $\eta_1, \eta_2 \in [\underline{\eta}, \bar{\eta}]$, conditional on sending a message $m_i \in \mathbb{R}$ the expected performance of agent 1 is

$$\tilde{\Pi}_1^c(m_1, \theta_1, e_2, \boldsymbol{\eta}) = e_2\mathbb{E}_{\theta_2}\left[\tilde{\pi}_1^c(m_1, \theta_2, \boldsymbol{\eta})\right] + (1-e_2)\tilde{\pi}_1^c(m_1, \emptyset, \boldsymbol{\eta}),$$

where

$$\tilde{\pi}_1^c(m_1,\theta_2,\boldsymbol{\eta}) = K - \left(\frac{(\lambda(1-\lambda)+\delta)\theta_1 - (\lambda(1-\lambda)+\lambda\delta)\cdot(m_1/t^c)}{\lambda(1-\lambda)+\delta} - \frac{(1-\lambda)\delta}{\lambda(1-\lambda)+\delta}\cdot\theta_2\right)^2 - \delta\left(\frac{\lambda(1-\lambda)\cdot(m_1/t^c-\theta_2)}{\lambda(1-\lambda)+\delta}\right)^2$$

and $\tilde{\pi}_1^c(m_1,\emptyset,\boldsymbol{\eta}) = \tilde{\pi}_1^c(m_1,\theta_2,\boldsymbol{\eta})|_{\theta_2=0}$. Differentiating with respect to m_1, we have, $\forall\theta_2 \in \mathbb{R}$,

$$\begin{aligned}
&\frac{\partial\mathbb{E}_{\theta_2}\left[\tilde{\pi}_1^c(m_1,\theta_2,\boldsymbol{\eta})\right]}{\partial m_1} \\
=&\mathbb{E}_{\theta_2}\left[2\left(\frac{(\lambda(1-\lambda)+\delta)\theta_1 - (\lambda(1-\lambda)+\lambda\delta)\cdot(m_1/t^c) - (1-\lambda)\delta\theta_2}{\lambda(1-\lambda)+\delta}\right)\left(\frac{(\lambda(1-\lambda)+\lambda\delta)/t^c}{\lambda(1-\lambda)+\delta}\right)\right] \\
&- \mathbb{E}_{\theta_2}\left[2\delta\left(\frac{\lambda(1-\lambda)\cdot(m_1/t^c-\theta_2)}{\lambda(1-\lambda)+\delta}\right)\left(\frac{\lambda(1-\lambda)/t^c}{\lambda(1-\lambda)+\delta}\right)\right] \\
=&2\left(\frac{(\lambda(1-\lambda)+\delta)\theta_1 - (\lambda(1-\lambda)+\lambda\delta)\cdot(m_1/t^c)}{\lambda(1-\lambda)+\delta}\right)\left(\frac{(\lambda(1-\lambda)+\lambda\delta)/t^c}{\lambda(1-\lambda)+\delta}\right) \\
&- 2\delta\left(\frac{\lambda(1-\lambda)\cdot(m_1/t^c)}{\lambda(1-\lambda)+\delta}\right)\left(\frac{\lambda(1-\lambda)/t^c}{\lambda(1-\lambda)+\delta}\right),
\end{aligned}$$

which is also equal to $\partial\tilde{\pi}_1^c(m_1,\emptyset,\boldsymbol{\eta})/\partial m_1$. Hence, we further have

$$\begin{aligned}
&\frac{\partial\tilde{\Pi}_1^c(m_1,\theta_1,e_2,\boldsymbol{\eta})}{\partial m_1} \\
=&2\left(\frac{(\lambda(1-\lambda)+\delta)\theta_1 - (\lambda(1-\lambda)+\lambda\delta)\cdot(\hat{m}_1/t^c)}{\lambda(1-\lambda)+\delta}\right)\left(\frac{(\lambda(1-\lambda)+\lambda\delta)/t^c}{\lambda(1-\lambda)+\delta}\right) \\
&- 2\delta\left(\frac{\lambda(1-\lambda)\cdot(\hat{m}_1/t^c)}{\lambda(1-\lambda)+\delta}\right)\left(\frac{\lambda(1-\lambda)/t^c}{\lambda(1-\lambda)+\delta}\right),
\end{aligned}$$

which is independent of e_2. This implies that the strategic communication incentives of agent 1 is independent of his belief about the effort exerted by agent 2. Thus, from now on we drop the variable e_2 from the function $\tilde{\Pi}_1^c$. We distinguish the following two cases:

Case 1: $s_i = \theta_i = 0$. It is straightforward to verify that

$$\left.\frac{\partial[q\tilde{\Pi}_1^c(m_1,\theta_1,\boldsymbol{\eta})]}{\partial m_1}\right|_{m_1=\theta_1=0} = 0 \quad \forall\eta_1,\eta_2 \in [\underline{\eta},\bar{\eta}].$$

Since $z(\hat{m}_1,0) \geq z(0,0) = 0\ \forall\hat{m}_1 \in \mathbb{R}$, it immediately follows that

$$q\mathbb{E}_{\boldsymbol{\eta}}\left[\tilde{\Pi}_1^c(0,0,\boldsymbol{\eta})\right] - z(0,0) \geq \mathbb{E}_{\boldsymbol{\eta}}\left[\tilde{\Pi}_1^c(\hat{m}_1,0,\boldsymbol{\eta})\right] - z(\hat{m}_1,0)\,\forall\hat{m}_1 \in \mathbb{R}.$$

Hence, when agent 1 learns that $\theta_i = 0$, he always prefer to send $m_1 = t^c\theta_1 = 0$ than any other message $\hat{m}_1 \in \mathbb{R}$.

Case 2: $s_i = \theta_i \neq 0$. Suppose first that $\theta_i > 0$. Consider any message that $m_1 \geq \theta_1$. By assumption 4.2, sending this message will incur a cost $z(m_1,\theta_1) = \kappa(m_1-\theta_1)^2$. Note that by construction, we have $t^c = \frac{1}{2} + \sqrt{\mathbb{E}\left[\frac{q\lambda(1-\lambda)[2\delta^2+\delta(\lambda^2+(1-\lambda)^2)]}{\kappa(\lambda(1-\lambda)+\delta)^2}\right] + \frac{1}{4}}$, which is larger than 1 for all $\delta \geq 0$, and

$$\left.\left[\frac{\partial\mathbb{E}_{\boldsymbol{\eta}}\left[q\tilde{\Pi}_1^c(m_1,\theta_1,\boldsymbol{\eta})\right]}{\partial m_1} - \frac{\partial z(m_1,\theta_1)}{\partial m_1}\right]\right|_{m_1=t^c\theta_1} = 0,$$

i.e., the first-order condition is satisfied exactly at $m_i = t^c\theta_i$.[3] This implies that if agent 1 learns that the true state is $\theta_1 > 0$ and the principal expects him to send messages according to the fully revealing communication rule $\check{m}_1^c$, then agent 1 will indeed prefer to send the message $m_1 = t^c\theta_1$ than any other messages $\hat{m}_1 \in [\theta_1, +\infty)$.

It remains to show that agent 1 will also prefer $m_1 = t^c\theta_1$ than any message $\hat{m}_1 \in [0, \theta_1)$. This is indeed the case, because $\forall m_1 \in [0, \theta_1)$ and $\forall \eta_1, \eta_2 \in [\underline{\eta}, \bar{\eta}]$,

$$\frac{\partial \tilde{\Pi}_1^c(m_1, \theta_1, \boldsymbol{\eta})}{\partial m_1} \geq 2\left(\frac{(\lambda(1-\lambda)+\delta)(\theta_1 - m_1/t^c)}{\lambda(1-\lambda)+\delta}\right)\left(\frac{(\lambda(1-\lambda)+\lambda\delta)/t^c}{\lambda(1-\lambda)+\delta}\right) > 0,$$

and, thus, $\forall \theta_1 > 0$ and $\hat{m}_1 < \theta_1$,

$$\begin{aligned} q\mathbb{E}_{\boldsymbol{\eta}}\left[\tilde{\Pi}_1^c(t^c\theta_1, \theta_1, \boldsymbol{\eta})\right] - z(t^c\theta_1, \theta_1) &\geq q\mathbb{E}_{\boldsymbol{\eta}}\left[\tilde{\Pi}_1^c(\theta_1, \theta_1, \boldsymbol{\eta})\right] - z(\theta_1, \theta_1) \\ &= q\mathbb{E}_{\boldsymbol{\eta}}\left[\tilde{\Pi}_1^c(\theta_1, \theta_1, \boldsymbol{\eta})\right] \\ &> q\mathbb{E}_{\boldsymbol{\eta}}\left[\tilde{\Pi}_1^c(\hat{m}_1, \theta_1, \boldsymbol{\eta})\right] \\ &\geq q\mathbb{E}_{\boldsymbol{\eta}}\left[\tilde{\Pi}_1^c(\hat{m}_1, \theta_1, \boldsymbol{\eta})\right] - z(\hat{m}_1, \theta_1). \end{aligned}$$

In sum, we have shown that for all $s_1 = \theta_i > 0$, agent 1 would prefer sending $m_1 = t^c s_1$ than any other message $\hat{m}_1 \in \mathbb{R}$. By symmetry, the same conclusion also holds for all $s_1 < 0$.

Since the effect on the actions chosen by the principal are the same for $\hat{m}_1 = 0$ and $\hat{m}_1 = \emptyset$, and we allow the cost $z(m, 0)$ to be fully general, it also follows from the argument in Case 1 that agent 1 will not find it profitable to send $\hat{m}_1 = \emptyset$ when $s_1 \neq \emptyset$. As for the remaining scenario $s_1 = \emptyset$, it is straightforward to verify that the expected performance of division 1 conditional on agent 1 sending any message $\hat{m}_1$ is the same as when he actually receives a non-null signal of $s_1 = \theta_1 = 0$. Hence, this expected performance is maximized when $\hat{m}_1 = \emptyset$. Trivially, the communication cost is also minimized when $\hat{m}_1 = \emptyset$. Therefore, it is optimal for agent 1 to report $m_1 = \emptyset$ to the principal whenever $s_1 = \emptyset$ is observed.

By the symmetry of distribution F, the incentive problem of agent 2 is analogous. Thus, given that agent 1 will be fully revealing his private information to the principal, it is also a best response for agent 2 to follow the communication strategy $\check{m}_2^c$.

Finally, we take both the decision rules $(\check{y}_1^c, \check{y}_2^c)$ and the communication strategies $(\check{m}_1^c, \check{m}_2^c)$ as given and consider the information acquisition problem for agent i. Given an arbitrary effort profile (e_1, e_2), the expected payoff of agent i is now given by

$$U_i^c(e_i, e_j) - e_i\mathbb{E}\left[\kappa(t^c\theta_i - \theta_i)^2\right],$$

where $U_i^c(e_i, e_j)$ was defined in the proof of Proposition 4.3. Thus, with costly exaggeration, the first-order condition at the information acquisition stage is

$$\left(1 - \mathbb{E}\left[\frac{\delta^2(\lambda^2 + (1-\lambda)^2) + 2\delta\lambda^2(1-\lambda)^2}{2(\lambda(1-\lambda)+\delta)^2}\right]\right) q\sigma_\theta^2 - c'(e_i) - \kappa(t^c - 1)^2\sigma_\theta^2 = 0.$$

Solving the above equation, we obtain the proposed equilibrium effort level $\check{e}_F^c$ as a unique solution. Therefore, given the above-mentioned decision rules and communication strategies, choosing $e_i = \check{e}_F^c$ is indeed optimal for agent i. ■□

[3] To arrive at the expression of t^c, we exploit that the symmetry of the distribution of λ and observe that $\mathbb{E}_\lambda\left[\frac{\lambda(1-\lambda)\delta^2 + \lambda(1-\lambda)^3\delta}{(\lambda(1-\lambda)+\delta)^2}\right] = \mathbb{E}\left[\frac{\lambda(1-\lambda)[2\delta^2 + \delta(\lambda^2 + (1-\lambda)^2)]}{2(\lambda(1-\lambda)+\delta)^2}\right]$.

E Appendix: Chapter 5

E.1 Proof of Proposition 5.2

Our proof essentially extends the proof of Propositions 1, 2, and 3 in Mookherjee and Reichelstein (1992) to Bayesian settings. For the if statement, note that agent i does not deviate from the truth-telling Bayes-Nash equilibrium if and only if

$$\begin{aligned} U_i(x_i) &\geq E_{\mathbf{x}_{-i}}\left(v_i(q(x_i', \mathbf{x}_{-i}), x_i) + t_i(x_i', \mathbf{x}_{-i})\right) \\ &= U_i(x_i') + E_{\mathbf{x}_{-i}}\left(v_i(q(x_i', \mathbf{x}_{-i}), x_i) - v_i(q(x_i', \mathbf{x}_{-i}), x_i')\right) \end{aligned} \tag{E.1}$$

for all $x_i, x_i' \in X_i$. Using (5.2), this is equivalent to require that for all $x_i, x_i' \in X_i$,

$$\begin{aligned} &\int_{x_i'}^{x_i} E_{\mathbf{x}_{-i}}\left(v_{ix}(q(s, \mathbf{x}_{-i}), s)\right) ds \\ &\geq E_{\mathbf{x}_{-i}}\left(v_i(q(x_i', \mathbf{x}_{-i}), x_i)\right) - E_{\mathbf{x}_{-i}}\left(v_i(q(x_i', \mathbf{x}_{-i}), x_i')\right), \end{aligned}$$

which is true under the condition that $E_{\mathbf{x}_{-i}}(v_{ix}(q(s, \mathbf{x}_{-i}), x_i))$ is non-decreasing in s for all $x_i \in X_i$.

For the only if statement, suppose mechanism (q, t) is BIC. We then have

$$U_i(x_i) = \max_{x_i' \in X_i}\left(E_{\mathbf{x}_{-i}}\left(v_i(q(x_i', \mathbf{x}_{-i}), x_i) + t_i(x_i', \mathbf{x}_{-i})\right)\right).$$

Since v_i is absolutely continuous in x_i and has a bounded derivative with respect to type x_i equation (5.2) follows from the envelope theorem (Milgrom and Segal, 2002). It remains to show that BIC also implies the monotone-expected-marginal condition. Suppose, in contradiction, we have $E_{\mathbf{x}_{-i}} v_{ix}(q(y, \mathbf{x}_{-i}), z) > E_{\mathbf{x}_{-i}} v_{ix}(q(x, \mathbf{x}_{-i}), z)$ for some agent i and $x, y, z \in X_i$, with $y < x$. Since v_i satisfies the increasing differences over distribution property, this implies that the difference $E_{\mathbf{x}_{-i}} v_{ix}(q(y, \mathbf{x}_{-i}), z') - E_{\mathbf{x}_{-i}} v_{ix}(q(x, \mathbf{x}_{-i}), z')$ is strictly positive for all $z' \in X_i$. It then follows that $E_{\mathbf{x}_{-i}} v_i(q(y, \mathbf{x}_{-i}), z') - E_{\mathbf{x}_{-i}} v_i(q(x, \mathbf{x}_{-i}), z')$ is increasing in z' for all $z' \in X_i$. Therefore, we have

$$\begin{aligned} &E_{\mathbf{x}_{-i}}\left(v_i(q(y, \mathbf{x}_{-i}), x) - v_i(q(y, \mathbf{x}_{-i}), y)\right) > \\ &E_{\mathbf{x}_{-i}}\left(v_i(q(x, \mathbf{x}_{-i}), x) - v_i(q(x, \mathbf{x}_{-i}), y)\right). \end{aligned}$$

At the same time, the incentive compatibility implies

$$E_{\mathbf{x}_{-i}}\left(v_i(q(y, \mathbf{x}_{-i}), x) - v_i(q(y, \mathbf{x}_{-i}), y)\right) \leq U_i(x) - U_i(y)$$

and

$$E_{\mathbf{x}_{-i}}\left(v_i(q(x, \mathbf{x}_{-i}), x) - v_i(q(x, \mathbf{x}_{-i}), y)\right) \geq U_i(x) - U_i(y).$$

We thus reach a contradiction. □

E.2 Proof of Proposition 5.3

The sufficiency part is straightforward. Let us prove the necessity part. Consider some $x', y' \in X_i$ such that $x' > y'$ and let $\underline{a} \in \arg\min_{a \in A} (v_i(a, x') - v_i(a, y'))$ and $\overline{a} \in \arg\max_{a \in A} (v_i(a, x') - v_i(a, y'))$. Given our assumption that $v_i(a, x_i)$ is continuous in a, such $\underline{a}$ and $\overline{a}$ are guaranteed to exist. For each $a \in A$ we can then always find $\alpha(a, x', y') \in [0, 1]$ such that

$$\begin{aligned} v_i(a, x') - v_i(a, y') = & \alpha(a, x', y')\big(v_i(\overline{a}, x') - v_i(\overline{a}, y')\big) \\ & + (1 - \alpha(a, x', y'))\big(v_i(\underline{a}, x') - v_i(\underline{a}, y')\big). \end{aligned}$$

Let us consider distribution G that puts the unit mass on allocation a and distribution F that puts probability $\alpha(a, x', y')$ on $\overline{a}$ and probability $1 - \alpha(a, x', y')$ on $\underline{a}$. By construction, we have

$$\int v_i(a, x')dG - \int v_i(a, x')dF = \int v_i(a, y')dG - \int v_i(a, y')dF,$$

and the increasing differences over distributions implies that the difference $\int v_i(a, x)dG - \int v_i(a, x)dF$ is a constant function in x, which we denote as $g_i(a)$. Hence,

$$\begin{aligned} v_i(a, x) &= \alpha(a, x', y')v_i(\overline{a}, x) + (1 - \alpha(a, x', y'))v_i(\underline{a}, x) + g_i(a) \\ &= f_i(a)M_i(x) + m_i(x) + g_i(a) \end{aligned}$$

where $f_i(a) = \alpha(a, x', y')$, $M_i(x) = v_i(\overline{a}, x) - v_i(\underline{a}, x)$, and $m_i(x) = v_i(\underline{a}, x)$. The increasing differences over distributions and $v_i(\overline{a}, x') - v_i(\underline{a}, x') \geq v_i(\overline{a}, y') - v_i(\underline{a}, y')$ then implies that $M_i(x)$ is either an increasing or constant function. For the latter case, we redefine $\tilde{f}_i(a) = 0$, $\tilde{M}_i(x)$ to be any increasing function, and $\tilde{g}_i(a) = g_i(a) + f_i(a)M_i(x')$ to obtain expression (5.3). □

E.3 Proof of Theorem 5.1

Below, we will prove two leammas, which completes the proof of Theorem 1 as explained in the main text.

Lemma E.1. *Suppose, for all $i \in \mathcal{I}$, $X_i = [0, 1]$ and λ_i is the uniform distribution on X_i. Then, for any BIC allocation $\tilde{q}$ there exists a feasible allocation q satisfying (5.4) with $f_i(q(\cdot, \mathbf{x}_{-i}))$ being non-decreasing for all $i \in \mathcal{I}$ and $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$.*

Proof. The proof essentially repeats the proof of Lemma 2 in Gershkov et al. (2013), and we only sketch it here. We consider a partition $[0, 1]^I$ to 2^{nI} cubes of equal size. For each cube S in this partition, we approximate $\mathbf{f}(\tilde{q}(\mathbf{x}))$, $\mathbf{x} \in S$, by its average defined by

$$\mathbf{f}(\tilde{q}(S)) = 2^{nI} \int_S \mathbf{f}(\tilde{q}(\mathbf{x}))d\mathbf{x}.$$

Note allocation $\tilde{q}(S) \in A$ is well-defined, because mapping $\mathbf{f}$ is convex-valued. In addition, discrete allocation $\tilde{q}(S)$ inherits non-decreasing expected marginals from $\tilde{q}$. Lemma 5.1 then ensures that there exists an allocation $q(S)$ with non-decreasing marginals that can also be extended to piecewise constant functions over $[0, 1]^I$. Taking the limit with respect to the size of partition, we obtain the result of the lemma. For the details of the construction, we refer to Gershkov et al. (2013). □

Lemma E.2. *Suppose, for all $i \in \mathcal{I}$, $X_i \subseteq \mathbb{R}$ and λ_i is some distribution on X_i. Then, for any BIC allocation $\tilde{q}$ there exists a feasible allocation q satisfying (5.4) with $f_i(q(\cdot, \mathbf{x}_{-i}))$ being non-decreasing for all $i \in \mathcal{I}$ and $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$.*

Proof. The proof repeats the proof of Lemma 3 in Gershkov et al. (2013). Its main idea is to relate the uniform distribution covered by Lemma E.1 to the case of a general distribution. In particular, if random variable Z_i is uniformly distributed, then $\lambda_i^{-1}(Z_i)$ is distributed according to λ_i, where $\lambda_i^{-1}(z_i) = \inf\{x_i \in X_i | \lambda_i(x_i) \geq z_i\}$. Hence, for a given BIC allocation $\tilde{q}$ we use transformation λ_i^{-1} to construct an allocation $\tilde{q}'$ defined on uniformly distributed types that also has a non-decreasing expected marginals. For allocation $\tilde{q}'$, we then apply the results of Lemmas 5.1 and E.1 to obtain an allocation q' with non-decreasing marginals defined on uniformly distributed types. We then use transformation λ_i to recover an allocation q with non-decreasing marginals defined on types distributed according to λ_i. For the details of the construction, we refer to Gershkov et al. (2013). □

E.4 Non-Convex-Valued Mappings: An Example

We now show that the assumption that mapping $\mathbf{f}$ being convex-valued is generally indispensable for the equivalence result of Theorem 5.1.

Consider a two-agent example with the set of possible allocations $A = [0, 1]$. Each agent i's type x_i is drawn independently from the uniform distribution over $[0, 1]$. For an allocation $q \in A$ and transfers $t_1, t_2 \in \mathbb{R}$, agent 1's utility equals to $qx_1 + t_1$, and agent 2's utility is $q^2 x_2 + t_2$. This environment satisfies all conditions of Theorem 5.1 except for the assumption that mapping (f_1, f_2) is convex-valued, where $f_1(q) = q$ and $f_2(q) = q^2$. Let us consider the following allocation rule:

$$q(x_1, x_2) = \begin{cases} 1 & \text{if } \max\{x_1, x_2\} \leq \frac{1}{2} \text{ or } \min\{x_1, x_2\} > 1/2, \\ 0 & \text{otherwise.} \end{cases}$$

This allocation rule is Bayesian implementable because its expected marginals $\int_0^1 f_i(x_i, x_j) dx_j$ are non-decreasing everywhere. It is, however, not dominant-strategy implementable because marginals $f_i(x_1, x_2)$ are strictly decreasing for some $(x_1, x_2) \in \mathbf{X}$. We now show that there does not exist an equivalent DIC mechanism for any BIC mechanism with allocation rule q.

Suppose, in contradiction, that for some BIC mechanism (q, t) there exists an equivalent DIC mechanism $(\hat{q}, \hat{t})$. Let $U_i(x_i)$ and $\hat{U}_i(x_i)$ be agent i's interim expected utilities in mechanisms (q, t) and $(\hat{q}, \hat{t})$, respectively. Since the two mechanisms are equivalent, we have $U_i(x_i) = \hat{U}_i(x_i)$ for all $x_i \in X_i$ and $i = 1, 2$. The envelope formula then implies that $\forall x_i, x_i' \in X_i$,

$$\begin{aligned} U_i(x_i) &= U_i(x_i') + \int_{x_i'}^{x_i} \int_0^1 f_i(q(s, x_j)) dx_j ds \\ &= \hat{U}_i(x_i') + \int_{x_i'}^{x_i} \int_0^1 f_i(\hat{q}(s, x_j)) dx_j ds = \hat{U}_i(x_i). \end{aligned}$$

Therefore, we have for almost all $x_i \in [0, 1]$, and for all $i, j \in \{1, 2\}$, $i \neq j$,

$$\int_0^1 f_i(\hat{q}(x_i, x_j)) dx_j = \int_0^1 f_i(q(x_i, x_j)) dx_j = \frac{1}{2}. \tag{E.2}$$

Integrating (E.2) over x_j, we have for all $i \in \{1, 2\}$,

$$\int_0^1 \int_0^1 f_i(\hat{q}(x_1, x_2)) dx_1 dx_2 = \frac{1}{2}, \tag{E.3}$$

which further implies that

$$\begin{aligned} 0 &= \int_0^1 \int_0^1 [f_1(\hat{q}(x_1, x_2)) - f_2(\hat{q}(x_1, x_2))] dx_1 dx_2 \\ &= \int_0^1 \int_0^1 \left[\hat{q}(x_1, x_2) - (\hat{q}(x_1, x_2))^2\right] dx_1 dx_2. \end{aligned} \tag{E.4}$$

Since $q(x_1, x_2) \in A = [0, 1]$, equation (E.4) implies that $\hat{q}(x_1, x_2) \in \{0, 1\}$ for almost every type profile $(x_1, x_2) \in \mathbf{X}$. In addition, allocation $\hat{q}$ being dominant strategy implementable implies that $f_2(\hat{q}(x_1, x_2)) = (\hat{q}(x_1, x_2))^2$ must be non-decreasing in x_2. The equal-expected-marginal condition (E.2) for agent 1 then implies that for almost all $x_1 \in [0, 1]$, $\hat{q}(x_1, x_2) = 0$ for $x_2 \in [0, 1/2]$ and $\hat{q}(x_1, x_2) = 1$ for $x_2 \in (1/2, 1]$.[1]

This allocation rule, however, does not satisfy the equal-expected-marginal condition (E.2) for agent 2. In particular, $\int_0^1 (\hat{q}(x_1, x_2))^2 dx_1 = 0$ for all $x_2 \in [0, 1/2]$, and $\int_0^1 (\hat{q}(x_1, x_2))^2 dx_2 = 1$ for all $x_2 \in (1/2, 1]$. We thus reach a contradiction. □

E.5 Proof of Theorem 5.2

Consider an arbitrary BIC mechanism $(\tilde{q}, \tilde{t})$ and the corresponding DIC mechanism (q, t) constructed in Theorem 5.1. Since equation (5.7) holds for any $\mathbf{g}$, the first part of Theorem 5.2 immediately follows. The idea behind the proof of the second part of the theorem is to show that if functions $\check{f}_i$ and g_i satisfy conditions (i) or (ii), the DIC mechanism constructed in Theorem 5.1 also satisfies

$$E_{\mathbf{x}} \left(\sum\nolimits_i g_i(q(\mathbf{x}))\right) \geq E_{\mathbf{x}} \left(\sum\nolimits_i g_i(\tilde{q}(\mathbf{x}))\right). \tag{E.5}$$

Suppose condition (i) is satisfied. Let us first consider the case where types are discrete and uniformly distributed (as in Lemma 5.1). If the marginals of allocation $\tilde{q}$ are not non-decreasing, then $\check{f}_j(\tilde{q}_j(x'_j, \mathbf{x}_j)) < \check{f}_j(\tilde{q}_j(x_j, \mathbf{x}_{-j}))$ for some j, $x'_j > x_j$, and $\mathbf{x}_{-j}$. Using the construction of the algorithm in Lemma 5.1 we then obtain an allocation $\hat{q} \in A$ satisfying the equal-marginal conditions in (5.4) and delivering strictly smaller value to objective $E_{\mathbf{x}}||\mathbf{f}(\cdot)||^2$. Since function $\check{f}_j$ is non-decreasing and concave (or non-increasing and convex), we also have

$$\begin{aligned} \hat{q}_j(x_j, \mathbf{x}_{-j}) = \hat{q}_j(x'_j, \mathbf{x}_{-j}) &\leq \frac{1}{2}\tilde{q}_j(x_j, \mathbf{x}_{-j}) + \frac{1}{2}\tilde{q}_j(x'_j, \mathbf{x}_{-j}), \\ \hat{q}_j(x_j, \mathbf{x}'_{-j}) &\leq (1-\delta)\tilde{q}_j(x_j, \mathbf{x}'_{-j}) + \delta\tilde{q}_j(x'_j, \mathbf{x}'_{-j}), \\ \hat{q}_j(x'_j, \hat{\mathbf{x}}_{-j}) &\leq (1-\delta)\tilde{q}_j(x'_j, \mathbf{x}'_{-j}) + \delta\tilde{q}_j(x_j, \mathbf{x}'_{-j}). \end{aligned}$$

Since g_i is non-increasing and concave in each component, this further implies

$$\begin{aligned} g_i(\hat{q}(x_j, \mathbf{x}_{-j})) + g_i(\hat{q}(x'_j, \mathbf{x}_{-j})) &\geq g_i(\tilde{q}(x_j, \mathbf{x}_{-j})) + g_i(\tilde{q}(x'_j, \mathbf{x}_{-j})), \\ g_i(\hat{q}(x_j, \mathbf{x}'_{-j})) + g_i(\hat{q}(x'_j, \mathbf{x}'_{-j})) &\geq g_i(\tilde{q}(x_j, \mathbf{x}'_{-j})) + g_i(\tilde{q}(x'_j, \mathbf{x}'_{-j})), \end{aligned}$$

[1] Because of the monotonicity of the allocation rule and $\hat{q} \in \{0, 1\}$, the only indeterminacy in $\hat{q}(x_1, x_2)$ could happen at $x_2 = 1/2$.

for each $i \in \mathcal{I}$ and, hence, $E_{\mathbf{x}}(\sum_i g_i(\hat{q}(\mathbf{x}))) \geq E_{\mathbf{x}}(\sum_i g_i(\tilde{q}(\mathbf{x})))$. We iterate this procedure to obtain a sequence of allocations $q^n \in A$ and a decreasing numerical sequence $s^n = E_{\mathbf{x}}||\mathbf{f}(q^n(\mathbf{x}))||^2$, $n = 1, 2, ...$ If we find that $\check{f}_j(q_j^n(\cdot, \mathbf{x}_{-j}))$ is non-decreasing for all j and $\mathbf{x}_{-j}$, we set $q^{n+1} \equiv q^n$ and $s^{n+1} \equiv s^n$. Since s^n is a weakly decreasing sequence bounded below by 0, it has a limit, which we denote as s. Since set A is compact, there also exists a convergent subsequence q^n with a limit q such that $q(\mathbf{x}) \in A$ for all $\mathbf{x} \in X$. Clearly, $s = E_{\mathbf{x}}(||\mathbf{f}(q(\mathbf{x})||^2)$ and $\check{f}_j(q_j(\cdot, \mathbf{x}_{-j}))$ is non-decreasing for each j and $\mathbf{x}_{-j}$. Since functions g_i are continuous, we also have $E_{\mathbf{x}}\left(\sum_i g_i(q(\mathbf{x}))\right) \geq E_{\mathbf{x}}\left(\sum_i g_i(\tilde{q}(\mathbf{x}))\right)$.

The result can then be further extended to continuous space with an arbitrary distribution similar to Lemmas E.1 and E.2. We then use equation (5.6) to define payment rule t delivering the same interim expected utilities. Finally, we derive that the social surplus in the constructed allocation

$$\begin{aligned} E_{\mathbf{x}}\left(\sum\nolimits_i v_i(q(\mathbf{x}), x_i)\right) &= E_{\mathbf{x}}\left(\sum\nolimits_i f_i(q(\mathbf{x}))M_i(x_i) + m_i(x_i) + g_i(q(\mathbf{x}))\right) \\ &\geq E_{\mathbf{x}}\left(\sum\nolimits_i f_i(\tilde{q}(\mathbf{x}))M_i(x_i) + m_i(x_i) + g_i(\tilde{q}(\mathbf{x}))\right) \\ &= E_{\mathbf{x}}\left(\sum\nolimits_i v_i(\tilde{q}(\mathbf{x}), x_i)\right), \end{aligned}$$

where the inequality follows from the equal-marginal conditions in (5.4) and inequality (E.5). This establishes the claim of the theorem. The proof is analogous when condition (ii) is satisfied. □

E.6 Proof of Corollaries

Proof of Corollaries 5.1 and 5.5. The statements follow from Theorem 5.1. □

Proof of Corollaries 5.2, 5.3, and 5.4. The statements follow from Theorem 5.2. □

Proof of Corollary 5.6. Consider any BIC mechanism $(\tilde{q}, \tilde{t})$ and the equivalent DIC mechanism (q, t), constructed in Theorem 5.1. Since we have $g_i(q) = 0$ for each $i \in \mathcal{I}$ in the public good provision setting, the same ex ante expected utilities in both mechanisms implies that both mechanism yield the same expected transfers, i.e., $E_{\mathbf{x}}\left(\sum_{i \in \mathcal{I}} t_i(\mathbf{x})\right) = E_{\mathbf{x}}\left(\sum_{i \in \mathcal{I}} \tilde{t}_i(\mathbf{x})\right)$.

To prove the claim of the corollary, we need to show that the expected costs for the DIC mechanism is lower than the expected costs for the BIC mechanism, i.e., $E_{\mathbf{x}}(K(q(\mathbf{x}))) \leq E_{\mathbf{x}}(K(\tilde{q}(\mathbf{x})))$. This statement follows from applying the argument of the proof of Theorem 5.2 to function $-K$ instead of functions g_i, $i \in \mathcal{I}$. In particular, consider the sequence of allocation q^n constructed in the algorithm of Theorem 5.1. Since function K is non-decreasing and convex, the expected cost of allocations q^n is non-increasing in n, i.e., $E_{\mathbf{x}}(K(q^{n+1}(\mathbf{x}))) \leq E_{\mathbf{x}}(K(q^n(\mathbf{x}))) \leq E_{\mathbf{x}}(K(\tilde{q}(\mathbf{x})))$. The continuity of function K then implies that the inequality holds in the limit. Finally, the result further extends to continuous type space with an arbitrary distribution similar to Lemmas E.1 and E.2. □

E.7 Proposition E.1

Proposition E.1. *If function v_i violates the increasing differences property for some agent $i \in \mathcal{I}$, then there exists a dominant-strategy incentive compatible mechanism (q, t) that does not have non-decreasing marginals $v_{ix}(q(\cdot, \mathbf{x}_{-i}), x_i)$ for all $\mathbf{x}_{-i} \in \mathbf{X}_{-i}$ and $x_i \in X_i$.*

PROOF. Suppose $v_i(a, x)$ does not satisfy the increasing differences property. There must exist $a, a' \in A$, and $x, y, z \in X$ with $x < y < z$ such that either

$$\begin{cases} v_i(a,x) - v_i(a',x) \le v_i(a,y) - v_i(a',y) \\ v_i(a,y) - v_i(a',y) \ge v_i(a,z) - v_i(a',z) \end{cases}, \tag{E.6}$$

with at least one inequality being strict, or

$$\begin{cases} v_i(a,x) - v_i(a',x) \ge v_i(a,y) - v_i(a',y) \\ v_i(a,y) - v_i(a',y) \le v_i(a,z) - v_i(a',z) \end{cases}, \tag{E.7}$$

with at least one strict inequality. We consider only case (E.6). Case (E.7) can be treated similarly.

Let us assume that the utility of agent i satisfies (E.6). We consider a mechanism with an allocation rule q and a payment rule t that are functions of agent i's reports only, i.e., $q : X_i \to A$ and $t : X_i \to \mathbb{R}^I$. In particular, we assign $q(x) = q(z) = a', q(y) = a$, and $\forall s \neq x, y, z$,

$$q(s) = \begin{cases} a & \text{if } v_i(a,s) - v_i(a',s) \ge \bar{t}_i \\ a' & \text{otherwise} \end{cases},$$

where $\bar{t}_i = v_i(a,y) - v_i(a',y)$. Agent i receives no transfers if allocation a is chosen and $\bar{t}_i$ otherwise, i.e., $t_i(s) = 0$ if $q(s) = a$ and $t_i(s) = \bar{t}_i$ if $q(s) = a'$. All other agents receive no transfers, i.e., $t_j(s) \equiv 0$ for all $j \neq i$ and $s \in X_i$. It is straightforward to check that (q,t) is dominant-strategy incentive compatible.

We now show that agent i's marginals induced by allocation rule q cannot be all non-decreasing. Suppose, in contradiction, that $v_{ix}(q(\cdot), s)$ is non-decreasing for all $s \in X_i$. Then, we have

$$v_{ix}(q(x), s) \le v_{ix}(q(y), s) \le v_{ix}(q(z), s), \ \forall s \in X_i$$

or, equivalently, $v_{ix}(a', s) \le v_{ix}(a, s) \le v_{ix}(a', s), \ \forall s \in X_i$. But then $v_{ix}(a', s) = v_{ix}(a, s), \forall s \in X_i$, and by integration over s we have

$$v_i(a',y) - v_i(a',x) = v_i(a,y) - v_i(a,x) \text{ and } v_i(a',z) - v_i(a',y) = v_i(a,z) - v_i(a,y),$$

which contradicts (E.6). □

E.8 Proposition E.2

Proposition E.2. *Suppose that there exist two agents whose type distributions are absolutely continuous. If function v_i violates the increasing differences over distributions property for some agent $i \in \mathcal{I}$, then there exists a Bayesian incentive compatible mechanism (q,t) that does not have non-decreasing expected marginals $E_{\mathbf{x}_{-i}}[v_{ix}(q(\cdot, \mathbf{x}_{-i}), x_i)]$ for all $x_i \in X_i$.*

Proof. For any $G, F \in \Delta(A)$ and any $s \in X_i$, let

$$\Delta(G,F,s) = \int v_i(a,s)dG - \int v_i(a,s)dF.$$

Suppose $v_i(a,x)$ does not satisfy the increasing differences over distributions property. Then, there must exist $G, F \in \Delta(A)$, and $x, y, z \in X$ with $x < y < z$ such that either

$$\Delta(G,F,x) \le \Delta(G,F,y) \text{ and } \Delta(G,F,y) \ge \Delta(G,F,z) \tag{E.8}$$

with at least one inequality being strict, or

$$\Delta(G, F, x) \geq \Delta(G, F, y) \text{ and } \Delta(G, F, y) \leq \Delta(G, F, z) \tag{E.9}$$

with at least one strict inequality. We consider only case (E.8). Case (E.9) can be treated similarly.

Assume that the utility of agent i satisfies (E.8). Let $a_G, a'_G, a_F, a'_F \in A$, and G_α (F_β) be the binary probability distribution that puts a weight α (β) on the allocation a_G (a_F) and the remaining weight $1-\alpha$ ($1-\beta$) on the allocation a'_G (a'_F), where $\alpha, \beta \in [0,1]$. We establish the following lemma.

Lemma E.3. *There exists a pair of binary distributions G_α, F_β such that*

$$\Delta(G_\alpha, F_\beta, x) \leq \Delta(G_\alpha, F_\beta, y) \textit{ and } \Delta(G_\alpha, F_\beta, y) \geq \Delta(G_\alpha, F_\beta, z) \tag{E.10}$$

with at least one inequality being strict.

Proof of Lemma E.3 Since both G_α and F_β can be deterministic, the claim of the lemma is clearly true if the increasing differences property is violated. Thus, it is without loss to assume that this property is satisfied by v_i. We want to first show that $\exists a, a', a'' \in A$ that satisfy the two following conditions simultaneously:

(i) $v_i(a,x) - v_i(a'',x) \neq v_i(a,y) - v_i(a'',y) \neq v_i(a,z) - v_i(a'',z)$.[2]

(ii) $\nexists \lambda \in \mathbb{R}$ such that

$$\begin{aligned}&(v_i(a,y) - v_i(a',y)) - (v_i(a,x) - v_i(a',x))\\ =\,&\lambda[(v_i(a,y) - v_i(a'',y)) - (v_i(a,x) - v_i(a'',x))],\end{aligned}$$

and

$$\begin{aligned}&(v_i(a,z) - v_i(a',z)) - (v_i(a,y) - v_i(a',y))\\ =\,&\lambda[(v_i(a,z) - v_i(a'',z)) - (v_i(a,y) - v_i(a'',y))].\end{aligned}$$

From the contrary, suppose that such a triple of allocations does not exist. Then, $\forall a, a', a'' \in A$, either

$$v_i(a,x) - v_i(a'',x) = v_i(a,y) - v_i(a'',y) = v_i(a,z) - v_i(a'',z),$$

or there exists $\lambda_{aa'a''} \in \mathbb{R}$ such that

$$\begin{aligned}&(v_i(a,y) - v_i(a',y)) - (v_i(a,x) - v_i(a',x)\\ =\,&\lambda_{aa'a''}[(v_i(a,y) - v_i(a'',y)) - (v_i(a,x) - v_i(a'',x))]\end{aligned}$$

and

$$\begin{aligned}&(v_i(a,z) - v_i(a',z)) - (v_i(a,y) - v_i(a',y))\\ =\,&\lambda_{aa'a''}[(v_i(a,z) - v_i(a'',z)) - (v_i(a,y) - v_i(a'',y))].\end{aligned}$$

[2]Note that because of the increasing differences property, $\forall a, a'' \in A$ we can only have either $v_i(a,x) - v_i(a'',x) \neq v_i(a,y) - v_i(a'',y) \neq v_i(a,z) - v_i(a'',z)$ or $v_i(a,x) - v_i(a'',x) = v_i(a,y) - v_i(a'',y) = v_i(a,z) - v_i(a'',z)$.

Fix any $a', a'' \in A$ such that $v_i(a', x) - v_i(a'', x) \neq v_i(a', y) - v_i(a'', y) \neq v_i(a', z) - v_i(a'', z)$.[3] Let us consider a set

$$A_{a''} = \{a \in A : v_i(a, x) - v_i(a'', x) = v_i(a, y) - v_i(a'', y) = v_i(a, z) - v_i(a'', z)\},$$

and $\bar{A}_{a''} = A \setminus A_{a''}$. Note that $\forall s, s' \in X_i$, we have

$$\begin{aligned}
&\Delta(G, F, s) - \Delta(G, F, s') \\
&= \int_{A_{a''} \cup \bar{A}_{a''}} \int [(v_i(a, s) - v_i(\tilde{a}, s)) - (v_i(a, s') - v_i(\tilde{a}, s'))] dF_{\tilde{a}} dG_a.
\end{aligned}$$

Hence,

$$\begin{aligned}
&\Delta(G, F, y) - \Delta(G, F, x) \\
=& \int_{\bar{A}_{a''}} \int \lambda_{a\tilde{a}a''} [(v_i(a, y) - v_i(a'', y)) - (v_i(a, x) - v_i(a'', x))] dF_{\tilde{a}} dG_a \\
&+ \int_{A_{a''}} \int [(v_i(a'', y) - v_i(\tilde{a}, y)) - (v_i(a'', x) - v_i(\tilde{a}, x))] dF_{\tilde{a}} dG_a \\
=& \int_{\bar{A}_{a''}} \int -\lambda_{a\tilde{a}a''} \lambda_{a''aa'} [(v_i(a'', y) - v_i(a', y)) - (v_i(a'', x) - v_i(a', x))] dF_{\tilde{a}} dG_a \\
&+ \int_{A_{a''}} \int \lambda_{a''\tilde{a}a'} [(v_i(a'', y) - v_i(a', y)) - (v_i(a'', x) - v_i(a', x))] dF_{\tilde{a}} dG_a \\
=& [(v_i(a'', x) - v_i(a', x)) - (v_i(a'', y) - v_i(a', y))] K,
\end{aligned}$$

where

$$K = \int_{\bar{A}_{a''}} \int \lambda_{a\tilde{a}a''} \lambda_{a''aa'} dF_{\tilde{a}} dG_a - \int_{A_{a''}} \int \lambda_{a''\tilde{a}a'} dF_{\tilde{a}} dG_a,$$

and, similarly,

$$\begin{aligned}
&\Delta(G, F, z) - \Delta(G, F, y) \\
=& \int_{\bar{A}_{a''}} \int \lambda_{a\tilde{a}a''} [(v_i(a, z) - v_i(a'', z)) - (v_i(a, y) - v_i(a'', y))] dF_{\tilde{a}} dG_a \\
&+ \int_{A_{a''}} \int [(v_i(a'', z) - v_i(\tilde{a}, z)) - (v_i(a'', y) - v_i(\tilde{a}, y))] dF_{\tilde{a}} dG_a \\
=& \int_{\bar{A}_{a''}} \int -\lambda_{a\tilde{a}a''} \lambda_{a''aa'} [(v_i(a'', z) - v_i(a', z)) - (v_i(a'', y) - v_i(a', y))] dF_{\tilde{a}} dG_a \\
&+ \int_{A_{a''}} \int \lambda_{a''\tilde{a}a'} [(v_i(a'', z) - v_i(a', z)) - (v_i(a'', y) - v_i(a', y))] dF_{\tilde{a}} dG_a \\
=& [(v_i(a'', y) - v_i(a', y)) - (v_i(a'', z) - v_i(a', z))] K.
\end{aligned}$$

Since $v_i(a'', s) - v_i(a', s)$ is monotone in s $\forall s \in X_i$, we have

$$sign\left[\Delta(G, F, y) - \Delta(G, F, x)\right] = sign\left[\Delta(G, F, z) - \Delta(G, F, y)\right],$$

which violates (E.8). Hence, there must exist $a, a', a'' \in A$ that satisfy both (i) and (ii). Note that for

[3]If such allocations do not exist, we will have $\Delta(G, F, x) = \Delta(G, F, y) = \Delta(G, F, z)$, which violates (E.8).

any such a triple of allocations (a, a', a''), we must also have

$$v_i(a,x) - v_i(a',x) \neq v_i(a,y) - v_i(a',y) \neq v_i(a,z) - v_i(a',z) \text{ and}$$
$$v_i(a'',x) - v_i(a',x) \neq v_i(a'',y) - v_i(a',y) \neq v_i(a'',z) - v_i(a',z),$$

since otherwise the two equations of (ii) will hold for either $\lambda = 0$ or $\lambda = 1$. Consequently, any triple of allocations that is a permutation of (a, a', a'') will also satisfy conditions (i) and (ii), which suggests that the order of selecting a, a' and a'' does not matter. Hence, without loss of generality, we can assume further that

$$v_i(a,y) - v_i(a,x) < \min\{v_i(a',y) - v_i(a',x), v_i(a'',y) - v_i(a'',x)\}. \tag{E.11}$$

Next, for all $s \in X_i$, let us denote

$$\begin{aligned}\Delta(\alpha,\beta,s) &= [\alpha v_i(a'',s) + (1-\alpha)v_i(a,s)] - [\beta v_i(a',s) + (1-\beta)v_i(a,s)]\\ &= \alpha[v_i(a'',s) - v_i(a,s)] + \beta[v_i(a,s) - v_i(a',s)]\end{aligned}$$

and

$$\begin{aligned}\hat{\Delta}(\alpha,\beta,s) &= [\alpha v_i(a',s) + (1-\alpha)v_i(a,s)] - [\beta v_i(a'',s) + (1-\beta)v_i(a,s)]\\ &= \alpha[v_i(a',s) - v_i(a,s)] + \beta[v_i(a,s) - v_i(a'',s)].\end{aligned}$$

Given (E.11) and the increasing differences property, $\Delta(\alpha,\beta,y) - \Delta(\alpha,\beta,x) \geq 0$ if and only if

$$\alpha \geq \beta \left[\frac{(v_i(a',y) - v_i(a,y)) - (v_i(a',x) - v_i(a,x))}{(v_i(a'',y) - v_i(a,y)) - (v_i(a'',x) - v_i(a,x))}\right], \tag{E.12}$$

while $\Delta(\alpha,\beta,y) - \Delta(\alpha,\beta,z) \geq 0$ if and only if

$$\alpha \leq \beta \left[\frac{(v_i(a',y) - v_i(a,y)) - (v_i(a',z) - v_i(a,z))}{(v_i(a'',y) - v_i(a,y)) - (v_i(a'',z) - v_i(a,z))}\right]. \tag{E.13}$$

Similarly, we have $\hat{\Delta}(\alpha,\beta,y) - \hat{\Delta}(\alpha,\beta,x) \geq 0$ if and only if

$$\alpha \geq \beta \left[\frac{(v_i(a'',y) - v_i(a,y)) - (v_i(a'',x) - v_i(a,x))}{(v_i(a',y) - v_i(a,y)) - (v_i(a',x) - v_i(a,x))}\right], \tag{E.14}$$

while $\hat{\Delta}(\alpha,\beta,y) - \Delta(\alpha,\beta,z) \geq 0$ if and only if

$$\alpha \leq \beta \left[\frac{(v_i(a'',y) - v_i(a,y)) - (v_i(a'',z) - v_i(a,z))}{(v_i(a',y) - v_i(a,y)) - (v_i(a',z) - v_i(a,z))}\right]. \tag{E.15}$$

Note that again because of the increasing differences property, the R.H.S. of the inequalities (E.12), (E.13), (E.14) and (E.15) are all positive. Hence, if

$$\begin{aligned}&\frac{(v_i(a',y) - v_i(a,y)) - (v_i(a',x) - v_i(a,x))}{(v_i(a'',y) - v_i(a,y)) - (v_i(a'',x) - v_i(a,x))} < \\ &\frac{(v_i(a',y) - v_i(a,y)) - (v_i(a',z) - v_i(a,z))}{(v_i(a'',y) - v_i(a,y)) - (v_i(a'',z) - v_i(a,z))},\end{aligned}$$

one can always find $\alpha, \beta \in [0,1]$ such that both (E.12) and (E.13) are satisfied, and with at least one of them holds strictly. Otherwise, if the above strict inequality holds the other way round, then one can always find $\alpha, \beta \in [0,1]$ such that both (E.14) and (E.15) are satisfied, and with at least one of them holds strictly. In conclusion, we can always construct a pair of binary probability distributions G_α, F_β that satisfy $\Delta(G_\alpha, F_\beta, x) \leq \Delta(G_\alpha, F_\beta, y)$ and $\Delta(G_\alpha, F_\beta, y) \geq \Delta(G_\alpha, F_\beta, z)$, with at least one inequality being strict. ■

Lemma E.3 shows that if v_i violates the property of increasing differences over distributions for some probability distributions (G, F), it must also violate this property for some binary probability distributions (G_α, F_β). Given this important observation, we now construct a Bayesian incentive compatible mechanism that violates the monotone-expected-marginal condition.

Let (G_α, F_β) be a pair of binary distributions that satisfies (E.10). By assumption, there must exist an agent $j \neq i$ whose type distribution is absolutely continuous (and hence atomless). By continuity, we can always find transfers $t_j^G, t_j^F \in \mathbb{R}$, and partitions $X_j^G \cup X_j^{G'} = X_j$ and $X_j^F \cup X_j^{F'} = X_j$ such that

(i) $\Pr\left(x_j \in X_j^G\right) = 1 - \Pr\left(x_j \in X_j^{G'}\right) = \alpha$,
$\Pr\left(x_j \in X_j^F\right) = 1 - \Pr\left(x_j \in X_j^{F'}\right) = \beta$;

(ii) $v_j(a_G, x_j) \geq v_j(a'_G, x_j) + t_j^G \ \forall x_j \in X_j^G$,
$v_j(a_G, x_j) \leq v_j(a'_G, x_j) + t_j^G \ \forall x_j \in X_j^{G'}$;

(iii) $v_j(a_F, x_j) \geq v_j(a'_F, x_j) + t_j^F \ \forall x_j \in X_j^F$,
$v_j(a_F, x_j) \leq v_j(a'_F, x_j) + t_j^F \ \forall x_j \in X_j^{F'}$.

Consider a mechanism with an allocation rule q and a payment rule t that are functions of the reports of agents i and j. In particular, we let

$$q(x_i, \mathbf{x}_{-i}) = \begin{cases} a_G & \text{if } x_i = y, \text{ and } x_j \in X_j^G, \\ a'_G & \text{if } x_i = y, \text{ and } x_j \in X_j^{G'}, \\ a_F & \text{if } x_i \in \{x, z\}, \text{ and } x_j \in X_j^F, \\ a'_F & \text{if } x_i \in \{x, z\}, \text{ and } x_j \in X_j^{F'}, \end{cases}$$

and $\forall s \neq x, y, z$,

$$q(s, \mathbf{x}_{-i}) = \begin{cases} a_G & \text{if } \Delta(G_\alpha, F_\beta, s) \geq \bar{t}_i \text{ and } x_j \in X_j^G, \\ a'_G & \text{if } \Delta(G_\alpha, F_\beta, s) \geq \bar{t}_i \text{ and } x_j \in X_j^{G'}, \\ a_F & \text{if } \Delta(G_\alpha, F_\beta, s) < \bar{t}_i \text{ and } x_j \in X_j^F, \\ a'_F & \text{if } \Delta(G_\alpha, F_\beta, s) < \bar{t}_i \text{ and } x_j \in X_j^{F'}, \end{cases}$$

where $\bar{t}_i = \Delta(G_\alpha, F_\beta, y)$. Agent i receives $\bar{t}_i$ if either allocation a_F or a'_F is chosen, and $t_i = 0$ otherwise. Agent j receives t_j^G (t_j^F) if allocation a'_G (a'_F) is chosen, and $t_j = 0$ otherwise. For all agents $k \neq i, j$, $t_k(\mathbf{x}) = 0 \ \forall \mathbf{x} \in \mathbf{X}$. It is straightforward to check that (q, t) is a Bayesian incentive compatible mechanism.

We now show that agent i's expected marginals induced by allocation rule q cannot be always non-decreasing. In contradiction, suppose $E_{\mathbf{x}_{-i}} v_{ix}(q(\cdot, \mathbf{x}_{-i}), s)$ is non-decreasing for all $s \in X_i$. Then, for any $s \in X_i$ we have

$$E_{\mathbf{x}_{-i}}[v_{ix}(q(x, \mathbf{x}_{-i}), s)] \leq E_{\mathbf{x}_{-i}}[v_{ix}(q(y, \mathbf{x}_{-i}), s)] \leq E_{\mathbf{x}_{-i}}[v_{ix}(q(z, \mathbf{x}_{-i}), s)],$$

or, equivalently,

$$\int v_{ix}(a,s)dF_\beta \leq \int v_{ix}(a,s)dG_\alpha \leq \int v_{ix}(a,s)dF_\beta,$$

which implies $\int v_{ix}(a,s)dG_\alpha = \int v_{ix}(a,s)dF_\beta$ for all $s \in X_i$. Then, by the integration over s we have

$$\Delta(G_\alpha, F_\beta, x) = \Delta(G_\alpha, F_\beta, y) \text{ and } \Delta(G_\alpha, F_\beta, y) = \Delta(G_\alpha, F_\beta, z),$$

which contradicts to (E.10). $\square$

Part IV

Bibliography

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Part V

Curriculum Vitae

Curriculum Vitae

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Education

08/2013 – 07/2019	PhD studies at the Zurich Graduate School of Economics University of Zurich, Switzerland
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